



Porosity prediction using cokriging with multiple secondary datasets

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Abstract

The prediction of porosity is essential for the identification of productive hydrocarbon reservoirs in oil and gas exploration. Numerous useful technologies have been developed for porosity prediction in the subsurface, such as multiple attribute analysis, kriging, and cokriging. Kriging allows us to create spatial maps from point information such as well log measurements of porosity. Cokriging combines well log measurements of porosity with seismic attributes recorded between the wells to improve the estimation accuracy of the overall map. However, the traditional cokriging for porosity estimation is limited to only one seismic attribute. To introduce more geological information and improve the accuracy of prediction, we develop a new cokriging system that extends traditional cokriging to two secondary variables. The Blackfoot seismic data from Alberta is applied to examine the new cokriging system and "leave-one-out" cross-validation is employed to evaluate the accuracy of porosity prediction with traditional cokriging and our new approach. Compared to kriging and traditional cokriging, an improved porosity map, with higher lateral geological resolution and smaller variance of estimation error, was achieved using the new cokriging system. We believe that the new approach can be considered for porosity prediction in any area of sparse well control.

Introduction

Porosity prediction plays an essential role in predicting elastic rock properties and planning production operations (Doyen, 1988). Many techniques have been introduced to predict porosity in subsurface reservoirs, for instance, kriging, cokriging, multi-attribute analysis. The kriging system uses only high vertical resolution well log data in the spatial interpolation, but well logs are poorly sampled laterally. However, the advantage of kriging is that the well values are honored perfectly. On the other hand, multi-attribute analysis gives good spatial resolution if 3D seismic data is used, but it is hard to match the exact well values, since these values are predicted using a least-squares algorithm. Doyen (Doyen, 1988) applied cokriging to predict porosity by using acoustic impedance extracted as secondary variable from 3D seismic data. Cokriging produces maps that contain the spatial trends constructed by the spatial correlation function to model the lateral variations of the reservoir properties (Doyen et al., 1996).

The traditional cokriging system combines well log data and seismic attribute data, but only one secondary dataset is allowed in calculation. It is necessary to corroborate more than one seismic attribute to support the prediction because every attribute has particular useful information about reservoir and to predict rocks properties (Guerrero et al., 1996). To optimize the secondary data, numerous methods have been proposed. Russell et al. (2002) combine cokriging and multi-attribute transforms. As Russell et al. (2002) illustrated, the secondary input of cokriging is an improved map generated by multi-attribute analysis. Babak and Deutsch (1992) improved the cokriging model by merging all secondary data into a single super secondary dataset and then implementing the cokriging system with the single merged secondary dataset. Nevertheless, those super secondary data were obtained under assumptions which are unpractical.

In this paper, to satisfy those assumptions and improve the estimation, we present a new approach that introduces two secondary variables in the cokriging. Two advantages are achieved with the new

cokriging system. First, the lateral geological resolution of the final produced maps is increased at the locations away from the well locations because the secondary variable brings in extra geological information. Secondly, the addition of the second seismic attribute offers an opportunity to decrease the variance of the estimation error.

Methodology

The traditional cokriging method consisted of one primary and one secondary variable. To introduce more seismic attributes into the estimation, a new cokriging system consisting of one primary and two secondary variables is implemented. The new algorithm exploits the crosscorrelation not only between the primary and secondary variables, but also between the two secondary variables. As with the traditional cokriging algorithm (Isaaks and Srivastava, 1989), the cokriging system containing one primary and two secondary variables is defined as,

$$\hat{u}_0 = \sum_{i=1}^n a_i \cdot u_i + \sum_{j=1}^m b_j \cdot v_j + \sum_{k=1}^p c_k \cdot x_k \quad (1)$$

where \hat{u}_0 is the estimation of \mathbf{U} at location 0. u is the primary data; v and x , both are the secondary data. a, b, c are cokriging weights vectors to be determined.

Then the estimation error can be written in matrix form as

$$\mathbf{R} = \mathbf{w}^t \mathbf{Z} = \hat{u}_0 - u_0 = \sum_{i=1}^n a_i \cdot u_i + \sum_{j=1}^m b_j \cdot v_j + \sum_{k=1}^p c_k \cdot x_k - u_0 \quad (2)$$

where $\mathbf{w}^t = (a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_m, c_1, c_2, \dots, c_p)$, $\mathbf{Z}^t = (u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_m, x_1, x_2, \dots, x_p, \mathbf{u}_0)$.

Similarly to the traditional cokriging method, two conditions must be satisfied. First, the weights in equation (1) must be unbiased. Secondly, the error variances in equation (2) must be as small as possible. To tackle the unbiasedness condition, the expected estimation value in Equation (1) is computed as below,

$$E(\hat{U}_0) = E\left\{ \sum_{i=1}^n a_i u_i + \sum_{j=1}^m b_j v_j + \sum_{k=1}^p c_k x_k \right\} = \tilde{m}_U \sum_{i=1}^n a_i + \tilde{m}_V \sum_{j=1}^m b_j + \tilde{m}_X \sum_{k=1}^p c_k \quad (3)$$

where $E\{U_i\} = \tilde{m}_U$, $E\{V_j\} = \tilde{m}_V$, and $E\{X_k\} = \tilde{m}_X$. To make this function to be unbiased, we need $\sum a = 1$, $\sum b = 0$, and $\sum c = 0$ as the unbiased condition.

To honor the second condition, we need to minimize the error. The Lagrange multiplier method (Ito and Kunisch, 2008) is used to minimize a function with three constraints. Therefore, the error variance can be expressed as (three additional term do not contribute to the error variance)

$$\mathbf{Var}\{\mathbf{R}\} = \mathbf{w}^t \mathbf{C}_z \mathbf{w} + \mu_1 \left(\sum_{i=1}^n a_i - 1 \right) + \mu_2 \left(\sum_{j=1}^m b_j \right) + \mu_3 \left(\sum_{k=1}^p c_k \right) \quad (4)$$

Then, the new cokriging system can be obtained by variance by setting the partial derivatives of error variance with respect to weight vectors to be zero. And the matrix form can be written as

$$\begin{pmatrix} C_{uu} & C_{vu} & C_{xu} & 1 & 0 & 0 \\ C_{uv} & C_{vv} & C_{xv} & 0 & 1 & 0 \\ C_{ux} & C_{vx} & C_{xx} & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} C_{u_0u} \\ C_{u_0v} \\ C_{u_0x} \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

Case study

This case study predicts porosity using the new cokriging estimation system described in the previous section and compares the result with maps generated by traditional cokriging. Twelve wells (Figure 1a) are involved in this study area (Blackfoot from Alberta), all of which contain calculated porosity logs. The two secondary datasets consist of two structure slices extracted from the acoustic impedance inversion of the stacked P-wave seismic data. The horizon slice of the P-wave impedance inversion was computed by using an arithmetic average over a 10 ms window below the picked channel top from the 3D inverted volume. Similarly, we extracted three data slices, seismic amplitude, amplitude envelope, and instantaneous phase, by calculating a 10 ms RMS average over the zone of interest. The cokriging estimation system requires a strong correlation between the primary and secondary variables.

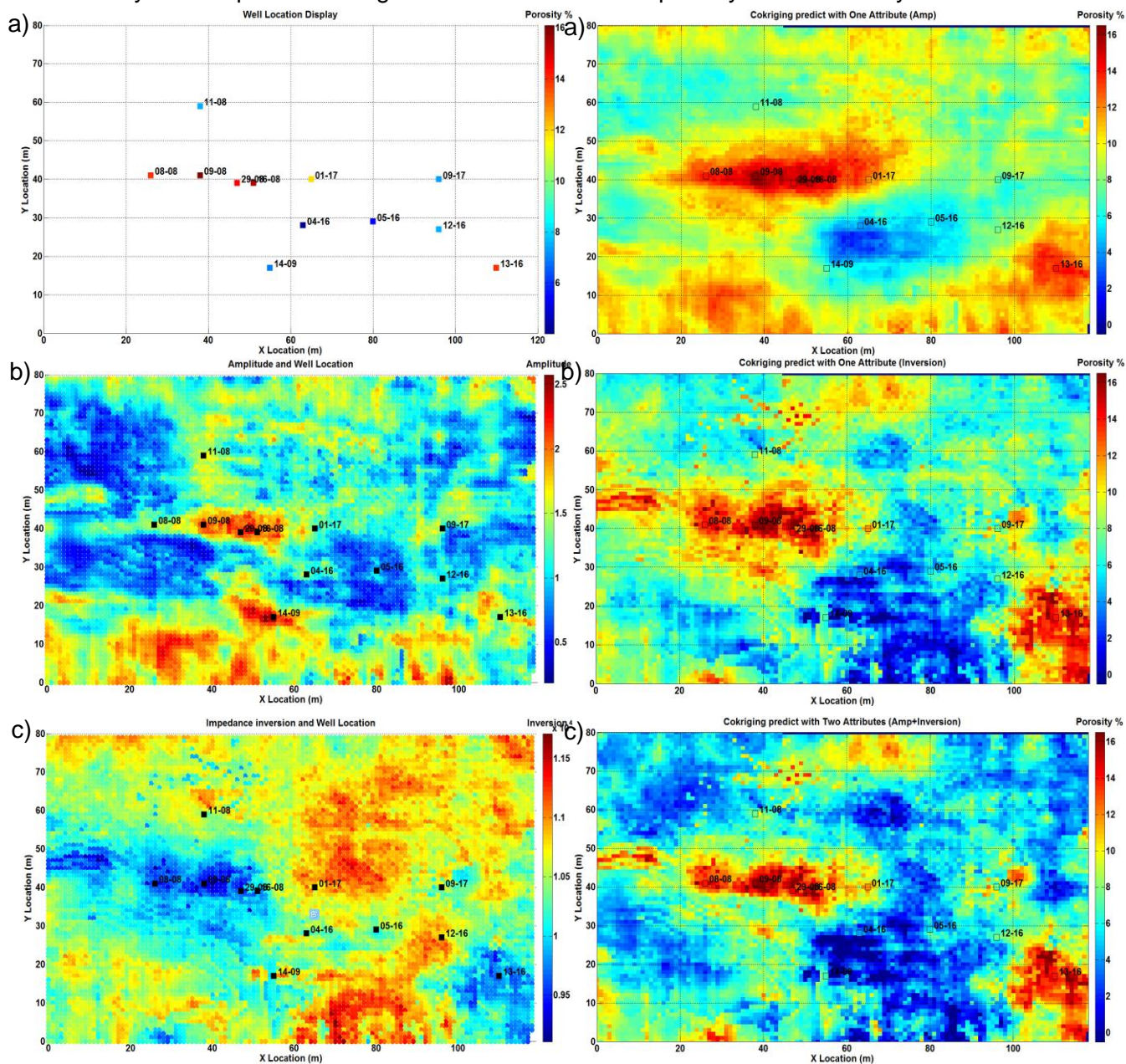


FIG. 1. a) well location, b) seismic raw amplitude slice, c) impedance inversion slice.

FIG. 2. a) cokriging with 1a, b) cokriging with 1b, c) new cokriging with 1a and 1b.

Thus, we calculated correlation coefficients between the porosity values and all four data slices. The best two correlation coefficients are calculated from the inversion slice and seismic amplitude slice, which are -0.65 and 0.41, respectively. Thus, we use the seismic amplitude slice (Figure 1b) and impedance inversion slice (Figure 1c) as the two secondary inputs.

The map of predicted porosity can be generated from matrix Eq. (5) after determining the weights vector (a, b, c). To evaluate the predicted result under the new cokriging system, the estimates from the traditional cokriging were calculated and compared. Figure 2a shows the result generated by traditional cokriging with only amplitude data utilized and Figure 2b shows the estimation with only the impedance inversion slice as the secondary input. The final produced porosity map (Figure 2c) was constructed by implementing new cokriging system, and including both impedance inversion and seismic amplitude attributes.

Compared to the traditional cokriging result, there is no significant difference in using two attributes at good well-distributed locations. However, the results using the new cokriging approach show higher lateral resolution and a remarkable difference in those areas where there is little well control. For a more quantitative, "leave-one-out" cross-validation (Voltz and Webster, 1990) was employed to calculate RMS errors for the traditional cokriging and the new cokriging system. RMS errors of traditional cokriging with inversion, traditional cokriging with seismic amplitude, and the new cokriging system are given by the histograms shown in Figure 3. It is worth noting that the new approach shows a lower RMS error than other approaches. In other words, the new cokriging system, involving two well correlated secondary datasets, gives a better estimation of the porosity.

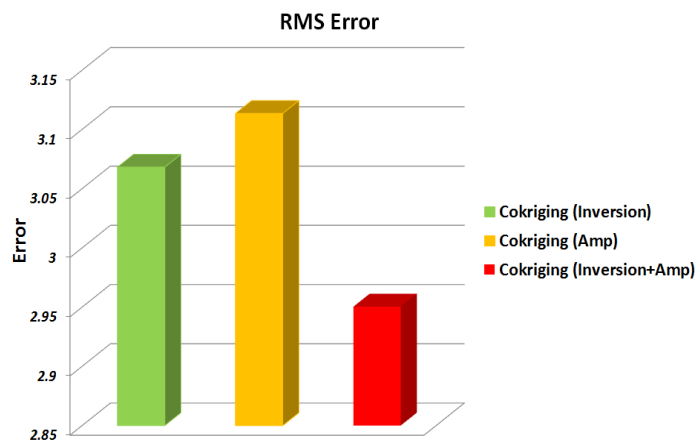


FIG. 3. Leave-one-out cross-validation

Conclusions

In this paper, we have derived and presented a new cokriging estimation system with one primary and two secondary variables, which is designed to bring extra geological information into the estimation process. The case study shows that the new approach is able to improve the spatial lateral resolution at locations away from the well values when compared with the traditional cokriging estimation system. The "leave-one-out" cross-validation method was applied to validate the accuracy of the new cokriging results. The new cokriging system gives a lower RMS error than the RMS errors of kriging and traditional cokriging. Furthermore, the new cokriging system offers us a new way to include more than one seismic attribute into the estimation of porosity with cokriging, and could be extended to three or four variables.

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References

- Babak, O., and Deutsch, C., 1992, Collocated cokriging based on merged secondary attributes: *Math Geosci*, 41, 921–926.
- Doyen, P. M., 1988, Porosity from seismic data -a geostatistical approach: *Geophysics*, 53, 1263–1257.
- Doyen, P. M., den Boer, L. D., and Pillet, W. R., 1996, Seismic porosity mapping in the ekofisk field using a new form of collocated cokriging: *SPE Annual Technical Conference and Exhibition*, Denver, Colorado.
- Guerrero, J., Vargas, C., and Montes, L., 1996, Reservoir characterization by multiattribute analysis: The orito field case: *Earth Sci. Res. J.*, 14, 173–180.
- Isaaks, E. H., and Srivastava, R. M., 1989, *An Introduction to Applied Geostatistics*: Oxford University Press, New York.

- Ito, K., and Kunisch, K., 2008, Lagrange Multiplier Approach to Variational Problems and Applications: Matheron, G., 1965, Porosity from seismic data -a geostatistical approach.
- Russell, B., Hampson, D., Todorov, T., and Lines, L., 2002, Combining geostatistics and multi-attribute transforms: a channel sand case study, black foot oilfield: *Journal of Petroleum Geology*, 21, 97–117.
- Voltz, M., and Webster, R., 1990, A comparison of kriging, cubic splines and classification for predicting soil properties from sample information: *J. Soil Sci*, 41, 473–490.