Full waveform inversion: a synthetic test using PSPI migration
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Summary

Full Waveform Inversion (FWI) is a tool used for estimating P-wave velocity. We demonstrate, for a 2D acoustic marine synthetic case, that the methodology can be applied with reasonable results; using PSPI migration with a deconvolution imaging condition plus a line search or a direct computation to calculate the step length to invert for P-wave velocity. We analyze the importance of the initial velocity model for convergence and obtain good responses for velocity models with smoothed flat layers, based on simulated wells in the real velocity model. Our results suggest that we can obtain a high resolution inversion if an initial model is created by interpolation of two or more wells. A test using the Marmousi 2D model shows encouraging results but more tests are necessary.

Introduction

Full waveform inversion, or FWI, is an inverse method that uses the least-square technique to minimize the objective function (the difference between a real seismic data and a synthetic seismic data) by updating the initial velocity model until convergence (Margrave et al., 2011). The update is obtained from a reverse-time migration of the data residuals.

Full waveform inversion was proposed in the early 80s (Pratt et al., 1998) but the technique was considered computationally too expensive. Lailly (1983), and Tarantola (1984) simplified the methodology by using the steepest-descent (or gradient) method in the time domain to minimize the objective function without calculating explicitly the partial derivatives. They computed the gradient of the misfit between real and synthetic data by pre-stack migration of the data residuals with reverse-time migration (RTM). Pratt et al. (1998) developed a matrix formulation for the full waveform inversion in the frequency domain and showed how to compute the inverse of the Hessian matrix (the step length for convergence in the FWI) by calculating the full-Hessian (Gauss-Newton method) or by an approximation of the Hessian (quasi-Newton). The Hessian computation is one of the study areas for the FWI improvement, as it has a large matrix and its computation requires substantial computer power. An overview of the FWI theory and studies were compiled by Virieux and Operto (2009). Margrave et al. (2010) used well control to calibrate the model update (known as step length) and pre-stack wave-equation migration (Ferguson and Margrave, 2005) instead of the RTM, so the iterations are done in the time domain; but only selected frequency bands are migrated, using a deconvolution imaging method (Margrave et al., 2011; Wenyong et al., 2013) as a better reflectivity estimation.

Warner and Guasch (2014) use deviation of the Wiener filters, of the real and estimated data, as the object function with great results.

In this work we investigate the FWI with a similar methodology proposed by Margrave et al. (2010), but not using well control as a model calibrator. The velocity residual is calculated using wave-equation migration with a deconvolution image condition, so the inversion is done in the time domain and only selected frequency bands are migrated. With the objective to find a better procedure, selection of the frequency bands to migrate is also studied in this paper.

Theory and Methodology

FWI is a least square methodology (Tarantola, 1984) that aims to find a velocity model that minimizes the objective function:

\[
\Phi (m) \equiv \|\Delta d\|^2 = \|u - d\|^2
\]  

(1)

where \(m\) is the velocity model, \(u\) is synthetic data generated by forward modeling of model \(m\), \(d\) is the observed data and \(\Delta d\) is the data residuals. Minimization of equation (1) is done by derivation of the objective function around a velocity perturbation, i.e., deriving \(\Phi (m_0 + \Delta m)\) and equalizing to zero (Margrave et. al., 2011; Virieux and Operto, 2009). The solution leads to a model update in the form:

\[
m_{n+1} = m_n + \alpha_n H^{-1} g_n
\]  

(2)

where: \(g\) is the gradient, or the back-propagated time-reversed data residuals, \(H\) is the Hessian matrix (the second derivative of the misfit function), \(\alpha\) is a scale factor determined by using a line search (searching for a scale factor that minimizes the objective function) and \(n\) is the iteration number. Equation 2 suggests that the correct velocity model can be determined iteratively by updating the model with the migrated data residuals having proper gain. For each iteration, we must do forward modeling at each shot point using the current velocity model, and repeat the process until convergence. Margrave et al. (2011) suggests that the Hessian calculation can be avoided by applying a deconvolution imaging condition in the migrated data residuals. This gives a better estimation of the reflections. Equation 2 can be reduced to:

\[
m_{n+1} = m_n + \alpha_n g_n
\]  

(3)

Pica et al. (1990) computed the scale factor \(\alpha\) directly by minimizing the objective function:

\[
S (m) = S(m_n + \alpha_n g_n)
\]  

(4)
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The result for the scale factor is:

\[ \alpha_n = \frac{[F_ng_n]^T[d_n]}{[F_ng_n]^T[F_ng_n]} \]  \hspace{1cm} (5)

where \( F \) is a linear operator that takes the derivative of \( g \). This is solved by doing a single forward modeling in a model with a perturbation \( \epsilon \) over the gradient, or \( m = m_n + \epsilon g_n \).

Examples

The tests are done using synthetic data. We created a velocity (Figure 1a) to be used in the forward modeling and used synthetic shots as real shots. The simulation uses 96 shots at 100m spacing and 30m depth, and 401 receivers at 10m spacing on the surface. The shot gather has a split-spread geometry. The dominant frequency of the wavelet is 10Hz.

For the first test, the initial velocity model (Figure 1b) is a smoothed version with a Gaussian window of the real velocity model. This model is used in the forward modeling of 96 synthetic shots (one for each shot point) and the results compared to the real shots to obtain the data residuals. The data residuals are each migrated with PSPI migration with a deconvolution imaging condition. The migration is applied to selected frequency bands, starting from low frequencies to avoid a local minimum (Pratt et al., 1998; Plessix et al., 2010), and increasing to higher frequencies for higher iterations. A mute is applied to avoid far offset artifacts. All the migrated residuals are stacked to generate the gradient, but results in small values. A scale factor is determined by a line search, which consists of applying different scale factor values to create temporary velocity models, then do forward modeling (only on the shot point in the center of the model), calculate the differences to the real shot, and chose the scale factor that yields the lowest difference, or least error. The first update is shown on Figure 1c. Layer interfaces started to show up, and after 60 iterations (Figure 1d) the velocity model is inverted with high resolution. This result was expected since we started with an initial guess very close to the global minimum.

An initial velocity model (Figure 2b) based on a sonic log simulation (dashed line on Figure 2a) was used in the second test. After the first iteration it is possible already to note some interfaces appearing, such as the high velocity body at the bottom. A high resolution velocity model resulted from 55 iterations, correctly inverting the layer interfaces. However, even by inverting the borders of the high velocity body, we could not ‘fill’ it with the correct velocity.

The Marmousi model (Figure 3a), used in the third test, utilized 102 shots placed at the surface. The only difference in the processing is that the gradient is smoothed in each iteration. The estimate model (Figure 3b) is a smoothed version of the Marmousi model. After 18 iterations (Figure 3c) the inverted model has good resolution, showing some of the major structures in the model. After 66 iterations (Figure 3c) the resolution of the shallow part (above 2000m) is increased but the deep part loses resolution. This can be explained higher iterations inverting higher frequencies that are not present in the real seismic data. However, the results are encouraging.

Figure 4 shows the results of inversion if the scale factor is computed using equation 5, which requires less computer power. For Figure 4c one scale factor (scalar) was used per iteration, selecting the shot in the center of the survey as the pilot. Figure 4d is the inversion of a vector of scale factors, where equation 5 was used for all the shots. Both show similar quality, comparable to the line search result, but use of the vector scale factor improves the borders of the model. The processing time for both cases (scalar and vector) is very similar since parallel processing is used in the forward modeling step.

Conclusions

A full waveform inversion, using PSPI migration, with a deconvolution imaging condition, when applied to synthetic data resulted in high resolution velocity models when a simple velocity model was used. The result improves when the initial guess is close to the real answer, but we obtained very good results using a starting model based on a single sonic log simulation. This suggests that an initial model resulting from an interpolation of information from two or more wells can be used to provide a starting model closer to the real answer.

The resulting velocity model for the Marmousi model is impressive but is constrained by the absence of higher frequencies in the simulated data that precludes proper inversion of thinner layers.

The scale factor computed by least squares proved to be a good tool, with results in quality comparable to the line search method, but this step required fewer forward modeling iterations to converge, and hence requires less computer power. The vector scale factor shows higher stability in the borders of the model when compared to use of a scalar scale factor.

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Figure 1: a) Real velocity model, b) initial model, c) first update, and d) iteration 60.

Figure 2: a) Real velocity model, b) initial model (based in a well log simulation), c) first update, and d) iteration 55.
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Figure 3: a) Marmousi velocity model, b) initial model, c) first update and d) iteration 66.

Figure 4: a) Real model, b) initial model, c) inversion using a scalar scale factor and d) a vector scale factor.