Application of 2D crosscorrelation and Radon transform for analysis of double couple microseismic source

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Summary

Microseismic events radiate a complex wavefield and are recorded at offset receivers commonly located in a borehole. The mapping of these individual events provides an estimate of the fracture geometry and indicates areas of deformation. In this paper, a double couple microseismic source is considered whose fracture plane is oriented horizontally. The waveform generated from this event contains both a P- and S-wave at the time of origin which propagate outward radially to produce both hyperbolic and linear arrival times. The failure mechanism is purely shear and is modeled by using two force couples.

The resulting waveform which includes P- and S-wave arrival times and opposing polarities is analyzed using two correlation techniques. The first technique requires 2D crosscorrelation of the modeled data obtained from ray tracing and the observed data to determine the similarity of the two signals. The second approach assumes the event as hyperbolic, and applies a time shifted hyperbolic Radon Transform to search for a best fit travel time match between the modeled and observed data. In the former method, the polarity of the P- and S-wave above and below the fracture plane had to be accounted for in the ray tracing results in order to match the pattern of the observed data. In the latter approach, the first motion of the arrival times had to be made consistent in order to avoid the canceling of the wave due to the summation process. Both approaches have been implemented to determine the time lag, which consequently can give the origin time of the events.

Introduction

Microseismic monitoring is used in the petroleum industry for the mapping of hydraulic fractures. The fracture openings created, known as mode I failure, help to increase permeability in unconventional reservoirs such as in gas shales and tight sands. The microseismic events are a side effect of this process and occur as a result of pore pressure changes (Warpinski, 2004) and shear movements of a limited area of rock surface (Pearson, 1981). These events generate a wavefield containing both a P- and S-wave and evolve with time to produce a complex radiation pattern. The source time and location of these events is unknown, which poses a challenge in the imaging of these hypocenters. Their spatial positions are commonly located by finding a best fit match between the observed and modeled arrival times. This is an approach used in earthquake seismology and further discussed by Nelson and Vidale (1990).

In this study, elastic wave forward modeling is performed using a double couple source. The resulting waveform which includes P- and S-wave arrival times and polarity is analyzed using two correlation techniques. The first technique requires 2D crosscorrelation of the modeled data obtained from ray tracing and the observed data to determine the similarity of the two signals. This process has the advantage of determining the time lag between two traces (Lines and Newrick, 2004). The second approach applies a hyperbolic Radon Transform to search for a best fit travel time match between the modeled and observed data. Both of these techniques calculate lag-time which can be used to calculate the origin time and spatial location of the microseismic event.
Theory and Method

**Modeling of double couple source:** In the 2D problem, equation (1) can be expressed in Cartesian coordinates as a pair of equations that represent the continuous, elastic wave equation for a homogeneous, isotropic medium:

\[
(\lambda + 2\mu) \frac{\partial^2 U_z}{\partial z^2} + (\lambda + \mu) \frac{\partial U_z}{\partial z} + \mu \frac{\partial^2 U_x}{\partial x^2} = \rho \frac{\partial^2 U_z}{\partial t^2}, 
\]

where \(\lambda\) and \(\mu\) are Lamé parameters, \(U\) is displacement, and \(\rho\) is the density of the medium. Equation (1) expresses the displacement in the \(z\)-direction and the following equation expresses the displacement in the \(x\)-direction (Manning, 2007),

\[
(\lambda + 2\mu) \frac{\partial^2 U_x}{\partial x^2} + (\lambda + \mu) \frac{\partial U_x}{\partial x} + \mu \frac{\partial^2 U_z}{\partial z^2} = \rho \frac{\partial^2 U_x}{\partial t^2}. 
\]

The double couple failure mechanism, as shown in figure (1a), has compressional and tensional components in the \(U_x\) and \(U_z\) directions. The force pairs, which point in the opposite direction and are a small distance apart (Stein and Wysession, 2003), are modeled in order to avoid net torque on the fault plane. The force couples explain the radiation pattern, the amount of seismic energy radiated in different directions and indicate first motion of the amplitudes. This provides a framework for modeling the double couple source using simultaneous sets of wavelet injections. To do so, a wavelet is simultaneously injected into the \(U_z(x, z, t)\) and \(U_x(x, z, t)\) directions and stepped forward in time using equations (1) and (2) by a step increment \(\Delta t\). The injected wavelet is introduced into the \(U_x\) and \(U_z\) matrix at two adjacent positions and illustrated in Figure (1b). The assigned (+1) and (-1) are the amplitude factors which are multiplied by the wavelet components to model the double couple source during each time step.

To determine the time lag of the event, we use a 2D crosscorrelation, which is a pattern recognition approach. This technique requires observed data \(U_{\text{data}}\) which is scanned by the modeled data \(U_{\text{model}}\) and a best fit data match along with the lag time is determined using (Hale, 2006):

\[
C(i, j) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} U_{\text{data}}(k, l) U_{\text{model}}(k + i, l + j),
\]

Figure 1: Double couple source failure mechanism showing (a) P-wave radiation pattern: the (+) sign indicates compression and the (-) sign indicates dilatation, (b) injection pattern of a source displacement for force couples into the wavefield matrix and (c) first motion behavior for each \(U_x\) and \(U_z\) displacement component; the solid blue wavelets indicate the polarity in the \(U_x\) direction and the red dashed wavelets indicate the polarity in the \(U_z\) direction.
where, $C(i,j)$ is the crosscorrelation coefficient. In another technique, we utilized the application of a time shifted hyperbolic Radon transformation (Stoffa, 1981). At defined time lags it intends to find the best match between the true velocity and collapsed point in the $\tau - p$ domain. Considering the radiation pattern of the double couple source, the summation path is defined by:

$$S(p, \tau) = \sum_{i=1}^{N} s(z_i, t) = \sqrt{(\tau - e)^2 + p^2 z_i^2} + e.$$

where, $S(p, \tau)$ represents the wave in the Radon domain, $s(z_i, t)$ is the recorded microseismic data, $\tau$ is the two-way travel time intercept, $t$ is the two-way travelt ime, $z$ is the depth to the apex of the hyperbola and $p$ is defined as $1/v_i$ and $e$ is the lag time to be solved.

**Numerical Example**

**Geologic model:** A three layered symmetric geologic model is used that contains both density and velocity ($\rho$ and $v$). The center layer is fast and is bounded by two lower velocity zones. A snap of an elastic wavefield of a double couple source containing $U_s$ and $U_i$, traveling through a three layer homogeneous, elastic medium at a sample time snap is shown in Figure (2a). The arrows indicate the source position of the fracture plane (solid arrows) and force couple (dashed arrow). The full colored circle legend indicates the direction of particle motion: the inner circle corresponds to the S-wave and the outer circle to the P-wave.

A vertical receiver array is positioned offset to the double couple source which is located at the center of the middle layer. Since the origin time and location are unknown in the field, a time lag is introduced to a single modeled event and random noise is added to simulate real data. Various methods are used to determine origin time; however the most common approach is to find the best fit time match by calculating minimum travel-time residual. This method is adopted here and the 2D crosscorrelation and Radon transform are implemented to find the corresponding time-lags.

**2D crosscorrelation:** Figure 4b shows a sample ray path (p-wave only) generated by a double couple source traced through the velocity model to each receiver. The modeled travel times from each grid point to the receivers, for both the p- and s-wave, are stored as travel time templates. A best fit arrival time match is obtained between the template and real data. This is shown in Figure 4c where the recorded wavefield is overlain by the template of the modeled data, given by black points. Finally, the modeled waveform is crosscorrelated with the real data to determine the relative time lag between the two signals, as shown by the correlation results in Figure 4d.

![Figure 2](image_url)

**Figure 2:** a) A sample time snap of an elastic wavefield. b) Ray tracing simulation result c) P- and S-wave arrival times (black points) through a three layer medium overlain with the recorded $U_s$ component of the wavefield d) crosscorrelation results of modeled event with observed data.
As expected, the highest correlation occurs between the event modeled at the same location as the original location of the observed data. The maximum amplitude correlation is 9.80 which results in a time lag of 0.04s.

**Hyperbolic time shifted Radon transform:** In this approach, first the polarity of the upper and lower parts of the hyperbola is corrected to avoid canceling effects. Figure (3) shows a case where the most optimum time lag applied, \( \varepsilon = 0.033s \), results in two focused points in the \( \tau - p \) domain which align well with the true P- and S-wave velocities.

![Figure 3](image)

**Discussion**

The crosscorrelation of modeled data and the observed data gives a time and depth lag of \( \varepsilon_{\text{true}} = 0.044s \). This value agrees reasonably well with the true lag introduced into the original data as summarized in table 1.

The time shifted hyperbolic Radon transform, compared to the crosscorrelation method, produced a more accurate time lag of \( \varepsilon_{\text{true}} = 0.033s \). The accuracy of the result depends to the increment of \( \varepsilon \) and knowledge about the velocity of the medium, which is often the most difficult parameter to accurately determine when processing microseismic data.

**Table 1:** Summary of the best time lag results compared to the true time shift introduced into the original forward modeled event

<table>
<thead>
<tr>
<th></th>
<th>Time lag (s)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>Crosscorrelation</td>
<td>0.044</td>
<td>29%</td>
</tr>
<tr>
<td>Radon Transform</td>
<td>0.033</td>
<td>3%</td>
</tr>
</tbody>
</table>

One must be aware that in real world microseismic acquisition, limitations on how deep the receivers can be placed, number of geophones deployed and aperture of the array do not always favor ideal sampling of the hyperbolic move-outs. This can have a significant impact in determining the lag times and thus predicting accurate origin times.

**Conclusions**

In this work, the signal of a microseismic event produced from a horizontally oriented fracture plane is analyzed. The analysis started by forward modeling of the wave P- and S-waves using an elastic finite difference modeling technique. The double couple source mechanism, which radiates P- and S-waves simultaneously, provides the direction in which the source wavelet could be injected. This generated the wavefield to be analyzed during each time step.
For the numerical analysis, an observed data with unknown origin time is generated, by time shifting and adding noise to a forward modeled event. The 2D crosscorrelation technique is implemented to determine the time lag of the modeled data. The polarities of the modeled data are adjusted based on the radiation pattern of the double couple source mechanism to allow for pattern matching.

Assuming the medium velocities are known and the microseismic wavefield is hyperbolic, a time shifted hyperbolic Radon transform scheme is proposed for estimation of the time lag. In this method, the velocity intercept of the focused events in the Radon domain depends on the amount of time lag applied. The points/events that focus at the true velocity in the $\tau-p$ domain correspond to the true lag.

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References