Simultaneous multiple source acquisition using m-sequences

Joe Wong
CREWES, University of Calgary
(wongjoe@ucalgary.ca)

Summary

The increasing demand for high-resolution 3D seismic imaging is driving the search for techniques of efficiently acquiring field datasets with hundreds of millions or even billions of seismic traces. The common method is to deploy as many geophones as possible for recording. Acquisition productivity can be further enhanced if multiple simultaneous sources can be used. When the sources are controlled sources such as land vibrators, simultaneous sourcing can be done efficiently with minimal crosstalk if they are driven by a set of quasi-orthogonal pilot signals. A quasi-orthogonal set in this context has the following properties: (1) the autocorrelation of any member in the set closely approximates the delta function; and (2) the cross-correlation between any two different members in the set is very nearly zero with no oscillatory side lobes. Maximal length sequences, or m-sequences, are periodic mathematical periodic entities with values of -1 and 1. A single m-sequence can be used to construct multiple shifted m-sequences that form a set of pilots with the desired properties. We illustrate the use of shifted m-sequences for simultaneous sourcing through a numerically-generated example, as well as with a laboratory experiment.

Introduction

Recently, Pecholcs et al. (2010) reported on using 24 vibrators operating simultaneously in close proximity to acquire data for a land 3D seismic survey. The use of 24 simultaneous sources resulted in more than 40,000 shot points per 24-hour period. This represents a very significant improvement in acquisition productivity. The multiple vibrators were controlled by a set of modified Gold codes (Sallas et al., 2011; Sallas and Gibson, 2008).

Gold codes are closely related to maximal length sequences, or m-sequences. A set of many related Gold codes can be derived from a pair of optimal m-sequences using the theory published by Gold (1967). Since m-sequences have a prior existence, they may be considered to be simpler and more fundamental than Gold codes.

Gold codes and m-sequences are widely used in many fields of science and engineering. Gold codes are popular for wireless communications because thousands or even millions of weakly-correlated forms can be easily produced from different preferred (or optimal) pairs of m-sequences. M-sequences have been used successfully in earth-science applications; these include ocean acoustic thermometry (Dushaw et al., 1999), electromagnetic exploration techniques (Duncan et al., 1980; Ziolkowski, 2011), and crosswell seismic scanning (Wong et al., 1983, 1987; Wong, 2000). Both Gold codes and m-sequences are periodic and binary-valued with values of -1 and 1. All members have autocorrelations that approximate the delta function, and so are often called pseudorandom binary sequences (PRBSs). Members of a related set of Gold codes are weakly coupled under cross-correlation, i.e., the cross-correlations at all lags oscillate about zero with predictable absolute values significantly less than the autocorrelation peaks. We may say that the set of related Gold codes is quasi-orthogonal (each member in a quasi-orthogonal set has an autocorrelation that approximates a delta function, but its cross-correlation with other members produces absolute values much less than the autocorrelation peak value).
Sets of m-sequences also can be constructed to be quasi-orthogonal under certain conditions. Quasi-orthogonality makes both types of PRBS suitable for controlling multiple vibratory sources simultaneously. Quasi-orthogonality allows the combined signals from multiple vibrators operating simultaneously to be separated by cross-correlation into individual traces with little crosstalk. We will show that the cross-correlations between specifically-designed m-sequences are better approximations to zero than those of Gold codes. M-sequences are easily generated using software equivalents of linear feedback shift registers, or LFSRs (Golomb, 1967). Since m-sequences are simpler than Gold codes, and since they are more nearly orthogonal, they may be better alternatives for driving multiple simultaneous seismic vibrators than the modified Gold codes suggested by Sallas et al. (2011).

Correlation of periodic functions such as m-sequences and Gold codes is circular. Complete correlation must include wrap-around if only one period or cycle of any periodic sequence is involved. In the following discussion, all operations involving correlation, convolution, and shifting are assumed to be circular.

Characteristics of m-sequences

Mathematically, an m-sequence is parameterized by its degree $m$, its fundamental length $L$, and its base period $t_B$. The fundamental length $L$ is related to the degree $m$:

$$L = 2^m - 1,$$

where $m$ is an integer. The exponential form of Equation 1 means that the fundamental length grows very quickly with the degree. In most practical geophysical applications, we limit $m$ to values of 10 to 16. If $m = 11$, then $L = 2047$; when $m = 15$, $L = 8191$.

In terms of real time, an m-sequence is periodic with period $T_M$ equal to:

$$T_M = L \cdot t_B,$$

(2)

For practical application, we include two further parameters: the sample time $t_S$ and the shift time $t_{\text{Shift}}$. An up-sampled version of the m-sequence is obtained when it is digitized with sample time $t_S$. The ratio of the base period to sample time of the digitized m-sequence is the up-sample ratio:

$$r = t_B / t_S,$$

(3)

where $r$ typically has integer values of 1, 2, 4, 8, or 16.
Correlations of m-sequences and Gold codes

The autocorrelation of an up-sampled m-sequence is also periodic, and has the appearance of a series of isosceles triangles separated by time $T_M$. The peak values on the triangles are equal to $rL$, and the triangles rest on a constant DC level equal to $-r$. Dividing the factor $rL$, we obtain scaled peak and DC values of 1 and $-1/L$, respectively. The base of each triangular peak has width equal to $2t_B$. The base widths compared period $T_M$ are very narrow when the degree $m$ is greater than 11. In effect, each triangle on the autocorrelations approximates the delta function. Figure 1 is a plot of an example m-sequence with its autocorrelation.

The autocorrelation of a Gold code has similar characteristics, with one crucial difference. The off-peak values are not constant, but oscillate about zero with small (but predictable) values. Figure 2 displays an example of a Gold code and its autocorrelation. If the degree $m$ is an odd integer, the off-peak values oscillate between three theoretical values: $-(2^{(m+1)/2} + 1)/L$, $-1/L$, and $(2^{(m+1)/2} - 1)/L$. If $m$ is even, the theoretical values are $-(2^{(m+2)/2} + 1)/L$, $-1/L$, and $(2^{(m+2)/2} - 1)/L$. For the example with degree 7, these are equal to $-17/127$, $1/127$, and $15/127$, respectively.

The oscillations constitute correlation noise that would mask weak reflection events on extracted seismograms. The correlation noise decreases with the degree $m$, but quite slowly. For Gold codes with $m = 11$, the correlation noise is on the order of $-30$dB compared to the autocorrelation peak value. Even for very long Gold codes with $m=20$ (which are too long for practical seismic acquisition), the correlation noise is still only about $60$dB down from the autocorrelation peak.

Defining shifted m-sequences

After constructing a single sampled m-sequence $S_1$, multiple shifted versions are easily produced by systematically applying a shift time $t_{Shift}$. If we want a set of $N$ shifted m-sequences, we define $t_{Shift}$ by

$$t_{Shift} \leq T_M / N .$$  

(4)

For example, if $T_M$ is $8188$ms and $N$ is four, then $t_{Shift}$ can be given a value of $2040$ms. The set $\{S_1, S_2, S_3, S_4\}$ is obtained by applying time shifts of 0ms, 2040ms, 4080ms, and 6120ms. $N$ is also the number of simultaneous vibrator sources that we wish to use in our seismic survey.

Within a correlation window restricted to lag times less than $t_{Shift}$, the scaled values of cross-correlations between different $S_j$ are all equal to $-1/L$, i.e., nearly zero compared to the scaled autocorrelation peak values of 1. It is this quasi-orthogonal property with non-oscillatory cross-correlation values that makes the set of shifted m-sequences $\{S_1, S_2, S_3, S_4\}$ eminently suitable for use as pilots for multiple vibrators operating simultaneously.
Driving multiple vibrators with quasi-orthogonal shifted m-sequences

The m-sequence with \( m = 7 \) shown on Figure 1 is too short for real-world seismic acquisition in most cases, and sequences with longer fundamental lengths \( L \) must be used. For example, for the crosswell seismic scanning instrumentation described by Wong (2000), an m-sequence PRBS with \( m = 11, L = 2047, t_g = 0.10\)ms, and \( t_s = 0.05\)ms was used for driving a piezoelectric vibrator source.

To demonstrate using shifted m-sequences for simultaneous-source acquisition, we will analyze a specific case where there are four vibrators. We start with an m-sequence with \( m = 11, L = 2047, t_B = 4\)ms, and sampling time \( t_s = 1\)ms. The sampled sequence will be 8188ms long. From this starting sequence \( S_1 \), we can construct three other sequences \( S_2, S_3, \) and \( S_4 \) by systematically shifting \( S_1 \) (with wrap-around) by 2040ms, 4080ms, and 6120ms. The initial 2040ms of the four shifted sequences \( S_1 \) to \( S_4 \) are shown on Figure 3. The important characteristic of this set of four sequences is that, when restricted to a correlation window with lags between 0ms and 2040ms, the (unscaled) autocorrelations approximate delta functions with peak values of 8188 while the cross-correlations between two different sequences have a constant value of -4, i.e., within the window of restricted lags, the four shifted m-sequences in this set are almost perfectly orthogonal under correlation. The auto- and cross-correlations for \( S_1 \) to \( S_4 \) for lag times 0ms to 1200ms are displayed on Figure 4.

If the four vibrators are each controlled by a pilot equal to a different \( S_i \), and if the four vibrators are driven simultaneously, the combined signal received at any geophone can be separated almost perfectly into four seismic traces simply by cross-correlation with each of the \( S_i \). Each separated trace will be associated only with the source being driven by the appropriate \( S_i \), with little if any crosstalk from the other sources.

Figure 5 shows two wavelets used to represent seismic arrivals in the seismic response of the earth due to an impulsive force on its surface. The weak second wavelet has an amplitude 10,000 times less than that of the first wavelet. Only the strong wavelet is visible on the normalized plot; both wavelets are visible on the AGC plot.

Convolving the wavelets with two cycles of each of the shifted m-sequences yields received signals \( R_1 \) to \( R_4 \), the first 4000ms of which are plotted on Figure 6. The received signals have been successively delayed by 100ms to represent increasing arrival times due to increasing separations between the four sources and a single receiver. The initial times of \( R_1 \) to \( R_4 \) have zero values because the receiver sees no energy until vibrations have travelled the distance between the sources and the receiver. At the single receiver, the detected signal will be the sum \( R_T \) of \( R_1 \) to \( R_4 \), since the four sources are operating simultaneously. Using values of \( R_T \) corresponding to the second cycle of the pilots, we extracted four seismograms by circular cross-correlation with each pilot \( S_1 \) to \( S_4 \). These are shown on Figure 7; both the weak event and the strong event have been recovered with excellent clarity.

The numerical example represented by Figures 6 and 7 has been duplicated in a laboratory physical experiment in which piezoelectric buzzers were used as four sources and as one receiver.

**Conclusion**

Numerical simulation has shown that shifted m-sequences can be used in place of Gold sequences for simultaneous multiple-source seismic acquisition. Comparison of Figures 1 and 2
indicates that shifted m-sequence may be better choices as pilot signals for controlling multiple vibrators than Gold codes, because the auto- and cross-correlations of m-sequences have much less correlation noise. This difference means that seismograms reconstructed using m-sequence pilots should be more effective in recovering weak reflection events.

**Acknowledgements**

The contents of this report have been contributed in part by JODEX Limited. We thank the industrial sponsors of CREWES and NSERC for supporting this research.

**References**


Golomb, S.W., 1967, Shift register sequences, Holden-day.


Wong, J., 2000, Crosshole seismic imaging for sulfide orebody delineation near Sudbury, Ontario, Canada: Geophysics, 65, 1900-1907.