Introduction

Computing shear wave velocity models for the near-surface is one of the most difficult tasks in the processing of converted wave data. One of the reasons for this is that multicomponent seismic data are usually acquired using P-wave sources. This prevents us from using S-wave critical refractions to compute near-surface velocity models, as is usually done in the processing of vertical-component data.

More recently, a technique based in interferometric principles has been developed for redatuming applications. Henley (2012) introduced a raypath-consistent method for solving S-wave statics using an interferometric approach. In this method delay times caused by the near-surface are retrieved by cross-correlating reflection amplitudes in the radial trace domain. Moreover, Cova et al. (2014) showed how the cross-correlation functions retrieved during this processing exhibit variations that follow the changes in the structure of the near-surface.

In this work we show how raypath-dependent traveltimes can be used to invert a velocity model for the near-surface. Here, we propose a reflection-based solution where the difference in traveltimes caused by changes in the structure of the near-surface between two receivers is used as the input. In this case, the problem becomes highly nonlinear and the derivatives needed for implementing a Newton’s inversion become very complicated. Instead, we will show a solution based on a simulated annealing algorithm that will provide us with a solution for this problem.

Traveltime differences between receiver locations

Figure 1 shows the geometry used for computing the upgoing travelt ime recorded at a fixed receiver location in terms of the raypath angles in both media. The model consists of a low velocity layer with a dipping base with angle $\phi$, overlying a medium with a flat base from which conversions are produced. Source-side near-surface effects are neglected assuming that P-wave statics have been removed. Therefore, the receiver-side travelt ime for this geometry can be computed as,

$$t(\theta) = \frac{Z_1}{V_1 \cos(\theta_1)} + \frac{Z_0 \cos(\phi)}{V_0 \cos(\theta - \phi)} \left( 1 - \frac{V_0 \cos(\theta)}{V_1 \cos(\theta_1)} \right)$$  \hspace{1cm} (1)

where $V_1$, $Z_1$ and $\theta_1$, are the velocity, depth and propagation angle in the underlying medium and $V_0$, $Z_0$ and $\theta$ are the parameters that control the propagation in the near-surface.

The first term in equation 1 represents the travelt ime from the reflector to the surface as if there were no low velocity layer present. Hence, the second term contains the near-surface effect on the upgoing travelt ime.

To isolate the second term in equation 1 we can subtract traveltimes recorded at two different receiver location and measured with the same raypath angle. If the reference interface is flat and the velocity of the underlying medium is laterally homogeneous, the difference in travelt ime between the two receiver

![Figure 1](image-url)  

Figure 1 Geometry used to compute receiver-side traveltimes from a reference interface to a fixed receiver location on the surface.
locations is a function of the changes in the structure of the near-surface layer. This is also true for a horizontally stratified medium as depicted in figure 2. In this model, for a fixed raypath angle, the travelt ime contribution of each layer is the same independent of the receiver location. Since the near-surface is the only layer assumed to have structure, the difference in traveltimes only responds to differences in the structure of this layer. This travelt ime difference can be computed as,

$$\Delta t(\theta) = \frac{\Delta Z_0 \cos(\phi)}{V_0 \cos(\theta - \phi)} \left( 1 - \frac{V_0 \cos(\theta)}{V_1 \cos(\theta_1)} \right)$$

(2)

This equation can also be written in terms of vertical ($q$) and horizontal slownesses ($p$) as,

$$\Delta t(p) = \frac{\Delta X \tan(\phi)}{V_0^2 (q + p \tan(\phi))} \left( 1 - \frac{V_0^2 q}{V_1^2 q_1} \right)$$

(3)

where $p$ and $q$ are defined by,

$$q = \frac{\cos(\theta)}{V_0} \quad \text{and} \quad p = \frac{\sin(\theta)}{V_0}$$

(4)

The main problem with this solution is the introduction of terms that depend on $\theta_1$ such as $q_1$. For a flat near-surface, this angle could be easily computed by using Snell’s law directly. However, due to the presence of dip at this interface the rayparameter values are going to differ. The magnitude of this difference depends on the velocity of both media and the dip of the interface.

The dependence of $\theta_1$ on the model parameters makes this inversion problem highly nonlinear. Moreover, Figure 3 shows a plot of the objective function ($L_2$ norm of the data misfit) for the inversion of equation 3 considering a fixed dip angle. There we can see, in orange, different series of local minima that may be a challenge for a gradient-based inversion method. Additionally, the partial derivatives of equation 3 are very complex due to the trigonometric functions and division operations involved. To avoid these problems, we chose a global optimization strategy using simulated annealing to invert for the parameters that define the structure of the near-surface.

**Simulated annealing inversion**

To initialize the simulated annealing inversion a quadratic temperature decay was used. Additionally, the probability distribution of the variables for which we are trying to invert is assumed to be normal. This means that in each iteration trial parameters are drawn from a Gaussian curve centred at the value of the previous iteration. The trial parameters are accepted or rejected following a metropolis rule included in
Figure 3: Objective function for a fixed dip value.

Figure 4: Evolution of the parameters involved in a simulated annealing realization (black lines). The red lines indicate the true parameters of the model.

The algorithm. This algorithm follows the basic steps explained by Kirkpatrick et al. (1983) to find the minimum of a function with many degrees of freedom.

Figure 4 shows one realization of the simulated annealing inversion. There we can see the progressive decay of the temperature and the objective function values. It is important to note that for the first half of the iterations the objective function displays a high variability. This is the part of the algorithm where trial solutions are accepted even if they lead to higher objective function values. In the bottom three panels we can also see how the parameter values used in every iteration also show a high variability. This is evidence of how the randomness that dominates the code at high temperatures allows it to explore a wide area of the model space.

In the later portion of the iterations shown in figure 4, the objective function variability decreases. During the last thousand iterations, only trial parameters that lead toward a lower value of the objective function are accepted. In the model parameter plots we can see how those trial values begin to converge toward the actual model parameter values (red lines in figure 4).
The downside of this method is the fact that it may provide different solutions for each realization. To avoid this problem one of the options is to perform several realizations and then use a statistical metric to retrieve the best solution. Figure 5 shows a histogram of the results for one hundred realizations. There we can see that the distributions are centred around the actual model parameter. In this case, we chose the median of the distribution as the best value for the estimation due to its robustness against outliers. The results show that the inverted parameters are very close to the actual solution.

Conclusions

When dealing with converted-wave data, additional parameters come into play in the problem of characterizing the near-surface. The method proposed in this study uses the differences in traveltimes between receivers in order to retrieve the changes in the structure of the near-surface. The complex topography of the objective function for this problem, and the complexity of the partial derivatives, led us toward a non-gradient based inversion method. The simulated annealing algorithm used here, proved to be effective in recovering the true parameters of the model.

Traveltime differences can be retrieved by using interferometric principles. Thus, by using the delay times retrieved by cross correlating receiver gathers in the rayparameter domain, we should be able to get the information needed to characterize the near-surface. Further analysis using synthetic and real seismic data is needed to confirm this idea.

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References