Time and offset domain internal multiple prediction with nonstationary parameters
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SUMMARY

Practical internal multiple prediction and removal is a high priority area of seismic processing technology, that has special significance for unconventional plays, where data are complex and sophisticated quantitative interpretation methods are apt to be applied. When the medium is unknown and/or complex, and move-out based discrimination is not possible, inverse scattering based prediction is the method of choice, but challenges remain for its application in certain environments. For instance, when generators are distributed up-shallow and within and below zones of interest, optimum prediction parameters are difficult to determine — in fact in some cases no stationary value of the search parameter $\varepsilon$ can optimally predict all multiples without introducing damaging artifacts. A re-formulation and implementation in the time-domain permits time-nonstationarity to be enforced in $\varepsilon$, after which a range of possible data-driven and geology-driven criteria for selecting a $\varepsilon(t)$ schedule can be analyzed. 1D and 1.5D versions of the time-nonstationary algorithm are easily derived and can be shown to add a new element of precision to prediction. Merging of these ideas with multidimensional plane-wave domain versions of the algorithm will provide 2D/3D extensions.

INTRODUCTION

The influence of internal multiples on primary reflections remains one of the most serious impediments to practical quantitative interpretation. This is especially true in unconventional onshore resource plays, in which subtle linkages between seismic amplitudes and rock physics and/or engineering relevant properties are sought (Iverson, 2014). Currently those linkages are being established with careful modelling of amplitude-variation-with-angle and are based on a primaries-only type of analysis. The continued development of multiple prediction and removal methods, especially those applicable in unconventional, onshore environments, is consequently a critical area of research. Several classes of wave equation-based removal of internal multiples exist (Weglein and Dragoset, 2005; Jakubowicz, 1998; Berkhour, 1999), however the inverse scattering series (ISS) internal multiple suppression algorithm (Araújo, 1994; Weglein et al., 1997, 2003; Otnes et al., 2004; Ramírez and Weglein, 2005) is optimal for predicting internal multiples in the absence of subsurface velocity or structural information, and when other primary/multiple discriminators (e.g., moveout) are unavailable. Two areas of research in ISS prediction are particularly active, the first being to move from an attenuation algorithm, in which the predicted amplitude is approximate, to an elimination algorithm, in which the predicted amplitude is exact (e.g., Zou and Weglein, 2015). The second concerns refining the prediction calculations to optimize them for certain high priority acquisition styles and environments.

Land application, in particular, remains challenging, for reasons outlined by Luo et al. (2011). Noisy traces with proximal and/or interfering primaries and multiples are common; on occasion the pre-subtraction prediction sections themselves are informative, but too noisy for subtraction to be advisable (Reshef et al., 2003; Hernandez and Innanen, 2014). However, the possible impact of even a small up-tick in the precision of multiple removal on land has been an incentive for investigation of new workflows (Fu et al., 2010; Wu et al., 2011; Sonika et al., 2012; Ras et al., 2012; de Melo et al., 2014, 2015).

A promising line of research is to seek optimum domains in which the basic numerics of prediction are carried out. The automated search for, and combination of, sub-events in a data record is fixed to occur in the pseudo-depth or vertical travel time domains (Weglein et al., 2003), but the output domains, i.e., the experimental variables on the left-hand side of the formula, can be varied quite widely. The standard form of the algorithm has the prediction emerging in the wavenumber/frequency domain, but formulations in the $\tau-\rho$ domain (Coates and Weglein, 1996) may have advantages in terms of reduction of artifacts (Sun and Innanen, 2015). This has motivated a new numerical analysis of 2D coupled plane-wave domain internal multiple prediction (Sun and Innanen, 2016). The output domain is critical also because it restricts and defines the variability we may assign to the search limiting parameter $\varepsilon$ (whose importance was first discussed by Coates and Weglein, 1996). For instance, high angle noise in 1.5D multiple prediction has been shown to be suppressed by setting the parameter $\varepsilon = k_{x}$, rather than giving it a fixed value (Innanen and Pan, 2015). This was possible with the standard form of the prediction algorithm because $k_{x}$ is one of the output variables of the formula. In contrast, a time-varying parameter $\varepsilon = \varepsilon(t)$, is not practically available in the standard $(k_{g}, \alpha)$ prediction algorithm.

To address this, in this paper we derive forms for 1.5D internal multiple prediction in several output domains, including the time, using the standard $(k_{g}, \alpha)$ domain as a starting point, and provide some numerical examples of some of them in action. Because the domain determines the type of allowable $\varepsilon$ nonstationarity, we may then proceed to investigate the consequences of allowing $\varepsilon = \varepsilon(t)$, and various basic criteria for selecting optimum $\varepsilon(t)$ schedules.

TIME DOMAIN FORMULAS

In standard notation the 1.5D version of the inverse scattering series internal multiple attenuation algorithm (Weglein et al., 1997, 2003) is

$$
b_{3}(k_{g}, \omega) = \int_{-\infty}^{\infty} dt' e^{ik_{g}t'} b_{1}(k_{g}, t') \int_{-\infty}^{t'} dt'' e^{-ik_{g}t''} b_{1}(k_{g}, t'') \times \int_{-\infty}^{\infty} dz'' e^{i\omega z''} \hat{b}_{1}(k_{g}, z''), \tag{1}
$$

where the $b_{1}$ are weighted versions of the input data expressed in the pseudo-depth domain. The weights maximize the match between predicted and actual multiples. In this paper we will neglect these weights and focus on the arrival time of predictions; input to the prediction procedure will be minimally pre-processed data (specifically, data from waves which are downgoing at the source, upgoing at the receivers, and free of surface multiples). We consider predictions in a variety of combinations of physical and Fourier domains. The 1D and 1.5D multiple predictions in the offset and/or time domains are denoted $m(t)$ and $m(x, t)$ respectively, and after transformation the 1.5D predictions are furthermore referred to as $\hat{M}(k_{g}, t)$ and $\hat{M}(k_{g}, \omega)$ respectively.

If the Fourier transform of unweighted data $s(x, t)$ with respect to $x$, namely $\hat{s}(k_{g}, t)$ are used as input to the prediction rather than the weighted data $b_{1}$, the standard form in equation (1) becomes

$$
\hat{M}(k_{g}, \omega) = \int_{-\infty}^{\infty} dt' e^{i\omega t'} \hat{s}(k_{g}, t') \int_{-\infty}^{t'} dt'' e^{-i\omega t''} \hat{s}(k_{g}, t'') \times \int_{-\infty}^{\infty} dz'' e^{i\omega z''} \hat{s}(k_{g}, z''). \tag{2}
$$

Here we have substituted $\omega t$ for $k_{x}z$, which is legitimate only if the ordering of sub-events in (total) traveltime $t$ is the same as ordering in vertical travel time (Nita and Weglein, 2009). 1D and 1.5D cases accommodate this, but to extend the results of this paper to 2D and 3D, the coupled $\tau - \rho$ domain, in which the time coordinate has the correct (vertical) interpretation, should be used. This extension is examined by Sun and Innanen (2016).
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The 1D form of equations (1)-(2) is obtained by setting \( k_y = 0 \). Letting the plane wave trace at normal incidence be \( s(t) \), it can be shown that time-domain form of the prediction is

\[
m(t) = \int_{-\infty}^{\infty} dt' s(t' - t) \int_{a(t',t')}^{\beta(t)} dt'' s(t'' - t') s(t''),
\]

where

\[
\alpha(t, t') = t' - (t - \varepsilon)\]

\[
\beta(t) = t - \varepsilon.
\]

The 1.5D prediction in equation (2) consists of repetitions of the 1D calculation, once for each contributing \( k_y \) value. The wavenumber-time domain version of the algorithm is, likewise,

\[
M(k_y, t) = \int_{-\infty}^{\infty} dt' S(k_y, t' - t) \int_{a(t',t')}^{\beta(t)} dt'' S(k_y, t'' - t'') S(k_y, t''').
\]

Finally, because the integrand in equation (5) contains \( S \) in a product with itself three times, the offset domain form can be interpreted as involving two spatial convolutions:

\[
m(x_g, t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' S(x_g - x', t' - t) \times \int_{-\infty}^{\infty} dx'' \int_{a(t', t'')}^{\beta(t)} dt'' S(x'' - x', t'' - t'') s(x'', t'').
\]

The basic output of the \((k_y, t)\) version of the algorithm is illustrated in Figure 1. In Figure 1a a data set \( s(x_g, t) \) consisting of an ongoing field with two primaries and a series of internal multiples is illustrated; the transform \( |S(k_y, t)| \) is illustrated in Figure 1b. The output of equation (5), inverse Fourier transformed over \( k_y \), is illustrated in Figure 1c. This example does not challenge the method particularly, but it is instructive to note that the spreading out of the two primary events along the \( t \) axis in the \((k_y, t)\) domain means their effective proximity at \( k_y \neq 0 \) is not the same as it is at \( k_y = 0 \).

THE PARAMETER \( \varepsilon \) AND NONSTATIONARITY

The only ad hoc quantity in the prediction formulas of the previous section is the parameter \( \varepsilon \) in the integral limits \( \alpha \) and \( \beta \), which limits the proximity of sub-events combined to estimate multiples. This parameter must be selected by the user of the algorithm with some care. Values of the parameter \( \varepsilon \) that are too large will lead to missed predictions, and values of \( \varepsilon \) that are too small will lead to the construction of artifacts which are correlated with primaries, and are for this reason very undesirable. An optimum value of \( \varepsilon \) trades off between these two negative extremes. We will refer to a prediction that involves a relatively low value of \( \varepsilon \), running the risk of generating artifacts, as “aggressive”; a prediction involving a relatively high value of \( \varepsilon \), running the risk of missing a multiple, will be referred to as “cautious”.

The two primary events in Figure 1b spread out in \( t \) as \( k_y \) increase. That is, their “size” along the vertical time or pseudo-depth coordinate axes, and their relative separation, are both nonstationary features of the input. Because the parameter \( \varepsilon \) is selected based on the proximity and extent in time/depth of the sub-events, it follows that the optimum value of \( \varepsilon \) too should be expected to be nonstationary with respect to \( k_y \) (Innanen and Pan, 2015). In fact, sensible arguments can be made in favour of \( \varepsilon \) being made a function of a range of output variables. Nonstationarity in \( \varepsilon \) with respect to output time, or output vertical time, appears to hold particular possible importance for the precision of multiple predictions. Qualitatively, as we scan down through a trace containing primaries and multiples, we often recognize that at this time point on the trace a more aggressive prediction is optimal, whereas at that time on the same trace a more cautious prediction is warranted.

The geological details of multiple generators and other bedding provide one justification for time-nonstationarity in optimum \( \varepsilon \). In Figure 2 some primary and multiple ray paths from such a situation are illustrated. The primaries (ray paths in dashed lines) are combined to predict the multiples (solid lines), and \( \varepsilon \) is selected such that only resolved events are so combined. When the primaries being combined are nearby in vertical time, as they are for the short path multiples, a selective \( \varepsilon \), varying rapidly from low to high, may be warranted; when the primaries are distant, a large \( \varepsilon \) with a low probability of generating artifacts is likely sufficient.

![Figure 2: A near-surface generator imposes a pattern on the output of the multiple prediction operator. Multiples at earlier times in the output prediction are predicted by combining sub-events that are proximal to one another in time (on the left, the sub-event ray paths, drawn with black and grey dashed lines, have similar lengths). Multiples at later times are predicted with sub-events that are distant from one another.](image)

Assuming a real need for time-nonstationarity in \( \varepsilon \) is identified, the formulas in equations (3), (5) and (6), because they involve the construction of the output one \( \varepsilon \) value at a time, admit \( \varepsilon \) values that may vary on any desired schedule \( \varepsilon(t) \).

PARAMETER SELECTION CRITERIA

With a search parameter schedule \( \varepsilon(t) \), \( \varepsilon(k_y, t) \) or \( \varepsilon(x_g, t) \) decided on (depending on which of formulas (3), (5) or (6) respectively are being used), the prediction proceeds straightforwardly. It remains to discuss
criteria by which such schedules can be arrived at. We will avoid being prescriptive here, as the main value of the algorithms is that they afford the user the freedom to choose an $\epsilon(t)$ suited to their particular problem. However, we discuss two plausible sources of external or a priori information by which a user might find guidance. In this paper we will focus on zero and near-offset traces and purely temporal variations, i.e., the 1D $\epsilon(t)$ case.

**Data driven**

The danger of a too-aggressive prediction (i.e., a relatively small $\epsilon$) is that artifacts correlated with primaries will be inadvertently constructed. If a multiple is predicted with an arrival time very close to that of a primary, and a small $\epsilon$ value has been used, we are in the position of having to decide if it is bona fide or an artifact. Especially on land, this ambiguity can be difficult to resolve.

\[
\epsilon(t) = \epsilon_0 + \lambda s_E(t), \quad (7)
\]

with $\epsilon_0$ being a constant corresponding with a maximally aggressive prediction, slightly larger than the dominant period of the wavelet, and with $\lambda$ chosen such that $\epsilon(t)$ reaches a maximum value of about $3 \times \epsilon_0$. In Figure 4 the result is plotted. In the right panel a scaled plot of $\epsilon(t)$ is superimposed, illustrating the relative aggressiveness versus caution we have enacted based on relative amplitudes in the data. In the left panel, a much cleaner and artifact free prediction is observed.

**Geology/well-log driven**

The previous example is illustrative of what can be done with a non-stationary $\epsilon$, but it is not a compelling case for the necessity of one. A physical modelling lab data set designed to test internal multiple prediction (Hernandez and Innanen, 2014) turns out to be more suitable. A 1D prediction (i.e., equation 3) is tested on nearly zero offset traces acquired over the model illustrated in Figure 5a. Because the medium is well-characterized, we have access to prior information akin to a blocked well-log (Figure 5b). From the log a distribution of times of expected primaries and expected multiples can be created, which is a second possible source of information for setting up a $\epsilon(t)$ schedule.

The input trace is illustrated in black in the right panel of Figure 7. The two panels in Figure 6 illustrate predictions carried out with two different fixed values of $\epsilon$, the left panel being relatively small (i.e., aggressive) and the right panel being relatively large (i.e., cautious). Two
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expected arrival times are indicated with circles in each trace, one, corresponding to the arrival time of a primary reflection, at roughly 1.2s and the other, corresponding to an internal multiple, at roughly 1.8s. The multiple has been predicted correctly in the left panel, suggesting that, at 1.8s, the aggressive \( \varepsilon \) value is suitable; however, artifacts can be seen to encroach on the primary, and so this same \( \varepsilon \) value is too small to be suitable at 1.2s. We naturally respond to this by shifting the fixed \( \varepsilon \) value up, just enough to remove the artifact at 1.2s, as illustrated in the right panel. However, we find having done this that we have lost the prediction at 1.8s. This is an example of multiple prediction where a single stationary \( \varepsilon \) value optimized for all output times is difficult, or even impossible, to find.

CONCLUSIONS

Internal multiple prediction encounters challenges in practice when the sub-events used in the prediction, and the multiples to be predicted, interfere with each other and overlap. In particular, optimal parameters for the inverse scattering series internal multiple prediction algorithm are very difficult to determine in such situations, and indeed there is reason to suspect in many circumstances that no stationary value of \( \varepsilon \) can draw a balance between correctly predicting short path and peg-leg multiples and suppressing prediction artifacts. Nonstationarity in \( \varepsilon \) can be applied, but only in the output domain of the algorithm; to permit time-nonstationarity, which seems to have the greatest promise for increasing the precision with which multiples are predicted, the algorithm must be re-formulated and implemented in this domain. Once this has been carried out, a wide range of data- and geology-driven (or any user specified) criteria for selecting schedules for \( \varepsilon (x,t) \) and/or \( \varepsilon (t) \) can be analyzed. The key next steps are to (1) merge this with a coupled multidimensional \( \tau - \rho \) implementation so that fully 2D and/or 3D output can be constructed with \( \varepsilon \) a function of output vertical time; (2) merge this with an implementation of a full multicomponent prediction framework. These efforts are currently in progress.

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