AVO/AVF analysis of thin-bed in elastic media
Wenyong Pan*, Kristopher A. Innanen, CREWES, Department of Geoscience, University of Calgary

Summary

The Zoeppritz equations, which describe reflection and transmission of plane waves incident on a single interface, are the basis for traditional AVO (Amplitude Variation with Offset) analysis. While Zoeppritz equations are not suitable extensions required to analyze the reflection and transmission responses of thin layers. In this study we simplify the elastic multi-layered medium formulation due to Breshkovskikh, to analyze the three-layer case. The amplitude responses are analyzed with varying incidence angle, dominant frequency and thin-bed thickness. We conclude: (1) thin bed effects are significantly different in elastic media and in acoustic media, especially for small incidence angles; (2) the amplitude response to a decrease in bed thickness is equivalent to that of an increase in dominant frequency; (3) for a highly attenuative target layer, the AVO curve obtained is close to the AVO curve calculated through use of the Zoeppritz equations acting on a single interface.

Introduction

Amplitude Variation with Offset (AVO) technology involves the use of amplitude variations as indicators of rock properties and/or fluids in subsurface reservoirs. It has been widely used in recent years for hydrocarbon detection. Traditional AVO analysis is based on the theory of Zoeppritz (1919), which describes how transmission and reflection coefficients vary with angle when plane wave impinge upon a single interface separating two infinite half-spaces (Bakke and Ursin, 1998).

However, the Zoeppritz equations do not permit quantitative interpretation of the response of thin beds. As a geological layer thins, the reflections from the top and bottom interfaces overlap, and the resulting wave is significantly different from that due to a single interface. Thin bed problems have been studied for several decades. An early examiner of the problem was Widess (1973), who studied reflections from the top and bottom interfaces of a thin layer at normal incidence. Studies of seismic detectability and resolution of thin beds has continued since that time (Kallweit and Wood, 1982; de Voogd and Rootijen, 1983; Gochioco, 1991). The generally accepted threshold for vertical resolution of a layer is a quarter of the dominant wavelength, which is called the “tuning thickness”. Chung and Lawton (1995) derived two analytical expressions for the normal incidence amplitude response, as a function of bed thickness, which are valid for weak reflectivity and for thicknesses below (1/8) λ, where λ is the dominant wavelength. Bakke and Ursin (1998) also analyzed thin-layer tuning effects, and present tuning correction factors for a general seismic wavelet and for both zero-offset and non-zero-offset data. Liu and Schmitt (2003) derived an exact analytical solution to model the reflection amplitude and analyzed the AVO responses of a single thin-bed in acoustic regime. Some scientists also studied the influences of attenuation to AVO responses (Samec and Blangy, 1992; Innanen, 2011).

In this research, we add to the foregoing studies, consider thin bed AVO responses for varying incidence angles, dominant frequencies, and bed thicknesses. A fully elastic scheme is adopted and compared with the AVO predicted with purely acoustic theory. We additionally investigate the influence of a changing Q in the target layer, thereby examining attenuation effects.

Model and Theory

Our geometrical configuration is illustrated in Figure 1: a three-layer model used in this research. The thin-bed (layer 2) is embedded between two infinite half spaces.

A plane harmonic and compressional wave (P-wave) impinges on interface 1, causing both a reflected P-wave and a reflected (converted) shear wave (S-wave), as well as a transmitted P-wave and S-wave. The transmitted P-wave and S-wave produce multiple reflections within layer 2. The total wave response involves overlapping reflected and transmitted waves at point C in Figure 1. This renders the Zoeppritz equations unsuitable as a tool for analysis of the upgoing field due to interface 1. In Figure 1, d is the thickness of thin-bed (layer 2). In this research, we use n= λ/d to analyze the AVO responses when varying d, where λ is the dominant wavelength.

![Figure 1: Three-layer model.](image)

The elastic parameters of the model are listed in Table 1. This model is consistent with a geological model involving shale (layer 1) overlying water-sand or gas-sand (layer 2) (Hilterman, 2001). The amplitude anomaly caused by
substituting water-sand in layer 2 with gas-sand is a gas reservoir indicator.

We simplify the layered elastic medium theory due to Breshkovskikh (1960), to coincide with three-layer elastic media. The displacements and stresses in layer 1 and layer 3 are connected by one coefficient matrix, as shown in equation (1). We assume that layer 1 and layer 3 have the same elastic properties for computational convenience.

We have

$$\begin{bmatrix} u_{x1}^3 \\ u_{z1}^3 \\ \sigma_{zz1}^3 \\ \tau_{zz1}^3 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{bmatrix} u_{x1}^1 \\ u_{z1}^1 \\ \sigma_{zz1}^1 \\ \tau_{zz1}^1 \end{bmatrix}, \quad \text{(1)}$$

where $u_{x1}^1$, $u_{z1}^1$, $\sigma_{zz1}^1$, $\tau_{zz1}^1$, $u_{x3}^3$, $u_{z3}^3$, $\sigma_{zz3}^3$, and $\tau_{zz3}^3$ represent the displacements and stresses in layers 1 and 3 respectively. Using these quantities we obtain:

$$\begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \\ v_{31} & v_{32} & v_{33} & v_{34} \\ v_{41} & v_{42} & v_{43} & v_{44} \end{bmatrix} \begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}. \quad \text{(2)}$$

According to Cramer’s rule, we can obtain four matrices $V_{pp}$, $V_{ps}$, $V_{pp}'$, and $V_{ps}'$ by replacing the first, the second, the third and then the fourth columns of matrix $V$ with $X$. The elements of $X$ and $V$ are listed in Appendix A.

We then express reflection and transmission coefficients as follows (e.g., Keys, 1989):

$$R_{pp} = \frac{\text{det}V_{pp}'}{\text{det}V}, \quad R_{ps} = \frac{\text{det}V_{ps}'}{\text{det}V}, \quad T_{pp} = \frac{\text{det}V_{pp}'}{\text{det}V}, \quad T_{ps} = \frac{\text{det}V_{ps}'}{\text{det}V}. \quad \text{(3)}$$

where $\text{det}V_{pp}'$, $\text{det}V_{ps}'$, $\text{det}V_{pp}'$, and $\text{det}V$ are the determinants of the matrices.

The reflection coefficient equation for thin bed in acoustic media is included in Appendix A of Liu and Schmitt’s (2003). It will be examined for comparison. Frayer (1978) gave the relationship of velocity and attenuation factor $Q$ which can be written as:

$$V \approx V_R + j\left(\frac{V_L}{2Q}\right), \quad \text{(4)}$$

where $V_R$ is the real part of velocity. Then we use equation (4) to substitute the velocity in equation (3) to analyze attenuation effects. This dispersion-free attenuation model removes an important part of the frequency dependence of the resulting reflection coefficients. Inclusion of a more realistic dispersive model is an important future addition.

Comparison of thin-bed AVO responses for elastic and acoustic media

Using equations (3) and Liu and Schmitt’s (2003) equation, it is possible for us to analyze and compare the AVO responses with varying incident angles and thin-bed thicknesses in both elastic and acoustic regimes when the dominant frequency is fixed. To examine the result of variation of $d$, we use $1/n$ values ranging from 0 to 1, which means that the thin-bed thickness lies between $(0, \lambda]$. Figures 2a and 2c show the absolute values of the elastic $P$-P and P-S reflection for model I, which corresponds to a water-sand thin-bed. In contrast, Figure 2(b) shows the absolute reflection coefficients.

Figure 2: AVO responses in elastic and acoustic regimes with varying thin-bed thickness when target layer contains water-sand for Model I at fixed dominant frequency of 30Hz. (a) Absolute $R_{pp}$ in elastic regime; (b) Absolute $R_{pp}$ in acoustic regime; (c) Absolute $R_{ps}$ in elastic regime. The color indicates the absolute reflection coefficients.

We extract vertical profiles from Figures 2a, 2b and 2c, corresponding to $n=1$, $\lambda$, $4$, $6$, $8$, $10$, $20$, $50$ respectively. In order to analyze the reflection coefficients’ variations. This is shown in Figure 3. Comparing Figures 3a with 3b, it can be seen that when $n=4$, in both the elastic and acoustic cases, the absolute reflection coefficients are maximal at normal incidence. However, when $n=1$ and 2, for both the elastic and acoustic cases, the absolute reflection coefficients are all 0 at normal incidence. For $n=4$, the reflections from interfaces 1 and 2 constructively interfere. Whereas, for $n=1$ and 2, the reflections from interfaces 1 and 2...
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destructively interfere. At normal incidence there is no elastic conversion, so the Rps reflection coefficient in Figure 3c is 0.

In Figure 3a, we can see that the absolute Rpp in elastic media decreases, firstly, to a local minimum and then increase quickly to near unity for \( n \geq 4 \). For small angles of incidence, the slopes of the AVO curves decrease with bed thinning. Meanwhile, in Figure 3b, we see that the absolute Rpp in acoustic media increases for \( n \geq 4 \).

Firstly, as indicated by the red boxes, acoustic and elastic P-P reflection coefficients are the same at normal incidence. However, as incidence angle increases, the AVO curves in acoustic and elastic regimes diverge significantly. After substituting water-sand with gas-sand in thin-bed, the acoustic “phase conversion” anomaly may change to a “bright spot” anomaly at about 40 degree. In elastic media, no such change occurs. These differences may cause misunderstanding in fluid prediction, indicating the importance of elasticity in thin bed AVO analysis.

AVF effects of thin-bed

The P-P reflection coefficient is also a continuous function of dominant frequency. We can analyze AVF (Amplitude Variation with Frequency) effects by systematically changing the dominant frequency.

Because gas reservoirs exhibiting low Poisson’s ratios accumulate beneath shale layers exhibiting high Poisson’s ratios, AVO analysis is particularly suitable for the detection and mapping of gas zones (Juhlin and Young 1993). If we substitute the water-sand in layer 2 with gas sand, the amplitude anomalies can assist in the prediction of gas zones. An example is included in Figure 4 for \( n=4 \) (Figure 4a) and \( n=8 \) (Figure 4b).

Figure 3: AVO responses in elastic and acoustic regimes when the thin-bed thickness is \( \lambda/12 \), \( \lambda/14 \), \( \lambda/16 \), \( \lambda/8 \), \( \lambda/10 \), \( \lambda/20 \), and \( \lambda/50 \). (a) Absolute Rpp in elastic regime; (b) Absolute Rpp in acoustic regime; (c) Absolute Rps in elastic regime. The black-dash lines are the AVO curves when thin-bed thickness is “tuning thickness”. The red-dash lines are calculated based on Zoeppritz equations.

Figure 4: AVO analysis when substituting water-sand in thin-bed with gas-sand for \( n=4 \) (a) and \( n=8 \) (b). The dashed lines are AVO curves in acoustic regime and the solid lines are AVO curves in elastic regime. And the black and blue colors mean water-sand and gas-sand containing in layer 2 respectively.

Figure 5 illustrates the AVF/AVO effects for elastic media at four fixed thin-bed thicknesses: 10m, 20m, 30m, and 50m. It can be seen that at fixed frequency with increasing incident angle, P-P reflection coefficients become smaller (the red-dash lines). And at fixed incident angle with increasing thin-bed thickness from 10m to 50m, Rpp curves experience greater fluctuations (the blue-dash lines).

Figure 6 shows Rpp with varying thin-bed thickness and dominant frequency at fixed incident angle of 20 degree (a) and 50 degree (b). We can see that Figure (a) and (b) are symmetric with the green lines. What’s more, the variations of Rpp are the same when increasing the frequency indicated by the blue arrows and decreasing the thin-bed thickness indicated by the red arrows. So, it is concluded that the amplitude responses of thin-bed thinning is equivalent to that of dominant frequency increasing at fixed incident angle.
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Figure 6: Elastic Rpp with varying bed thickness and dominant frequency, given a gas-sand target layer (model I). Two fixed incident angles of (a) 20° and (b) 50° are used. The color indicates Rpp.

Attenuation effects to AVO responses of thin-bed

We can simulate the energy absorption function by incorporating the attenuation factor Q, which describes the energy change of seismic wave within one wavelength (only the target, layer 2, is considered attenuative in this study).

Figure 7: Rpp in elastic regime with varying Q (Q=1,3,5,10,30,50,100) for n=1/4 (a) and n=1 (b) at fixed frequency of 30Hz. The bold-solid lines are Rpp with no attenuation and the red-dash line and green-dash line are calculated by Zoeppritz equation with no attenuation and with Q=1 respectively.

Figure 7 shows Rpp over model I, with Q varying from 5 to 100, with water-sand in layer 2, and n set at 1/4 (Figure 7a) and n=1 (Figure 7b). As expected, with increasing Q, the AVO curves approach their expected values in the elastic limit. In Figure 7b, decreasing Q is seen to have a smoothing influence on the AVO curve. If the thin-bed is very highly attenuative, the wave energy will be absorbed mostly and the reflections from interface 1 cannot reach Interface 2. In this pathological condition, the AVO curve calculated by the three-layer equation is expected to approach that calculated through the (single interface) Zoeppritz equations.

Conclusion

Inclusion of elasticity in AVO analysis of thin-beds is important, as acoustic approximations can cause erroneous changes in AVO anomaly types. The amplitude responses associated with bed-thinning is equivalent to that associated with an increase in dominant frequency. For very highly attenuative target layers, the AVO curve obtained by our three-layer approach approaches that calculated through the Zoeppritz equations.

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Appendix A

For matrices of V, X and C in equation (2), we have:

\[
C = \begin{bmatrix}
\sin\theta_1 & \cos\theta_1 & -\sin\theta_1 \\
-\sin\theta_1 & \cos\theta_1 & \sin\theta_1 \\
\mu_{12}\sin2\gamma_1 & -\mu_{12}\cos2\gamma_1 & 2\mu_{12}\gamma_1 \\
\mu_{31}\sin2\gamma_1 & -\mu_{31}\cos2\gamma_1 & 2\mu_{31}\gamma_1 \\
\mu_{32}\sin2\gamma_1 & -\mu_{32}\cos2\gamma_1 & 2\mu_{32}\gamma_1 \\
\mu_{33}\sin2\gamma_1 & -\mu_{33}\cos2\gamma_1 & 2\mu_{33}\gamma_1 \\
\end{bmatrix}
\]

where \( \theta_1 \) is the reflection or refraction angle of P-wave, \( \gamma_1 \) is the reflection or refraction angle of S-wave, \( \mu_1 \) and \( \beta_1 \) are the P-wave and S-wave velocity, \( \mu_1 \) is the shear modulus, \( \rho_1 \) is the density and \( Z_1 = \mu_1/\rho_1 \). \( H = \mu_1\cos\theta_2/\mu_2 \). \( R = \mu_2\cos\theta_2/\beta_2 \). \( \omega \) is angular frequency, \( d \) is thickness. \( \lambda_1 \) is Lame coefficient, \( \tau_1 = \kappa_1\cos\theta_1 \), where \( \kappa_1 \) and \( \kappa_2 \) are the wave-numbers of P-wave and S-wave, \( \sigma_1 \) and \( \tau_1 \) are the horizontal and vertical components of P-wave number respectively, \( \sigma_2 \) and \( \tau_2 \) are the horizontal and vertical components of S-wave number respectively, \( T = \lambda_2k_2 + 2\mu_2\tau_2 \), \( L = \tau_2 - \sigma_2 \) and \( d = \sigma_1 = \sigma_2 = \sigma_2' = 1,2 \).