Resolution enhancement after migration

Summary
The resolution of seismic data can be significantly improved after migration. This can be achieved with a simple trace deconvolution in areas with a simple geology such as a sedimentary basin, or a more complex deconvolution if the structure is complex. There are considerable objections to this process; some are identified and discussed, then reasons for its use are presented. Two examples of deconvolution after migration are presented.

Introduction
The improvement of resolution after migration is often a black-box process that hides the simplicity of the process. The major part of this process is deconvolution, but this is often met with resistance as it is considered to have a negative effect after migration.

Some objections to deconvolution after migration are:
1. Deconvolution should only be applied before migration to maintain the resolution of horizontal and dipping events,
2. Deconvolution will alias the frequencies of steeply dipping migrated events,
3. Deconvolution will increase noise,
4. Deconvolution will produce artifacts, and
5. Kirchhoff migration already uses whitening operators.

We will address these issues and present arguments that support the uses of a simple trace deconvolution after migration for data with reasonable dips.

Theory
Consider a zero-offset geophysical model where a wavelet is stationary and assumed to be defined by the source, and the raypaths are normal to the reflectors, as illustrated in Figure 1a. The reflection energy is plotted below the source/receiver location in (b) to form a seismic section, even when coming from a dipping reflector. These reflection events will contain the same wavelet, independent of the dip, and will have the same frequency content. After migration, (c), the dip wavelets are “rotated” back to the geological location. The effect of rotating the wavelets in Figure 1c lowers the frequency content of the vertical traces after migration.

It is contended that a deconvolution should be applied (and only applied) to the stacked data in Figure 1b, as that is the stage in processing where the wavelets are all similar.

Migration in Figure 1c lowers the frequency content of a dipping event to prevent aliasing. It is argued that deconvolution will alias these dipping events. That is true if the dips are significant, however it is not a problem with data that contains shallow dips. A special deconvolution may be required for structured data that have steeper dips.

Some processes overcome the time varying (non-stationary) nature of seismic data by processing parts of the data in time or spatial windows. These windows of data are then combined (by cut and paste) to create the total section. The edges of these windows have abrupt changes in the wavelets, so the data are high-cut filtered to remove this discontinuity. A deconvolution applied to these cut and paste sections will enhance the discontinuities to reveal the window boundaries and be unacceptable. If this is the case, then a deconvolution after migration may be more desirable to a client than to the processor.

Figure 1: Cartoon of a) raypaths, b) seismic traces, and c) the migrated traces.

A quality migration attenuates noise and may appear wormy to some interpreters. Noise may be deliberately added back to the section to make it appear less wormy. A deconvolution will reduce the wormy appearance by adding higher frequencies to the data, making it appear more interpretable, with the benefit of a higher resolution section.

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Kirchhoff modelling may be viewed as spreading energy along a diffraction with an amplitude defined by the reflectivity. This is a forward process and straight forward to implement. This can be represented in linear algebra with a diffraction matrix $D$, a reflectivity matrix $r$, and the resulting seismic matrix $s$. A convolution of the diffraction matrix with the reflectivity is expressed in linear algebra as

$$Dr = s.$$  \hspace{1cm} (1)

When Kirchhoff migrating, the “reverse process” sums energy along a diffraction. The summation is not a convolution process and we require some form of reversing or inversion to find a convolution type operator. In a constant velocity environment, this operator is a semi-circle. This semi-circle operator may be found by taking the transpose of the diffraction matrix. We now write the migration equation as

$$r = D^T s,$$  \hspace{1cm} (2)

where $D^T$ is the transpose of the $D$ matrix, and $\bar{r}$ is the migrated form of the reflectivity. We are being quite liberal with our notation of transpose and using the lower case to represent matrices, especially when the diffraction matrix may be a three or four dimensional volume. However, it can be shown that the multidimensional matrix $D$ can be reduced to a two dimensional matrix, and the two dimensional matrices $r$ and $s$ can be reduced to vectors, to satisfy the above equations, (Claerbout 1992 and 1998, Bancroft et al., 2011, Bancroft 2012, Yousefzadeh 2013).

Rather than trying to recover the reflectivity with a transpose process, it is more desirable to obtain a true inverse diffraction matrix $D^{-1}$ defined by

$$r = D^{-1} s,$$  \hspace{1cm} (3)

but that is not possible as the $D$ matrix is usually multi dimensional and not square. We can however use $D^T$ and the least squares solution to get a good estimate of $r$ using

$$\bar{r} = \left(D^T D\right)^{-1} D^T s.$$  \hspace{1cm} (4)

The $D^T D$ matrix can be inverted, especially if the diagonal is weighted in some manner. The $D^T D$ matrix may be diagonally dominant, and it may be assumed to be similar to the identity matrix $I$, which has the convenient inverse that equals $I$, i.e.,

$$\left(D^T D\right)^{-1} \approx (I)^{-1} = I.$$  \hspace{1cm} (5)

In this case, the inverted term in equation (4) is eliminated and we get the transpose solution of equation (2). This confirms our assumption that migration is a transpose type process.

Least squares migration (LSM) is an expensive process and not practical at this time with real seismic data. However it is used in an academic environment and is useful for identifying the features that can lead to an improved migration.

A true inversion was accomplished with Least Squares Migration (LSM). An example is shown in Figure 2, which contains in (a) the reflectivity, (b) a Kirchhoff prestack time migration, and (c) a LSM. These data were created using four source records, and the migration used the same diffraction shapes as the inversion.

![Figure 2: Illustration of a) a reflectivity structure, b) a prestack time migration, and c) a Least Squares Migration](image)

The least squares migration shows the recovery of the high frequency reflectivity, that is not present in the migration. The resolution of the migration could be improved significantly with a simple trace deconvolution.

A Kirchhoff migration may use a single valued function for a diffraction, and this is often considered to be a high frequency whitener. Linear algebra analysis of this type of migration and the LSM processes shows that the migration is still missing the inversion part of a wavelet (Bancroft 2012). The recovery of the frequencies of the wavelet may be approximated with a trace deconvolution.

It should be noted that the LSM recovers the resolution of all dips, but it is not practical for real data at this time.

Deconvolution essentially tries to flatten the amplitude spectrum of the seismic data. However, when we flatten the spectrum we flatten both the signal and the noise. We generally consider the bandwidth of the signal, or reflection energy, to be the area on an amplitude spectrum where the signal is greater than the noise, i.e., when the signal to noise ratio (SNR) is greater than one, SNR > 1. This is illustrated in Figure 3, which contains a cartoon sketch of the amplitude spectrum of seismic data and three levels of noise. The first noise level represents the noise level in the raw data, the second is the reduced noise after stacking, and the third is the noise level after migration. Each time we reduce noise we increase the frequency band where the signal to noise ratio is greater than one, i.e., from $F_s$ to $F_s$, and then to $F_{sn}$. Deconvolution tends to flatten the
Resolution enhancement after migration

amplitude spectrum in each of these bands where a broader bandwidth corresponds to an increase in resolution. Bandpass filters are designed to attenuate the energy when the SNR < 1.

Figure 3 illustrates the improved bandwidth at the higher frequencies. Similar improvements are also obtained at the lower frequencies, and this may be especially important to other inversion processes such as full waveform inversion (FWI) that require very low seismic frequencies that can be successfully mated with the low frequencies from velocity analysis.

A poststack migration benefits from the stacking process and it is common practice to apply a deconvolution at this stage of the processing, i.e. before the migration.

A prestack migration does not stack the data before the migration, thus does not benefit from the improved resolution of deconvolution after stacking but before migration. It tends to have spectral whitening over a bandwidth equivalent that of the “Raw noise” in Figure 3. However, the stack after a prestack migration does have an improved SNR due to the migration and stacking, and will benefit greatly from a deconvolution to whiten the spectrum over the increased bandwidth. Consequently the bandwidth gained through deconvolution after prestack migration is considerably greater than that for poststack migration, and should be considered essential in this case.

Data examples
Two data examples are shown that illustrate the improved resolution of applying deconvolution after migration. The first is from an area in North-East British Columbia in Canada, that were processed by the authors, and the second is an example of data from South America that were processed by a commercial processing center in Calgary, Alberta Canada.

A noisy 2D seismic line was chosen from the project in NE BC, Canada, that used a low-dwell sweep from 1 to 100 Hz, into the vertical component of a 3C phone. The data were processed to a flat datum at the central elevation. Deconvolution, gain recovery, and statics were applied to the prestack data. The data were then prestack time migrated using the Equivalent Offset method (EOM). The migrated section is shown in Figure 4a and the migration followed by a spiking deconvolution is shown in Figure 4b. The increase in resolution with deconvolution after migration is evident.

The amplitude spectra of the data are compared in Figure 5, with (a) the prestack migration, and (b) the same data with a spiking deconvolution applied after the migration. Note the flatter amplitudes of the deconvolved migrated section, and the broader pass band at the lower and higher frequencies.
Resolution enhancement after migration

results are shown in the following sections of Figure 6. Zoom displays are included in Figure 7 that focus on areas of significant improvement in resolution when using a spiking deconvolution after migration.

Deconvolution should be applied after a quality migration. Deconvolution after migration is even more important after a prestack migration.

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Comments and Conclusions

Least-squares migration performs a complete inversion with the wavelet matrix $W$, and is independent of dip, implying that it does the correct deconvolution for horizontal and for dipping events. We have only approximated this process with a trace deconvolution that is appropriate for data with shallow dips. A more elaborate multidimensional deconvolution that considers dip may be more appropriate in structured environments.

A simple spiking deconvolution was applied in the examples. A non-stationary or time varying deconvolution such as Gabor deconvolution (Margrave et al, 2011) should produce greater improvements over a larger time portion of the data.

A clean, noise reduced migration may appear wormy to an interpreter, and noise may be deliberately added it improve its appearance. A deconvolution after the migration will make the section appear less wormy, with the added benefit of increased resolution.

Figure: 6 Seismic sections: a) a prestack migrated section and b) deconvolution applied to the section in (a).

Figure: 7 Zoom of seismic sections: a) a prestack migrated section and b) deconvolution applied after migration.