Experimental Confirmation of “Reflections on Q”
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Summary

While seismic reflection amplitudes are generally dominated by acoustical impedance contrasts, there is recent interest in reflections due to contrasts in seismic absorption coefficients (or reciprocal-Q values). In this note, we compare the anelastic reflection coefficient computations with ultrasonic laboratory measurements for such reflections, showing that the laboratory measurements produce reflections, as predicted by the anelastic models. While theory, modeling and experiments produce reflections from Q contrasts, the experimental data produce amplitudes that are larger than our theories predict.

Introduction

Exploration seismologists normally consider seismic reflections to be caused by contrasts in real acoustical impedance (product of seismic velocity and density). However, in media where there is significant absorption of seismic energy, there can be reflections that are caused by contrasts in the seismic absorption coefficient, \( \alpha \). Contrasts in absorption can be considered as contrasts in the quality factor, which is inversely proportional to absorption. This interesting phenomenon was shown in the Ph.D. thesis by Bourbie’ (1982) and later published by Bourbie’ and Gonzalez-Serrano (1983) and by Bourbie’ and Nur (1984).

There has been much recent interest in the topic of Q reflections. A paper “Reflections on Q” by Lines, Vasheghani and Treitel (2008) derived the reflection coefficients for Q contrast and showed this effect on SH-wave synthetic seismograms. Other recent papers showing the Q-effects on seismic reflections include papers by Odebeatu et al. (2006), Morozov (2011), and Innanen (2011).

In the case of normal incidence (NI) P-wave reflections in an anelastic acoustic mediums, the boundary conditions require continuity of normal displacement and normal stress. The derivation of the displacement reflection coefficient for such media, as shown by White (1965), Lines, et al. (2008) and Morozov (2011) is given by:

\[
R = \frac{\rho_1 v_1 \left[ 1 + \frac{i}{2Q_1} \right] - \rho_2 v_2 \left[ 1 + \frac{i}{2Q_2} \right]}{\rho_1 v_1 \left[ 1 + \frac{i}{2Q_1} \right] + \rho_2 v_2 \left[ 1 + \frac{i}{2Q_2} \right]}
\]

(1)

Here \( v \) is the seismic velocity, \( \rho \) is the rock density and \( Q \) is the quality factor which is related to the seismic absorption coefficient and seismic wavelength \( \lambda \), by the following relationship.

\[
\alpha = \frac{\pi}{Q\lambda}
\]

(2)

Methodology

Equation (1) is the NI reflection coefficient for displacement, a vector quantity, describing reflections recorded by geophones. The reflection coefficient describing the pressure, a scalar quantity, as recorded by a hydrophone would have the opposite sign. For a complete description of displacement and pressure reflection coefficients, one can refer to Robinson and Treitel (2008, 196-197).

If there is no absorption in the medium (infinite \( Q \)) such that the \( \frac{1}{Q} \) terms vanish, equation (1) reduces to the difference of acoustical impedances divided by the sum as given by Robinson and Treitel (2008). On the other hand, if there is no acoustical impedance between medium 1 and medium 2, we can still obtain reflections due to a contrast in absorption (inverse Q). In such a case, the reflection coefficient is given by:
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\[
R = \frac{i}{2} \left[ \frac{1}{Q_1} - \frac{1}{Q_2} \right] \\
\left( 2 + \frac{i}{2} \left[ \frac{1}{Q_1} + \frac{1}{Q_2} \right] \right)^{-1}
\]  

(3)

Therefore, even with no impedance contrast, we can have reflections due to a contrast in the absorption properties of the two layers. Given the nature of finite Q in absorptive media, this is an important characteristic of R in equations (1) and (3) that is different from the non-absorptive case. Futterman (1962) shows that absorption is linked to wave dispersion through a Hilbert transform relationship, and that Q necessarily has a frequency dependence for absorptive media. Therefore, the reflection coefficient, as expressed in (1) also has a frequency dependence.

Examples

In Figure 1 we show the velocity models used in Lines et al. (2008). In this figure, the boundary between the layers is at the depth of 400 m. In initial tests, synthetic seismograms were computed with the following:

1. Test 1, layer 1 velocity = 2000 m/s, layer 2 velocity=3500 m/s, Q=40 for both layers 1.
2. Test 2, layer 1 velocity=2000 m/s, layer 2 velocity=2000 m/s, Q=40 for layer 1, Q=6.283 for layer 2.

![Figure 1](image1.png)

Figure 1. Models used for forward modeling. Model 1 shows two layers with different velocities and no contrast in absorption. Model 2 shows two layers with different quality factors.

The reflection coefficient for the first case (where Q is constant) would simply be equation (1) with the Q and density contributions cancelling out leaving the difference in velocity divided the sum of the velocities,

\[
\frac{v_1 - v_2}{v_1 + v_2} = \frac{1.5}{5.5} = 0.27.
\]

The reflection coefficient for the second case (with no impedance contrast) would be obtained by substituting into equation (3) to give:

\[
R = \frac{i}{2} \left[ \frac{1}{40} - \frac{1}{6.283} \right] \\
\left( 2 + \frac{i}{2} \left[ \frac{1}{40} + \frac{1}{6.283} \right] \right)^{-1}
\]

(4)

or

\[
R = -0.134i = -0.0015 - 0.0335i.
\]

(5)

This is a complex number whose amplitude is about 1/8 as big as the reflection coefficient for the impedance contrast, and there is a phase shift due to absorption. If we include both the impedance contrast and the Q contrast, the resulting seismic trace is not significantly different from that of the impedance contrast only. The synthetic seismograms for the case of impedance contrast only, the
case of Q-contrast only, and a case where there is both a Q and an impedance contrast are shown in Figure 2, as computed by Tad Ulrych. Ulrych’s computations are the exact reflections using equation (1), while the figure in the original paper of Lines et al. (2008) used approximate finite-difference calculations.

The reflection amplitudes for the impedance-contrast case and the (impedance plus Q) contrast case are nearly identical. The reflection amplitudes are slightly different and phase shifted but the differences are barely noticeable. For the case of Q-contrast only, the reflection is weak and phase shifted by about 90 degrees, as one would expect.

Our numerical models suggest that reflections due to a Q-contrast alone will generally be weak but can exist. This result has been verified by Carl Sondergeld in a rock physics experiment using ultrasonic frequencies near 500 kHz. As shown in Figure 3, Sondergeld chose materials which can emulate the numerical experiments of Figures 1 and 2. Sondergeld chose aluminum immersed in water to emulate the case of large contrast in the real part of the impedance (density X velocity) in equation (1). As shown in the Figure, this produces a very large reflection coefficient of amplitude -0.84022. In a second experiment, Sondergeld chose materials with almost no contrast in the real part of the impedance but with significant contrasts in their seismic absorption. The materials were Crisco (vegetable shortening) immersed in water. The real part of the reflection coefficient was very small (0.003413). However, while water has a low attenuation or very high Q of 210,000 at room temperature, Crisco has a high attenuation or low Q, as indicated by the experiments of Joe Wong using the CREWES Seismic Physical Modeling Facility. According to equation (1), we expect a reflection coefficient with a large imaginary component. The results of Sondergeld’s experiments for elastic (water/aluminum) and anelastic (water/Crisco) are shown in Figure 3. The amplitudes of the reflections are surprisingly similar in magnitude, but the anelastic reflection is phase shifted from the elastic case as one would expect from equation (1).
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Sondergeld’s interesting experimental result was recently confirmed in a set of experiments described by Wong and Lines (2011). The setup for Wong’s experiment is shown in Figure 4. As in the Sondergeld results, the anelastic reflection was similar to the elastic reflection but was phase shifted. There were slight differences in the waveforms since the seismic pulses in the experiments were not identical in the two sets of experiments. Also, the geometry of the reflectors was slightly different. However, the amplitudes for the case of high impedance-low Q contrast are very similar in magnitude to those from the low impedance-high Q contrast. Both unaltered Crisco and Crisco altered by melting and re-solidification were used. The amplitudes of the reflections in both the elastic and anelastic reflection experiments are very large. We expect this for water/aluminum given their well-known large contrasts in acoustical impedance. However, the amplitude of the water/Crisco reflection requires that there be an extremely large contrast in the Q for water and Crisco.

If we substitute different values of Q for Crisco and use the Q value for water, we can compute the amplitude of R by using equation (1). In doing these calculations we can obtain results similar to those shown in Figure 5. However, in order to obtain a calculated reflection amplitude of about 0.65 that matches the experimental value, we required an extremely low Q value of approximately 0.3!

Therefore, these experiments are encouraging in that they verify the existence of significant reflections due to contrast in seismic-Q, as predicted our previous modeling. However, the magnitude of the Q-reflections requires an extremely low (physically unrealistic) Q value for Crisco – much lower than other measurements described by Wong and Lines (2011). We need to examine the need for more general theories involving frequency-dependent reflection coefficients in order to explain the amplitude of these reflections.

In examining general theories, it would seem necessary to consider frequency variation of the reflection coefficient due to attenuation. Bourbie’ (1982) and describe the reflection coefficient, \( R' \) in terms of an expression which can be related to our original equation (1).

\[
R' \approx R + \frac{1}{2\pi} \left( \frac{1}{Q_1} - \frac{1}{Q_2} \right) \log \frac{\omega}{\omega_0}
\]

The expression of Bourbie’ and Nur (1984) is essentially given by our equation (1) plus the term \( \frac{1}{2\pi} \left( \frac{1}{Q_1} - \frac{1}{Q_2} \right) \log \frac{\omega}{\omega_0} \). This additional term multiplies the difference in the reciprocals of Q for the two layers by a term that depends on \( \log \frac{\omega}{\omega_0} \), the logarithm of the ratio of the frequency to the reference frequency (frequency at which the original phase velocity was measured). In other words, the reflection coefficient \( R' \) is frequency dependent and we should write it as \( R'(\omega) \). However, this term would require unrealistically large values of \( \log \frac{\omega}{\omega_0} \) in order to make the water/Crisco reflection have a reflection strength comparable to water/aluminum reflection coefficient. In summary, experiments confirm the existence of Q-contrast reflections but they are much bigger than anticipated.

Conclusions

Theory, numerical modeling, and two sets of laboratory measurements at ultrasonic frequencies all show that seismic reflections can arise from contrasts in seismic-Q and that these reflections are phase shifted compared to elastic reflections. While we have shown that Q-contrast reflections exist, their amplitude is significantly larger than predicted with conventional theory. These reflections from Q contrasts are scientifically interesting, and can possibly be related to the viscosity of fluids in reservoir rocks. A more general anelastic theory may be needed to account for the anelastic reflection.

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