Borehole Reverse Time Migration in Anisotropic Medium

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Summary

Reverse time migration (RTM) incorporating with the borehole environment is now applied in borehole acoustic reflection imaging. However, RTM in borehole environment with anisotropic media is yet to be discussed. In this paper, borehole RTM in vertical transversely isotropic (VTI) media was proposed to image near borehole structures. The second order time-space domain staggered-grid finite difference is used in the forward and backward extrapolation. The harmonic average equation is applied to solve the elastic parameters discontinuity between the borehole and formation. The benchmark of finite difference solution with analytical solution combining with the snapshots comparison with borehole and without borehole condition prove the precision of borehole RTM. The imaging results of synthetic fault models with different dip also prove our borehole RTM. For applications in real data, the long wavelength equivalent method is used in the determination of elastic parameters. Finally, the borehole RTM has been applied to the real data from East Asia. The real data processing result corresponds with core samples and the FMI data.

Introduction

Acoustic reflection imaging logging, first dating back to 1989 when Hornby (Hornby, 1989) presented the data processing and imaging methods for sonic tool BARS(borehole acoustic reflection survey) developed by Schlumberger, takes the leaky energy as incident waves, and probes waves reflected from near borehole fracture and micro-structures. By analyzing received waveform signals, we can obtain the structure information of nearby formations and evaluate small subtle and fractured reservoirs.

On the other hand, with an increasing interest in unconventional subtle reservoirs like fractures and vugs, anisotropy becomes an inevitable issue to tackle with both in seismic exploration and acoustic well logging. Though, attempts have been made to utilize borehole acoustic measurements to obtain an image of geological structures away from the borehole (Fortin et al., 1991; Coates et al., 2000; Li et al. 2002; Tang 2003, 2004; Tang and Patterson, 2009; Bolshakov and Patterson, 2011), under the circumstance of anisotropic medium, those migration and imaging methods used in isotropic medium will definitely give rise to imaging problems and positioning errors. Reverse time migration (RTM), first proposed by Whitemore et al. (1983) in the 53rd SEG conference, has the ability to migrate any type of multiples (surface and internal) to their correct location in the subsurface, can handle multi-pathing, image turning waves and steep dips. Although the application of RTM in acoustic reflection imaging logging was developed (Li, 2014), its validity in anisotropic media is yet to be discussed. However, as is known to all, the acoustic wave field propagation is greatly influenced by anisotropy commonly existing in the subsurface outside borehole, which in turn inevitably impedes the precision of borehole RTM in isotropic medium.

Theory and methodology for borehole RTM in VTI media

For formation whose properties are transversely isotropic (TI), particularly in vertical transversely isotropic (VTI) media, the velocity-stress elastic equations in 2D VTI media can be described as:

\[
\begin{align*}
\rho \frac{\partial V_x}{\partial t} &= \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}, \\
\rho \frac{\partial V_z}{\partial t} &= \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} , \\
\frac{\partial \tau_{xx}}{\partial t} &= C_{11} \frac{\partial V_x}{\partial x} + C_{13} \frac{\partial V_z}{\partial z} , \\
\frac{\partial \tau_{zx}}{\partial t} &= C_{13} \frac{\partial V_x}{\partial x} + C_{33} \frac{\partial V_z}{\partial z} , \\
\frac{\partial \tau_{zz}}{\partial t} &= C_{44} \frac{\partial V_x}{\partial x} + C_{44} \frac{\partial V_z}{\partial z}.
\end{align*}
\]

where \( \rho \) is density, \( V_x \) and \( V_z \) are velocities in \( x \) and \( z \) direction respectively, and \( C_{ij} \) is elastic parameters.
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To discretize the above equation, the 2nd order finite difference (FD) on temporal and 4th order staggered-grid on spatial derivatives is used. As shown in Figure 1, as a staggered-grid difference method (Madariaga, 1976) both in time and space, both velocity and stress iterative fields are put in one iterative grid space, where, $\tau_{xx}$ and $\tau_{zz}$ denote the normal stress components, whereas $\tau_{xz}$ and $\tau_{zx}$ ($\tau_{xz} = \tau_{zx}$) denote shear stress components; $v_x$ and $v_z$ are velocity components towards $x$ and $y$ direction respectively. For every iteration in staggered-grid space, the previous calculated point-in-time is used in the simulation iteration when we calculate its current derivative. The calculated results are then fully used and thus the precision of iteration is improved.

Therefore, the iterative equations of velocity-stress elastic equations in 2D VTI media can be described as:

$$
\frac{\rho}{\partial t} \frac{\partial v_x}{\partial x} + \frac{\rho}{\partial z} \frac{\partial v_z}{\partial z} = v_{x+\frac{1}{2},j} - v_{x-\frac{1}{2},j} + \frac{1}{\rho \Delta x} \left( \tau_{xx,\Delta t}^{n+1} - \tau_{xx,\Delta t}^{n+1} \right)
$$

(2)

$$
\frac{\partial \tau_{xx}}{\partial t} = C_{11} \frac{\partial v_x}{\partial x} + C_{13} \frac{\partial v_z}{\partial z} + \frac{\Delta t}{\Delta x} \left( \tau_{xx,\Delta t}^{n+1} - \tau_{xx,\Delta t}^{n+1} \right)
$$

(3)

where $\Delta x$ and $\Delta z$ are constant grid spacing, $\Delta t$ denotes time step, $n$ is the index of time steps and $i, j$ are index of each grid node respectively.

Since in the borehole environment, the source is placed in the borehole, which leads to the elastic parameters discontinuity between the borehole and formation outside when the source is fired, especially for the shear module. This parameter discontinuity severely hinders the accuracy of the staggered FD method introduced above. Therefore, average method is applied in this paper to eliminate this inaccuracy. Take elastic component $C_{44}$ as an example, its harmonic average equation can be determined,

$$
\frac{4}{C_{44,\Delta t}} = \frac{1}{C_{44,\Delta t+1/2,\Delta z+1/2}} + \frac{1}{C_{44,\Delta t-1/2,\Delta z-1/2}} + \frac{1}{C_{44,\Delta t+1/2,\Delta z-1/2}} + \frac{1}{C_{44,\Delta t-1/2,\Delta z+1/2}}
$$

(4)

Figure 2 shows comparison of wave field propagation snapshots when harmonic average equation isn’t applied in determination of elastic parameters (above) and when it is applied (below). As we can see, the total grid size is 260*320, and the center of wellbore is located at point 40 along the horizontal direction. When the travel time is 8e-4 s after the source is fired at point (60, 40) inside borehole, the wave field snapshots are shown in the left column of Figure 1, where no differences are detected compared the above snapshot with the below one. However, when the propagating time increases shown in middle column of Figure 2, the difference emerges and becomes more obvious in the right column of Figure 2. The major difference is resulted from the instability error occurs at the interface of borehole and formation, which can be found in the oval-shaped area. Besides, the wave fields in above snapshots on each side of the borehole don’t perform a symmetric shape, which can be detected in rectangle area. In the snapshots when harmonic average equation is applied in the calculation of elastic parameters, however, there is no instability error detected at the interface of borehole and formation and the wave fields on each side of the borehole are almost symmetric.

Synthetic data processing

In this section, the synthetic data is used to further testify our borehole RTM. The model is a fluid filled borehole model with an fault like interface lies on the one side of the borehole with a dip of 0, 15, 30, 45, and 60 degree, respectively.

![Figure 1: Layout of velocity and stress iterative fields in staggered-grid space.](image-url)
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Figure 2: Comparison of wave field propagation snapshots when harmonic average equation isn’t applied in determination of elastic parameters (above) and when it is applied (below).

Figure 3 shows the synthetic model with a dip angle of 45 degree. The model is 15 m high by 10 m long and the diameter of the borehole is 0.2 m. The borehole is located at the lateral depth of 1 m and denoted in blue colour. The rock formation in red stands for medium I and that in yellow for medium II. The acoustic reflection logging tool is designed to have a source-receiver spacing of 3 m, and thirteen receivers separated at 0.15 m intervals. The borehole and formation parameters are given in Table 1.

Table 1 Parameters of fault-like model away from the borehole

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>ρ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow</td>
<td>23.87</td>
<td>9.79</td>
<td>15.33</td>
<td>2.77</td>
<td>2.5</td>
</tr>
<tr>
<td>Borehole</td>
<td>1.5’2</td>
<td>1.5’2</td>
<td>1.5’2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Red</td>
<td>40</td>
<td>13.55</td>
<td>40</td>
<td>13.225</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Mimicking the field data acquisition, we move the source point from depth of 2 m to 10.85 m while firing total 60 shots at a depth interval of 0.15 m. 2D finite difference modeling is used to acquire the array acoustic full waveform at each source point. Figure 3 (left) shows the received full waveform for the borehole and formation model in Figure 3. The total length of recording time is 2 microseconds. As shown in Figure 3 (left), because of the slow formation outside the borehole, the first arrived signal is leaky P wave signal. The water wave (whose velocity is about 1500 m/s) signal propagating in the borehole fluid is behind the P wave signal. The signal behind the water wave is Stoneley wave. There is no shear head wave signal in the slow formation. It is interesting that the water wave is quite distinctive in this 2D simulation. However, it is yet hardly observed in 3D simulations. Figure 4 (Right) shows different reflections received: the signals in the upper and lower circles are P-P reflections from lower and upper straight vertical interfaces in Figure 3. The signal in the yellow rectangular area is P-P reflection from the interface with a dip angle of 45 degrees. The signals in red rectangular areas are P-S and S-P converted reflections, respectively. Usually, the P-S and S-P reflections will be overlapped when the interface vertical or when the dip angle is very small. However, when the dip angle of the interface is big enough, they will separate into two signals. The bigger the dip angle is, the further they will separate. And the signal in green rectangular area is S-S reflection. It’s noticed that the shear and converted reflection signals can only be recorded by a few receivers. This is because the dip angle of the interface imposes restrictions on the receivers to record shear and converted reflections in VTI media. The bigger the dip angle is, the fewer receivers will receive these signals. This is also the reason why there is no

Figure 4: Recorded full waveform (Left) and correspondent reflections (Right) after mode waves are removed when the dipping angle is 45 degree.
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shear reflections from upper and lower straight interfaces. After the reflections have been acquired, the finite difference borehole RTM can be applied to process the data. The space step of FDM is $\Delta x = \Delta z = 0.075 \text{ m}$, and the time step $\Delta t = 10^{-6} \text{ sec}$ satisfies the stability condition. Figure 5 shows the imaging results when dipping angle is 0, 15, 30, 45 and 60 degrees, respectively. When the dipping angle is smaller than 60 degrees, imaging results show enough details of the structure outside borehole. However, when dipping angle is 60, the dip interface is missing in the imaging result. This is because the dip angle is too sharp for the receivers to record reflection signals.

Figure 5: Imaging results when dipping angle is 0, 15, 30, 45 and 60 degree, respectively.

Field data processing

The field data used in this paper was acquired by an acoustic reflection imaging instrument from East Asia. The source to the first receiver distance of this tool is 3.6276 m. There are 8 receivers at an interval of 0.1524 m. The time interval recorded by receivers is 10 microsecond, and the total recording time is 8 ms. The basic processing flow is the same as that for the fault-like model. The core samples extracted from interval X830- X870 show that the formation is mainly composed of shaly limestone, which shows a transversely isotropic characteristic. Figure 5 (left) shows the acquired well log data, reflections and the analyzed P and S wave velocities. The first track shows the gamma ray. The second track is the depth. We processed the data from vertical depth of X830 m to X890 m containing total 395 shots. Full waveforms are shown in the third track. The next track shows the reflections extracted from the recorded waveforms. In this paper we apply the multi-scale slowness-time-coherence (MSTC) method (Tao et al. 2008) for reflection wave extraction. As we can see, there are some distinctive reflections in the upper zone whose lithology is shaly limestone. And the last track shows the P and S wave slowness curves. In order to derive the elastic parameters, expressions derived by Schoenberg (1983) for the long wavelength anisotropic equivalent moduli for a VTI material were used in this paper. Finally, borehole RTM is applied to get the imaging result, shown in Figure 5 (right). From the imaging result, the reflection signals emerge all along the depth interval, whereas the true effective reflectors should be in the depth ranges where the imaging energy distinctively exists. The structures are about 5 m way from the borehole.

Figure 6: Field data from East Asia (Left) and the imaging result (right).

Conclusions

We have developed an effective data processing flow and corresponding code for acoustic reflection image logging in the VTI medium. This paper also developed the RTM accommodating the borehole environment for near borehole imaging in VTI medium to fix imaging problems and positioning errors in anisotropic medium. In this borehole RTM algorithm, the harmonic average equation is applied to solve the elastic parameters coupling at the boundary of the borehole fluid and the solid formation. The synthetic data processing for the fault-like models shows that this borehole RTM is capable of imaging near the borehole structures. For applications to real field data, the long wavelength equivalent method is adopted in the determination of elastic parameters. Finally, this algorithm has been applied to the real field data from East Asia. The processing results correlate well with core samples and the FMI data.

Acknowledgments

This study is supported by the NSFC (Projects NO.50674098 and NO. 41174118) and one of the major state S&T special projects (NO. 2011ZX05020). The first author, Junxiao Li, would like to thank the Scholarship support from Shell.