AVAZ Inversion for Fracture Orientation and Intensity - A Physical Modeling Study
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Abstract
We present a pre-stack amplitude inversion of P-wave data to determine fracture orientation and intensity. We test the method on multi-azimuth multi-offset physical model reflection data acquired over a simulated fractured medium. This medium is composed of phenolic material with controlled symmetry planes, and its elastic properties have already been determined using traveltime analysis. This experimental model represents an HTI layer. We apply amplitude inversion of small incident angle data to extract the fracture orientation (direction of isotropic plane of the medium), and are able estimate the fracture orientation quite accurately. Knowing the fracture orientation, we modified the available linear PP reflection coefficient approximation to invert for the anisotropy parameters ($\varepsilon^{(V)}$, $\delta^{(V)}$, $\gamma$). The results for all three anisotropy parameters from AVAZ inversion compare very favorably to those obtained previously by a traveltime inversion. This result makes it possible to compute the shear-wave splitting parameter, directly related to fracture intensity, which historically had been determined from shear-wave data, from a quantitative analysis of the PP reflection data.

Introduction
The ultimate goal of using AVO in fracture-detection studies is to invert the pre-stack amplitude data for fracture orientation (strike) and the magnitude of fracture intensity. Fracture orientation is defined as the dominant direction of fracture faces. Fracture intensity is the number of fractures per unit volume. Assuming fractures can be described by a horizontal transverse isotropy (HTI) medium, we present the AVAZ inversion for fracture orientation, based on the method of Jenner (2002), and tested on physically modeled data acquired over a simulated fractured medium. For fracture intensity, we propose a pre-stack amplitude inversion of large-offset P-wave data, based on the Rüger (2001) equation, to invert for ($\varepsilon^{(V)}$, $\delta^{(V)}$, $\gamma$), where $\gamma$ is directly related to fracture intensity.

Theory
The plane-wave approximation for the PP reflection coefficient at a boundary between two HTI media with the same orientation of the symmetry axis (along $\phi_0$ azimuth) is given by Rüger (2001) as

$$R(\theta, \phi) = \frac{1}{2} \frac{\Delta Z}{Z} + \frac{1}{2} \left( \frac{\Delta \alpha}{\alpha} - \left( \frac{2\beta}{\alpha} \right)^2 \frac{\Delta G}{G} \right) \left[ \Delta \varepsilon^{(V)} + \left( \frac{2\beta}{\alpha} \right)^2 \Delta \gamma \right] \cos^2(\phi - \phi_0) \sin^2 \theta$$

$$+ \frac{1}{2} \left( \frac{\Delta \alpha}{\alpha} + \Delta \varepsilon^{(V)} \cos^2(\phi - \phi_0) + \Delta \delta^{(V)} \sin^2(\phi - \phi_0) \cos^2(\phi - \phi_0) \right) \sin^2 \theta \tan^2 \theta$$

(1)

where $\theta$ is the incident angle, $\phi$ is the source-receiver azimuth, $\alpha$ is the vertical P-wave velocity (fast P velocity), $Z = \alpha \rho$ is the P-wave impedance, $\beta$ is the vertical S-wave velocity (fast S velocity), $G = \rho \beta^2$ is the shear modulus. ($\varepsilon^{(V)}$, $\delta^{(V)}$, $\gamma$) are the Thomsen-style anisotropy parameters for HTI media, defined using stiffness coefficients by $\varepsilon^{(V)} = (c_{11} - c_{33})/2c_{33}$, $\gamma = (c_{44} - c_{55})/2c_{55}$, and $\delta^{(V)} = [(c_{13} + c_{35})^2 - (c_{35} - c_{55})^2]/[2c_{55}(c_{35} - c_{55})]$. We reformulate equation (1) as a function of P- and S-wave velocities, using $\Delta Z / Z = \Delta \alpha / \alpha + \Delta \rho / \rho$ and $\Delta G / G = 2\Delta \beta / \beta + \Delta \rho / \rho$, as
\[ R(\theta, \varphi) \equiv \frac{1}{2 \cos^2 \theta} \frac{\Delta \alpha}{\alpha} - \frac{4 \beta^2}{\alpha^2} \sin^2 \theta \frac{\Delta \beta}{\beta} + \frac{1}{2 \rho^2} \left( 1 - \frac{4 \beta^2}{\alpha^2} \sin^2 \theta \right) \frac{\Delta \rho}{\rho} + 1 \left( \cos^2(\varphi - \varphi_0) \sin^2 \theta \tan^2 \theta \right) \Delta e^{(\psi)} \]
\[ + \frac{4 \beta^2}{\alpha^2} \cos^2(\varphi - \varphi_0) \sin^2 \theta \Delta \gamma + \frac{1}{2} \cos^2(\varphi - \varphi_0) \sin^2 \theta \left( 1 + \sin^2(\varphi - \varphi_0) \tan^2 \theta \right) \Delta e^{(\psi)}. \]

(2)

\textbf{Estimation of anisotropy parameter}

Assuming the fracture orientation is known (\(\varphi_0\) is perpendicular to it), equation (2) can be written as a function of six parameters \((\Delta \alpha / \alpha, \Delta \beta / \beta, \Delta \rho / \rho, \Delta e^{(\psi)}, \Delta \delta^{(\psi)}, \Delta \gamma)\):

\[ R(\theta, \varphi) \equiv A \frac{\Delta \alpha}{\alpha} + B \frac{\Delta \beta}{\beta} + C \frac{\Delta \rho}{\rho} + D \Delta e^{(\psi)} + E \Delta \delta^{(\psi)} + F \Delta \gamma, \]

(3)

where the coefficients \(A, B, C, D, E, \) and \(F\) are functions of the incident angle, azimuth, and velocity, and defined as in equation (2). At each azimuth, we use PP ray tracing of a smooth background velocity at a given offset and depth to determine the incident angle, then to calculate the coefficients. Assuming that the pre-stack PP reflection amplitudes approximate reflection coefficients, incorporating “n” reflection amplitudes for up to large incident angles (e.g., far-offset up to 45\(^ \circ \)) from “m” different azimuths as the input data, we use equation (3) to form a linear system of equations to invert for the six parameters solved by a damped least-squares inversion.

\textbf{Fracture orientation estimation}

Considering only small incident angle data (e.g., less than 35\(^ \circ \)), for which the \(\sin^2 \theta \tan^2 \theta\) term can be neglected, equation (1) becomes

\[ R(\theta, \varphi) \equiv I + \left[ G_1 + G_2 \cos^2(\varphi - \varphi_0) \right] \sin^2 \theta, \]

(4)

where \(I = (0.5) \Delta Z / Z, \ G_1 = 0.5(\Delta \alpha / \alpha - (2 \beta / \alpha)^2) \Delta G / G, \) and \( G_2 = 0.5(\Delta \delta^{(\psi)} - 2(2 \beta / \alpha)^2 \Delta \gamma). \) Equation (4) has the form of AVO intercept and gradient. The gradient term \( Q = (G_1 + G_2 \cos^2(\varphi - \varphi_0)) \) is nonlinear with respect to three unknowns \((G_1, G_2, \varphi_0)\). The goal here is to invert the PP amplitude data for these three unknowns. The following describes how to bypass this non-linearity and to apply a linear inversion. Using the identity \(\sin^2(\varphi - \varphi_0) + \cos^2(\varphi - \varphi_0) = 1, \) the AVO gradient becomes \( Q = (G_1 + G_2) \cos^2(\varphi - \varphi_0) + G_1 \sin^2(\varphi - \varphi_0) \). If the AVO gradient does not change sign azimuthally, the gradient versus azimuth vector delineates a curve that closely resembles an ellipse (Rüger, 2001) with the semi-axes aligned with the symmetry plane directions of the fracture system (Figure 1). For the coordinate system aligned with the fracture system, every point on the gradient ellipse is \((y_1, y_2) = r(\cos(\varphi - \varphi_0), \sin(\varphi - \varphi_0))\), where \(r\) is the vector magnitude. In this coordinate system the gradient term can be written as:

\[ Q = r((G_1 + G_2)y_1^2 + G_1 y_2^2). \]

(5)

\textbf{Figure 1: Reference coordinate systems.} \((x_1, x_2)\) is the acquisition coordinate system, and \((y_1, y_2)\) is the coordinate system aligned with the fracture system. \(\varphi\) is the source-receiver azimuth, and \(\varphi_0\) is the fracture orientation azimuth.

On the other hand by expanding trigonometric terms, the gradient term can be written as \( Q = r(W_1 \cos^2 \varphi + 2W_2 \cos \varphi \sin \varphi + W_3 \sin^2 \varphi), \) where \(W_j\) are functions of \((G_1, G_2, \varphi_0)\). In the acquisition coordinate system, every point on the gradient ellipse is \((x_1, x_2) = r(\cos \varphi, \sin \varphi)\), so
the last equation becomes \( Q = r(W_{11}x_1^2 + 2W_{12}x_1x_2 + W_{22}x_2^2) \), a quadratic form that can be written in a matrix form, \( \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = X'WX \). If we wish to eliminate the mixed term \( x_1x_2 \) using the eigenvalues of matrix \( W (\lambda_1, \lambda_2) \), it reads

\[
Q = r(\lambda_1x_1^2 + \lambda_2x_2^2),
\]

where \((y_1, y_2)\) are the eigenvectors of matrix \( W \). Comparing coefficients from equations (5) and (6), we obtain: \( G_i + G_j = \lambda_i \) and \( G_iG_j \) can be obtained from eigenvalues. From the eigenvalue problem, we also know that an orthogonal rotation matrix, \( R_\beta \), relates the two coordinate system as \( \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = R_\beta \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \), where the rotation angle \( \phi_0 \) obeys (e.g., Lax, 1997)

\[
\tan 2\phi_\beta = \frac{2W_{12}}{W_{12} - W_{11}}. 
\]

This results in two values for \( \phi_0 \), where \( \phi_0^{(1,2)} = \phi_0^{(1)} + \pi/2 \), as

\[
\phi_0^{(1,2)} = \tan^{-1} \left( \frac{W_{11} - W_{22} \pm \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2}}{2W_{12}} \right). 
\]

Equation (7) is used by Jenner (2002), and is equivalent to that used by Grechka and Tsvankin (1998) in solving for fracture orientation from the azimuthal variation of NMO velocity. Incorporating the PP reflection amplitudes from different azimuths and small incident angles as the input data, we apply a linear inversion to solve for \((W_{11}, W_{12}, W_{22})\). Then, knowing \( W_\beta \) we use equation (7) to estimate the fracture orientation.

### Physical model reflection data

We test the proposed AVAZ inversions for fracture orientation and intensity on physical model reflection data acquired over a four-layered model which includes a HTI layer (simulating a fractured medium) with known symmetry axis direction composed of phenolic material (Figure 2). The initial characterization of the experimental phenolic layer is presented in Mahmoudian et al. (2012a). The five independent parameters required to describe our simulated fractured medium are listed in Table 1.

![Figure 2](image)

**Figure 2:** (left) The four-layered earth model used to acquire reflection data. The acquisition 0° azimuth is parallel to the fracture symmetry axis. (right) Fracture top corrected reflection amplitudes from all nine azimuths and incident angles before the critical angle.

We used reflection amplitudes from nine long offset common midpoint (CMP) gathers acquired along azimuths between 0° and 90°. The amplitudes reflected from the top of the simulated fractured layer are inputs to AVAZ inversion. We applied deterministic amplitude corrections to make amplitudes represent reflection coefficients required by an amplitude inversion; a detailed description is given in Mahmoudian et al. (2012b). The corrected reflected amplitudes from the top of the fractured layer are shown in Figure 2(right).

### Table 1: Elastic properties of the simulated fractured layer

<table>
<thead>
<tr>
<th>Incident angle (degrees)</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
</tr>
<tr>
<td>20</td>
<td>0.2</td>
</tr>
<tr>
<td>30</td>
<td>0.15</td>
</tr>
<tr>
<td>40</td>
<td>0.1</td>
</tr>
<tr>
<td>50</td>
<td>0.05</td>
</tr>
</tbody>
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As the symmetry of the simulated fractured medium is known and the physical model data acquisition coordinates were aligned with the fracture system, we rotate our acquisition coordinate system to arbitrary directions and use the proposed AVAZ inversion to estimate the fracture orientation. Table 2 shows the predicted fracture orientations.

Table 2: Estimated fracture orientation from the AVAZ inversion.

<table>
<thead>
<tr>
<th>True φ₀</th>
<th>0°</th>
<th>20°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated φ₀</td>
<td>-1.5°</td>
<td>18.5°</td>
<td>38.5°</td>
<td>48.5°</td>
<td>58.5°</td>
<td>78.5°</td>
<td>88.5°</td>
</tr>
</tbody>
</table>

To estimate the anisotropy parameters, knowing the fracture orientation of our simulated fractured medium, we test the proposed six-parameter AVAZ inversion on the physical model data. Figure 3 shows the six-parameter AVAZ inversion for different maximum incorporated incident angles. The six-parameter AVAZ inversion using small incident angles does not produce good estimates for any of the six parameters (Figure 3(left)). Incorporating large incident angles (e.g., 40°) within 10° of the critical angle can result in good estimates for the first three isotropic term, indicative of the greater influence of the isotopic terms on the reflection coefficients (Figure 3(middle)). Incorporating larger incident angles, closer to the critical angle, results in better estimates for the anisotropy parameters as the azimuthal anisotropy is more pronounced at larger incident angles (Figure 3(right)). However, it produces large errors for the estimates of the three isotropic terms.

In an effort to obtain better estimates for the anisotropic terms, we applied some constraints to the three isotropic terms using their estimated values from an isotropic AVA inversion of 90° (isotropic plane) azimuth data. With such constraints, we examine the three-parameter inversion results, inverting for anisotropy parameters only, for various maximum incorporated incident angles; the best results come from a large maximum incident angle but not close to critical angle (Figure 4). For the right choice of the maximum incorporated incident angle, the inversion results for the anisotropy parameters agree very favorably with those obtained previously by traveltime inversion.
Conclusions
We presented pre-stack amplitude inversion procedures to extract the anisotropy parameters ($\varepsilon^{(v)}$, $\delta^{(v)}$, $\gamma$), and fracture orientation from the azimuthal variations in the PP reflection amplitudes. As the $\gamma$ parameter is directly related to fracture intensity, we showed that it is possible to relate the difference in P-wave azimuthal AVO variations directly to the fracture intensity. The AVAZ inversion determines the fracture orientation with an inherent ambiguity, as it predicts both the directions of the isotropic plane and symmetry axis of a HTI medium. To resolve the ambiguity some other information is required, such as azimuthal NMO and shear-wave splitting.

References