Using well logs to estimate the effect of fine layering on Q estimation
Gary F. Margrave*
CREWES, University of Calgary

Summary
The long-standing prediction that a seismic wave propagating in a finely layered earth model displays an apparent attenuation is investigated. Called stratigraphic filtering, this effect looks much like constant-Q attenuation and adds to intrinsic attenuation to produce effective attenuation. Using a 1D synthetic seismogram algorithm, the seismic response is calculated for a sequence of finely layered models derived from well logs with assigned Q values. The models all have a finely-layered Q structure, representing intrinsic attenuation, derived from measured density and sonic logs by an empirical relation. The model properties are all sampled at 0.5 m intervals and averaged into constant thickness layers. Using 0.5 m layers, when the Q value is carefully measured using the spectral-ratio technique, the measured Q is always lower than that expected from the specified model. In a series of experiments in which various physical effects are turned off and on again, it is demonstrated conclusively that this Q bias is due to internal multiples. Using a series of models derived from the same logs but with progressively thicker layers (each model has constant thickness layers and each is sampled a 0.5m) it is demonstrated that there is significant measurement bias for layer thicknesses less than 10m but for thicknesses greater than this the bias disappears. The feasibility of estimating stratigraphic Q from such experiments and using these measurements to correct measurements from field data is suggested.

Introduction
In 1971, O’Doherty and Anstey published their now famous paper in which they showed that fine-layering in a stratigraphic sequence causes a seismic wave to attenuate as though there were Q involved even if each layer is perfectly elastic. They further showed that transmission losses alone would attenuate the primary arrival to near undetectability but the effect of short-path internal multiples is to reinforce the primary while selectively attenuating its higher frequencies just as would happen in a constant-Q medium. This effect has come to be called stratigraphic filtering. The implication is that, in a finely-layered, visco-elastic medium, there will be two types of attenuation: the intrinsic Q caused by the internal friction in rocks as modelled by constant-Q theory, and the effective Q caused by the fine layering. Any attempt to measure Q will necessarily measure the combined effect. Stratigraphic filtering can be simulated using a synthetic seismogram algorithm capable of handling a very finely layered model, and preferably one which can vary the Q value in each layer. Such an algorithm was published by Ganley (1981) and is used here with some straightforward modifications that allow various features to be turned off and on.

Ganley’s synthetic seismogram algorithm is a propagator matrix method that assumes 1D but can accommodate any number of layers with unique values of velocity, density, and Q. Additionally, receivers (and sources) can be placed at any depth so that a synthetic VSP can be generated which is the best seismic experiment for Q measurement. The propagator matrix technique produces a very accurate response over a broad frequency range which is important for the analysis of Q. The method also calculates all possible multiples. The anelastic behaviour follows the constant-Q model of Kjartanssen (1979). The restriction to 1D means that only p-waves are simulated and there is no wavefront spreading but the computation is sufficiently fast that many thousands of layers can be easily simulated. Earth models with such layer numbers can be prescribed velocity and density values using measurements from well logs; however, no such information regarding Q is available. Therefore, Q values were prescribed by assuming a linear relationship between velocity and a Q_v and a similar linear relation between density and a Q_ρ. Then layer Q’s were defined from $Q^{-1} = Q_v^{-1} + Q_ρ^{-1}$ for each layer. The precise specification of the assumed linear relations is not important here, the parameters were simply chosen to give low Q values (near 20) for low velocities and densities and high Q values (near 200) for high velocities and densities. What is important is that by this mechanism a finely layered intrinsic Q profile, with detail derived from and similar to both the velocity and density profiles, can be prescribed. This then permits a study of the ability of Q estimation methods to resolve this very detailed Q profile.

Q estimation
The spectral-ratio technique (Bath, 1974) was chosen due to its common familiarity and its clear theoretical basis. Assume receivers at two different depths for which the first-arrival times are $t_1$ and $t_2$ where the second is assumed to be the larger. Then, given a surface source, the amplitude spectrum of the first arrival waveform at level 1 can be modelled as

$$A_1(f) = A_0(f)T_1e^{-\pi f t_1/Q_1}$$

where $A_0(f)$ is the amplitude spectrum of the source, $T_1$ represented transmission losses, and $Q_1$ is the average Q.
Fine layering and Q estimation

value to level 1. Similarly, the receiver at level 2 has an amplitude spectrum modelled by

\[ A_2(f) = A_1(f)T_2 e^{-\pi f t_2/Q_2}. \]

Then the log-spectral ratio is given by

\[ lsr(f) = \ln \frac{A_2}{A_1} = \ln \frac{T_2}{T_1} - \pi f \Delta t \frac{Q_{int}}{Q_2} \]

where \( \Delta t = t_2 - t_1 \) and \( Q_{int}^{-1} = \Delta t^{-1}(t_2 Q_2^{-1} - t_1 Q_1^{-1}) \) is the interval Q between the two receivers. Thus the spectral ratio estimation method fits a linear relation to \( lsr(f) \), usually by least squares, with the slope providing an estimate of \( Q_{int} \) and the intercept estimating \( \ln T_2/T_1 \). It is important to restrict the frequency range of the least-squares fit to a strong signal band because, even with high quality synthetics, the strong exponential decay can easily exceed numerical dynamical range at even modestly high frequencies. Moreover, the slope and intercept of a linear fit are not independent meaning that an error in one causes a correlated error in the other. Thus a bias in Q estimation could arise from a bias in the estimation of transmission effects.

The fine-layered model, the synthetic VSP response, and Q measurement

Figure 1 shows the well data used in this computation and the empirically derived model. The coloured curves in this figure show the data at the finest blocking size used in this study which is 0.5m. Blocking sizes up to 80m were used and the black dotted lines show the data as blocked as 20m as an example. Here “blocking” refers to averaging the velocity and density logs to produce logs with constant-values over the stated block size. All logs, regardless of blocking size were sampled at the same 0.5m interval. Thus, in the synthetic seismograms to come, all had the same number of layers even though, for a strongly blocked log, many of these layers had identical values. Since the maximum depth is about 1700m, there were about 3500 layers. Blocking was done using Backus averaging (Backus, 1962) and Q values were empirically derived from the blocked logs in each case. The logs start at about

![Figure 1: Velocity, density, and intrinsic Q (times 10) curves for 0.5m blocking (colour) and 20m blocking (black dots) as used in this study.](image1)

![Figure 2: The VSP total field for the 0.5m blocking model shown in Figure 1.](image2)

![Figure 3: The upgoing field from Figure 2.](image3)

![Figure 4: The downgoing field from Figure 2.](image4)

200m depth and a smoothly varying overburden has been attached.

Figures 2-4 show the total VSP field and the upgoing and downgoing fields as generated by the algorithm of Ganley...
Fine layering and $Q$ estimation

(1981) for the model of Figure 1 with 0.5m layers. All possible multiples are included in this simulation and the up and downgoing fields are generated separately so that wavefield separation methods are not needed. Spectral-ratio estimates were then computed from the downgoing field in Figure 4 using selected pairs of receivers. For each measurement, if intrinsic $Q$ is the only effect, theory predicts that the measurement should estimate the interval $Q$ between the receivers given by

$$Q_{int}^{-1} = \Delta t^{-1} \sum_k \Delta t_k Q_k^{-1}$$

where $\Delta t$ is the time between receivers, $\Delta t_k$ and $Q_k$ are the time thickness and intrinsic $Q$ of the $k^{th}$ layer and the summation is over all layers between the two receivers. Shown in Figure 5 are the results of 7 spectral-ratio $Q$ estimates for selected pairs of receivers (red stars) and the expected interval $Q$ values between the receivers (blue circles). These and all other spectral-ratio computations in this paper were done with waveforms extracted in a 200ms window beginning at the first arrival times. The least-squares fit was performed over the 5-70 Hz band. As can be seen, the measurements are biased to be consistently lower than the expected values. This begs the question of the cause of this bias. Is it a failure of the synthetic seismogram algorithm, or perhaps a failure of the spectral ratio method, or is it truly due to the stratigraphic filtering effect caused by internal multiples.

To answer these questions, another synthetic VSP was calculated using the same input logs but with multiples and transmission losses turned off. The resulting downgoing field, which is not shown for lack of space, has first arrivals similar to Figure 4 but lacks the complex coda of following events caused by multiples. The first arrival waveforms also have subtle amplitude differences due to the lack of transmission losses. The result of spectral-ratio estimates on this much simplified downgoing wavefield is shown in Figure 6 where it is now apparent that measurement and theoretical expectations are now in agreement. So, it is concluded that the spectral-ratio software is not the cause of the bias in Figure 5 and it must therefore be due to either improper handling of transmission losses or to multiples.

The next experiment, which is not shown, was to turn transmission losses on while leaving multiples off. The resulting spectral-ratio estimates were essentially identical to those of Figure 6 confirming that transmission losses are not to blame. Subsequently, it was observed that turning on surface-related multiples while leaving internal multiples off did not introduce the bias. On the other hand, turning on internal multiples while leaving surface-related multiples off resulted in the spectral-ratio estimates in Figure 7. This confirms that internal multiples are causing the measurement bias. Subsequently, by turning off the $Q_{intrinsic}$ mechanism in Ganley’s algorithm, it was possible to measure the stratigraphic $Q_{strat}$ directly and confirm the relation of Richards and Menke (1983) that

$$Q_{eff}^{-1} = Q_{intrinsic}^{-1} + Q_{strat}^{-1}$$

where $Q_{eff}$ is the effective $Q$ that will be measured in any practical setting. Finally, let an overbar indicate an average value and define $Q_{bias} = Q_{strat}^{-1} - Q_{intrinsic}^{-1}$ as a measure of the stratigraphic filtering effect. Then Figure 8 shows the result of a series of simulations of increasing well-log blocking size. For each blocking size the log properties were averaged (Backus, 1962) of the stated size while still sampled at 0.5m. Thus the number of layers remained constant but as the blocking size increased there were more and more
layers with identical properties. Figure 8 shows the bias is very strong at blocking sizes near 0.5m (comparable to the sample size in the original logs) but this decreases rapidly to effectively vanish at about 20m which is roughly 1/5 of the dominant wavelength of the wavelet used in the simulations. Closer inspection suggests that the stratigraphic filtering effect is only significant for blocking sizes considerably less than 10m.

Conclusions

The stratigraphic filtering effect, as first described by O’Doherty and Anstey (1971), can be simulated by a relatively simple synthetic seismogram algorithm. Using such an algorithm which also allows the specification of intrinsic Q, this work has shown that Q measurements are always estimates of an effective Q which is a combination of intrinsic and stratigraphic effects. While the intrinsic Q may be considered as a rock property, the stratigraphic Q is fundamentally non local. Numerical experimentation shows that the stratigraphic Q arises from the earth’s fine layering and vanishes for earth models with layer thicknesses above 10m. When estimates of intrinsic Q are desired, it appears possible to predict stratigraphic Q from well log information and use this to correct Q measurements for the stratigraphic effect.

Acknowledgements

I thank the sponsors of CREWES for their support. I also gratefully acknowledge support from NSERC (Natural Science and Engineering Research Council of Canada) via two grants: CRDPJ 379744-08 and RGPIN 217032-2013.