Rapid estimates of converted wave velocities using prestack migration
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Summary

A new approach is presented for estimating the velocities of converted wave data that is based on prestack migration by equivalent offset to form common conversion point gathers. These gathers are used to form an initial estimate of the converted wave velocity $V_c$, that can then be used for the full accurate process of equivalent offset migration of converted wave data.

Equivalent offset common conversion point gathers are formed using the P-wave and S-wave velocities and the double-square-root equation. The formation of these gathers requires approximate values for the P-wave and S-wave velocities, but after their formation, accurate velocities can be picked and the prestack migration completed with moveout correction.

Introduction

Converted wave data are acquired with a P-wave source and convert to S-wave at a reflector and then recorded at the surface as a S-wave on a three component receiver. These P-S surveys use conventional sources, but require two more recording channels per receiving location and some special processing. The data quality of modern P-S sections approach and in some cases exceeds the quality of conventional P-P seismic data (Steward et al, 2002).

Converted wave data compliments the conventional P-P data with more refined applications that includes structural imaging, lithologic estimation, anisotropy analysis, subsurface fluid description and reservoir monitoring. In addition, they are relatively inexpensive, broadly applicable, and effective way to get S-wave velocity information (Stewart et. al., 2002).

The reflection and refraction of converted wave energy is illustrated in Figure 1 with raypaths at a boundary in an elastic medium. The incident P-wave is reflected as both a P-wave and S-wave, and transmitted energy is also both P-wave and S-wave. The angles of reflection and refraction obey Snell’s law with the corresponding P and S wave velocities. Converted wave data assume an incident P-wave with angle $\theta_1$ and velocity $V_{p1}$ and a reflected S-wave with angle $\phi$ and velocity $V_{s1}$ as

$$\frac{\sin \theta_1}{V_{p1}} = \frac{\sin \phi}{V_{s1}}. \quad (1)$$

Introduction

As the depth increases the conversion point moves towards an asymptote located at the distance $X_p$ from the source as illustrated in Figure 2b. The location of this asymptote is often used as the assumed location for all the conversion points. Since they are relatively inexpensive, broadly applicable, and effective way to get S-wave velocity information (Stewart et. al., 2002).
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points of the trace, enabling simplified processing of the converted wave data.

A more accurate method of processing is to go directly to prestack migration where each conversion point is defined and the energy scattered from that point is gathered and saved at the conversion point. This is repeated for all conversion points in the subsurface. This Kirchhoff approach computes traveltimes base on the P and S wave RMS velocities for a prestack time migration, or from ray or wavefront computations for a depth migration. Since converted wave data is usually acquired in areas with shallow geological structure, the prestack time migration is adequate for imaging and more cost effective.

The equivalent offset method (EOM) of prestack migration is defined by converting the double square root equation into a single square-root equation where the source and receiver are colocated. All prestack energy can be summed into a prestack migration gather at an offset defined by the equivalent offset. The fold in these gathers is very high and all reflection energy lies on a hyperbolic path. After the gathers are formed, accurate velocities are estimated, then moveout correction and stacking completes the prestack migration. No velocity in formation is required when forming an initial gather for P-P data, but use of approximate velocities will yield accurate RMS velocities (Bancroft et.al., 1998).

The principles of EOM can be applied to converted wave data where both the P and S wave velocities are used to define the equivalent offset. Converted wave EOM is more sensitive to velocities that conventional P-P data, and requires an initial velocity estimate. This initial estimate can be formed from the RMS velocities of P-P data with an assumed \( \frac{V_{RMS}}{\gamma} \) ratio. A more accurate initial estimate can be obtained directly from the data as presented in this paper.

Theory

The traveltimes from the source to the conversion point and from the conversion point to the receivers are computed for all points in the subsurface. The converted wave double-square-root (DSR) equation is

\[
t = t_s + t_r = \sqrt{\frac{T_{0P}^2}{4} + \left(\frac{x+h}{V_{P-RMS}}\right)^2} + \sqrt{\frac{T_{0S}^2}{4} + \left(\frac{x-h}{V_{S-RMS}}\right)^2}
\]

where \( t_s \) and \( t_r \) are the source and receiver raypath traveltimes, \( V_{P-RMS} \) is the P-wave velocity of the source raypath, \( V_{S-RMS} \) is the S-wave velocity of the receiver raypath, \( T_{0P} \) the vertical zero-offset traveltime for the source raypath, \( T_{0S} \) the vertical zero-offset traveltime for the receiver raypath, \( x \) the lateral distance of the source from the conversion point, and \( h \) the half offset between the source and receiver. The source displacement form the migration point \( x+h \) may be written as \( h_s \), and the receiver displacement \( x-h \) written as \( h_r \) as illustrated in the Figure 3. The depth of the conversion point is \( z_0 \) and corresponds to \( T_{0P} \) and \( T_{0S} \).

![Figure 3. Raypath diagram for converted wave EOM.](image)

If we assume the ratio of the RMS and average velocities, \( \frac{V_{RMS}}{\gamma} \) for the P and S wave velocities are constant,

\[
\frac{V_{RMS}}{V_{Ave}} = \frac{V_{P-RMS}}{V_{P-Ave}} = \frac{V_{S-RMS}}{V_{S-Ave}}
\]

then we can define a pseudo depth

\[
z_0 = z_0 \frac{V_{RMS}}{V_{Ave}}
\]

and write the equivalent offset DSR equation as

\[
t = \frac{1}{V_{P-RMS}} \sqrt{z_0^2 + h_s^2} + \frac{1}{V_{S-RMS}} \sqrt{z_0^2 + h_r^2}
\]

The square-root portions are equal, giving the hyperbolic traveltime equation

\[
t = \left(\frac{1}{V_{P-RMS}} + \frac{1}{V_{S-RMS}}\right) \sqrt{z_0^2 + h_s^2}
\]

If we drop the RMS designation of the velocity, use \( \gamma \), then defining \( V_c \) as the converted wave velocity we get

\[
t = \frac{1 + \gamma}{V_p} \sqrt{z_0^2 + h_s^2} = \frac{2}{V_c} \sqrt{z_0^2 + h_s^2}
\]

or

\[
V_c = \frac{2V_p}{1 + \gamma}
\]
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Equation 7 may be used to define the equivalent offset \( h_e \) for the converted wave data, and the prestack migration gathers formed will have converted wave reflection energy with hyperbolic moveout defined with the velocity \( V_c \). Once the gathers are formed, conventional velocity analysis will provide a more accurate estimate of \( V_c \). Our objective at this point is to find a quick and accurate method of estimating and initial value for \( V_c \).

One method of computing an initial velocity for \( V_c \) is to scale \( V_p \) with an assumed value for \( \gamma \). This requires scaling the amplitude of the velocity and shifting the T0 times \( T_{0s} \) to a greater value, i.e.,

\[
V_{c,RMS} (T_{0}) = \frac{2}{1+\gamma} V_{p,RMS} (T_{0p}),
\]

(9)

where

\[
T_{0s} = \frac{1 + \gamma}{2} T_{0p}.
\]

(10)

Tests can be run with different values of \( \gamma \) to establish a more accurate values that vary with time \( T_0 \).

A more accurate and simple method for estimating \( V_c \) is to use the equivalent offset method with short offsets. Consider again equation (5) but now using the midpoint location \( x \) and half offset \( h \), we have

\[
t = \frac{1}{V_p} \sqrt{\frac{z_0^2}{x+h} + \frac{x-h}{x+h}} + \frac{1}{V_S} \sqrt{\frac{z_0^2}{x-h} + \frac{x+h}{x-h}},
\]

(11)

and when \( x \) is small relative to \( h \) we have

\[
\hat{t} = \frac{1}{V_p} \sqrt{\frac{z_0^2}{h} + \frac{1}{x-h}} + \frac{1}{V_S} \sqrt{\frac{z_0^2}{h} + \frac{1}{x+h}} = \frac{2}{V_c} \sqrt{\frac{z_0^2}{x+h}}.
\]

(12)

This equation tells us that we can approximate an initial equivalent offset \( h_e \) with the source receiver offset \( h \) to form gathers with short displacements \( x \), independent of any velocity, and that the moveout of the reflection energy will have a moveout velocity \( V_c \). This is again similar to conventional data processing where a super CDP gather is formed by stacking neighbouring CDP gathers. The difference however is that the actual equivalent offset \( h_e \) is used where,

\[
h_e = \frac{h}{x} + \frac{h^2}{x}.
\]

(13)

The question now becomes: how short is the useable range of the CMP displacement \( x \). Previous work (Guirigay et al., 2010) has indicated that the sign of the time difference between \( t \) of equation (11) and \( \hat{t} \) of equation (12) is dependent on the polarity of \( x \), and will tend to remove any bias in the sum when \( x \) becomes larger allowing a greater range for gathering. Another question is if \( x \) is small, maybe we don’t need to use equation (13), simply use a super CMP gather to find the velocities. If the displacement \( x \) is too small, there may not be enough energy as the amplitude of a converted wave is zero when \( x = 0 \).

Examples

In 2011, the CREWES consortium acquired a 2D research data set in the area of Hassar, Alberta, Canada. Three component data were recorded with a number of different source types. A radial component line was preprocessed for source and receiver statics and the equivalent offset gathers formed with limited range of lateral displacements with a maximum \( x \) of \( x_{max} \) between the midpoint and vertical array of conversion points.

A test was conducted that extracted eight traces across the line using various values of \( x_{max} \) from 25 m to 2000 m, as indicated on the bottom of the panels in Figure 4. CSP gathers were formed for each \( x_{max} \), then stacked using the first estimate of \( V_{c,p} \) using the \( V_{p,RMS} \) velocities. The shorter offsets around 100 m tended to produce more coherent energy.

Figure 4. Ten panels formed with various \( x_{max} \). Each panel contains eight traces taken at equal increments across the converted wave line.

An example of one limited offset CSP gather (LO-CSP) is shown in Figure 5a that used an \( x_{max} = 100 \) m, and in the central portion of the line. While this data was being processed, an estimated \( V_{c,p} \) from \( V_{p,RMS} \) was used to apply moveout correction for testing purposes. A \( V_p/V_s \) ratio of 2.0 was used for this test. This result is shown in Figure 5b. Note that the moveout corrected data appears to only valid at approximately 0.5 seconds. This is possibly due to \( \gamma = 2 \) being valid only at that time.

Velocity analysis of the limited offset CSP gathers produced a more accurate velocity estimate of \( V_c \) than \( V_{c,p} \) computed from \( V_{p,RMS} \). These velocities are compared in Figure 6 that shows the original \( V_{p,RMS} \) velocity in blue, \( V_{c,p} \) computed from \( V_{p,RMS} \) with \( \gamma = 2 \) in green, and the more
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Accurate $V_r$ in red. $V_c$ shows a greater range of velocity than $V_{cP}$, and it is interesting to note that the two converted wave velocities are equal close to a time of 0.5 s, corresponding to the best horizontal fit of the moveout data in Figure 5b.

![Figure 5. Limited offset CSP gather in a), and b) with moveout correction using $V_{cP}$.](image)

The velocities $V_{P,RMS}$ and $V_c$ can be used to compute a new estimate of $\gamma$ relative to the S wave velocity times. The newly estimated $\gamma$ tends to lower than 2.0 near the surface, and higher than 2.0 deeper in the section as anticipated, though these are only initial estimates.

Two semblance plots are shown in Figure 8 for the same location as in Figures 5, 6, and 7. The first semblance plot (a) was created from a gather that used a super CMP gather with $s_{max}$ as for LO-CSP gather. Both are quite similar throughout the main part of the data but there is a difference at the shallow times. It appears that the difference is significant in estimating the velocities at shallow times.

![Figure 7. Comparison of the estimated $\gamma$ function with the defined constant value of $\gamma = 2$.](image)

Conclusions

The velocities obtained from the short displacement equivalent offset gathers provided a reasonable estimate of the converted wave velocities.

These gathers were formed using only the geometry of the trace, namely the source and receiver locations relative to the location at which the gather is formed.

The initial velocity estimates provide an initial estimate of the $V_p/V_s$ ratio and reliable starting converted wave velocity for the full equivalent offset method of prestack migration of the converted wave data.

Acknowledgements

We would like to thank all CREWES sponsors, staff and students for their support.