Inverse scattering series internal multiple prediction in the double plane wave domain

Jian Sun[1], Kris Innanen[1], Dept. of Geoscience, CREWES Project, University of Calgary, Canada*

Summary

The inverse scattering series (ISS) multiple prediction and attenuation algorithm, developed by Weglein and collaborators in the 1990s, estimates multiples from subevents which satisfy the lower-higher-lower criterion in the pseudo-depth or vertical traveltime domain. Prediction results that (1) have a high degree of numerical accuracy, (2) have a relatively stationary optimum search parameter, and (3) represent a significantly reduced computational burden, can be achieved in the plane wave domain. ISS prediction carried out in this domain may therefore be well suited as part of an effort to demultiple, effectively in, e.g., complex land environments, where such features become increasingly important. In this paper, we discuss and examine 2D internal multiple predictions calculated by an implementation of the ISS algorithm in the double plane wave domain. This requires a properly formulated prediction algorithm and data transformed to the coupled τ-p_s-p_g domain as input. A synthetic 2D case is examined to highlight some of the numerical features of this implementation.

Introduction

Multiple identification and removal is a critical seismic data processing technology, whose accuracy directly affects seismic imaging and interpretation. Multiples are classified as being either surface-related or interbed, distinguished by their interaction (or lack of interaction) with the free-surface. Surface-related multiples can be eliminated by taking advantage of their periodic character or by their deterministic predictability, and many innovative technologies have been developed in different domains (Taner, 1980, Treitel et al., 1982, Verschuur, 1991, Liu et al., 2000, Berkhout and Verschuur 2005, 2006). Removal of the interbed or internal class of multiple, although also provided for with powerful algorithm forms, remains in practice a major challenge, especially on land data, even though considerable progress has been made recently.

Kelamis et al. (2002) introduced a boundary-related/ layer-related approach to remove internal multiples in the poststack data (CMP domain). Berkhout and Verschuur (2005, 2006) extended the inverse data processing to attenuate internal multiples by considering them to be effective surface-related multiples through the boundary-related/layer-related approach in common-focus-point (CFP) domain. The same algorithm was applied by Luo et al. (2007), through re-datuming the top of the multiple generators and transforming internal multiples to be 'surface-related'. These approaches, while powerful, have in common that significant knowledge of the subsurface is required; thus if the possibility exists that multiple removal will have to take place with incomplete knowledge of the velocity structure and generators, the ISS approach will be optimal.

In the context of the inverse scattering series it was shown that all internal multiples can be estimated through the correct combination of primary and/or lower order multiple events (Weglein et al. 1997), with generators sought for in the data, in a stepwise and automatic way, rather than assumed. This means no subsurface information is required. Recently, research has been particularly active in (1) reformulating the prediction so that its amplitudes are exact rather than approximate (Zou and Weglein, 2013), and (2) applying the existing ISS algorithm in complex environments, such as land (e.g., de Melo et al. 2014). In practical implementation, the search parameter has been focused on as a key element (Hernandez and Innanen 2014; Innanen and Pan 2015; Pan 2015); in certain domains the non-stationarity of the optimum parameter choice can be a source of artifacts. Especially concerning artifact mitigation, implementation in the plane-wave domain appears to have some attractive features. Building on the τ-p framework of Coates and Weglein (1996) and Nita and Weglein (2009), Sun and Innanen (2015) demonstrated that an environment both with a relatively stationary search parameter and with a relatively low level of numerical artifacts is achieved. Because of these important features, we suggest that a 2D internal multiple prediction implementation, based on the ISS algorithm expressed in the coupled τ-p_s-p_g domain, will likely have a range of attractive features, practical and computational. In this paper we present a numerical implementation of this formula, and provide 2D numerical results.

ISS prediction in the double plane-wave domain

Araujo et al. (1994) and Weglein et al. (1997) showed that all internal multiples can be predicted by summing over events which satisfy the lower-higher-lower relationship. In the pseudo-depth domain, the attenuation term is

$$b_{3_{IM}}\left(k_g,k_s,\omega\right) = \frac{1}{(2\pi)^2}\iint dk_1 e^{-iq_1(\varepsilon_g-\varepsilon_s)} dk_2 e^{-iq_2(\varepsilon_g-\varepsilon_s)} \int_{-\infty}^{+\infty} dz\, e^{i(q_g+q_1)z} b_1\left(k_g,k_1,z\right) \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1\left(k_1,k_2,z'\right) \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1\left(k_2,k_s,z''\right) \quad (1)$$

where

$$q_X = \frac{\omega}{c_0}\sqrt{1-\frac{k_X{}^2 c_0{}^2}{\omega^2}}; \qquad k_z = q_g + q_s\,, \quad (2)$$

Internal multiple prediction in double plane wave domain

and where q_x, are vertical wavenumbers associated with the various lateral wavenumbers and the reference velocity. The left hand side of the Eq. 1 is inverse Fourier transformed over all three Fourier variables, and the result is added to the original seismic record to attenuate the multiples (normally with mismatches between the prediction and the data managed through adaptive subtraction).

Based on the relationship between pseudo-depth and intercept time (Nita and Weglein, 2009)

$$k_z z = \omega\tau \ , \tag{3}$$

Eq. (1) can be transformed into the plane wave domain (Coates and Weglein, 1996):

$$b_{3_{IM}}\left(p_g, p_s, \omega\right) = \frac{1}{(2\pi)^2}\iint dp_1 e^{-i\omega\left(\tau_{1g}-\tau_{1s}\right)} dp_2 e^{-i\omega\left(\tau_{2g}-\tau_{2s}\right)} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1\left(p_g, p_1, \tau\right) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1\left(p_1, p_2, \tau'\right) \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1\left(p_2, p_s, \tau''\right) \tag{4}$$

where p_g and p_s are source and receiver horizontal slownesses respectively (these are equal in 1D and 1.5D calculations). The input is the 3D volume calculated by scaling the double τ-p_s-p_g transformed data after sorting into source and receiver gathers, i.e., $b_1(p_g,p_s,\tau)=-2iq_sD(p_g,p_s,\tau)$. Figure 1 illustrates the ray-path relationships between primaries and predicted internal multiples, which can be labeled according to their contributing slownesses, i.e., p_g, p_s, p_1, and p_2 in Eq. 4. The quantities p_1 and p_2 represent horizontal slownesses at both source and receiver locations within the algorithm. This requires that suites of source and receiver slownesses be equal.

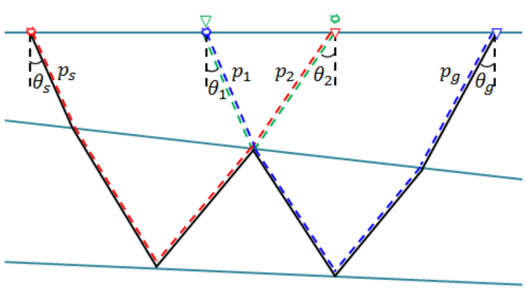

Figure 1: Ray-path schematic of primaries (red, blue and green) and internal multiple (black) with corresponding source and receiver locations (after Coates and Weglein, 1996).

Double τ-p_s-p_g transform

Traditional τ-p mapping, also known as slant-stack, was introduced by Ocola (1972) and Diebold and Stoffa (1981) and is traditionally implemented by considering travel times for varying offsets and fixed source locations,

$$t = p_g x + \sum_i z_{si}(q_{si} + q_{gi}) \tag{5}$$

or, varying offsets and fixed receiver locations,

$$t = p_s x + \sum_i z_{gi}(q_{si} + q_{gi}) \tag{6}$$

where t is the travel time and x is the offset. The variables q_{si} and q_{gi} are the vertical components of slowness in the i^{th} layer for source and receiver, respectively. The z_{si} and z_{gi} are the thicknesses of the i^{th} layer below the source and receiver locations respectively.

In order to implement τ-p mapping on CMP gather, Diebold and Stoffa (1981) also introduced a reference point M, located between source and receiver (M is the midpoint in the CMP gather), and applied the τ-p transform based on this reference point location,

$$t = p_s x_{s-M} + p_g x_{g-M} + \sum_i z_{Mi}(q_{si} + q_{gi}) \tag{7}$$

where, $x_{s\text{-}M}$ and $x_{g\text{-}M}$ are the distances between the source and geophone and the reference point respectively. z_{Mi} is the thicknesses of the i^{th} layer below the reference point. A schematic illustration of ray-paths is given in Figure 2. Using this scheme, an alteration is necessary in Eq. (4). Suppose the offset is positive (i.e., $x=x_g-x_s>0$) when source is on the left. From Eq. 7, we find that the horizontal slowness has the opposite sign for the same ray-path if the source and receiver locations are exchanged, i.e., $p_s'=-p_g$, $p_g'=-p_s$, which means we must set p_1 and p_2 in the integration over τ' in Eq. (4) to negative.

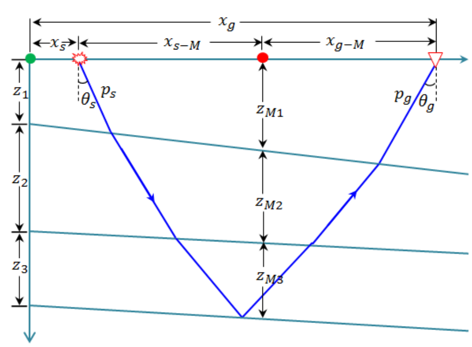

Figure 2: The ray-path geometry schematic of τ-p mapping with respect to the reference point (after Diebold and Stoffa 1981).

To manage this, the origin, which is located on the one side of source and receiver, is selected as the reference point. By proper transforming, the τ-p transform with respect to source and receiver locations can be expressed as

$$t = p_s x_s + p_g x_g + \sum_i z_i(q_{si} + q_{gi}) \tag{8}$$

where x_s and x_g are the source and receiver locations respectively, and z_i is the thicknesses of i^{th} layer below the origin. Eq. 8 indicates that double τ-p_s-p_g transform is a variant of slant-stack, with a particular phase shift carried out over source and receiver locations respectively (Stoffa et al. 2006). This can be expressed as

$$D\left(p_s, p_g, \tau\right) = \iint d\left(x_s, x_g, \tau + p_s x_s + p_g x_g\right) dx_s dx_g \quad (9)$$

The forward double τ-p_s-p_g transform for a fixed frequency can be expressed as

$$\tilde{D}\left(p_s, p_g, \omega\right) = \iint \tilde{d}\left(x_s, x_g, \omega\right) e^{+i\omega(p_s x_s + p_g x_g)} dx_s dx_g \quad (10)$$

with the inverse transform as

$$\tilde{d}\left(x_s, x_g, \omega\right) = \omega^2 \iint \tilde{D}\left(p_s, p_g, \omega\right) e^{-i\omega(p_s x_s + p_g x_g)} dp_s dp_g \quad (11)$$

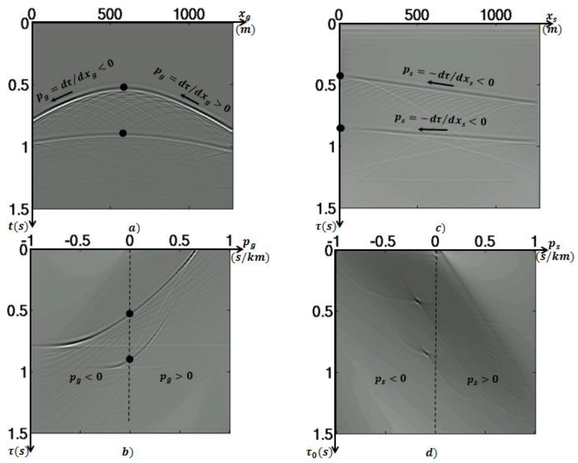

Figure 3: Double τ-p procedure. a) Record for a fixed source location, i.e., data matrix with varying (x_g,t) with x_s=636m. b) Reflections in a) mapped into the (p_g,τ) domain. c) A common p_g=0 gather extracted from the data volume in b), i.e., the data matrix (x_s,τ). d) Common p_g gather extracted from the data volume D(p_g,p_s,τ), i.e., data matrix with varying (p_s,τ) with p_g=0.

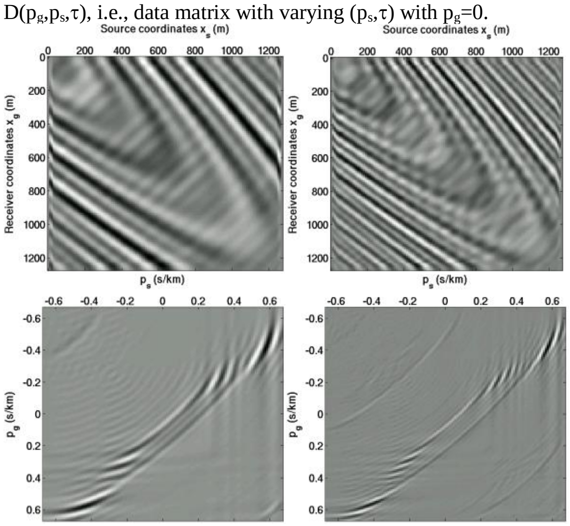

Figure 4: The comparison of data matrices at fixed frequency. The first row shows data matrix (x_s,x_g) and the second row represents data matrix (p_s,p_g), at 15 Hz and 25 Hz, respectively.

Figure 3 illustrates the τ-p_s-p_g transform procedure. The data in the (x_s,x_g,ω) and (p_s,p_g,ω) domains at fixed values of frequency (15Hz and 25Hz) are compared in Figure 4. We note that the energy in the (p_s,p_g,ω) domain is more compactly represented than it is in the (x_s,x_g,ω) domain. Furthermore, the bandwidth of the sparse matrix (p_s,p_g) can be connected with the maximum dipping angle of contributing interfaces, as demonstrated by Liu et al. (2000).

Synthetic example

To examine the characteristics of the double plane wave ISS internal multiple prediction algorithm, we applied it to data from a 3-layer model including two reflectors, one dipping and one flat (Figure 5). 128 geophones at 10m intervals were embedded just below the surface, and shot records were generated for source locations moving from left to right and occupying each geophone location. Slices through the multi-shot record's source/receiver/time volume is illustrated in Figure 6.

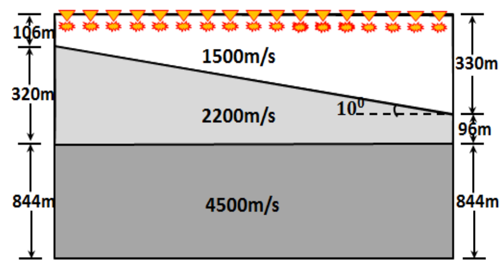

Figure 5: 3-layer synthetic model with 1500m/s in top, 2200m/s in middle, and 4500m/s in bottom layer.

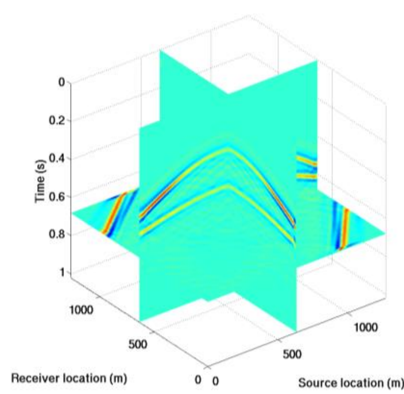

Figure 6: Multi-shot records in source-receiver coordinates.

Figure 7 shows three common shot gathers extracted from the 3D volume at 310m, 630m, and 950m respectively. Two primaries can be easily identified; the first-order internal multiple in each of those gathers is indicated with arrows. The 2D nature of the multiples caused by the dipping layer is clearly visible. With the double τ-p_s-p_g

transform, a 3D volume in horizontal slowness (for source and receiver, separately) and intercept time is obtained. Three common p_s gathers extracted at p_s values of -0.3, 0.0, and 0.3 are plotted in Figure 8.

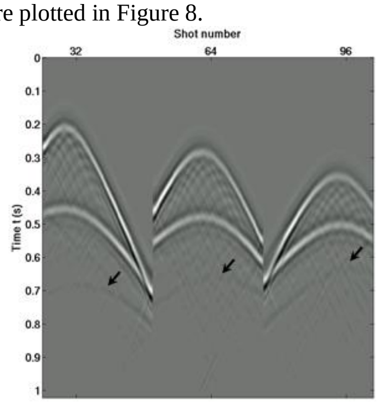

Figure 7: Common shot gathers at locations: 310m, 630m, 950m.

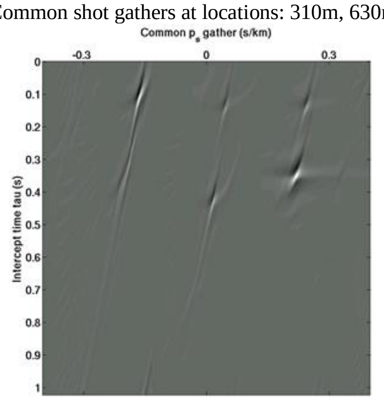

Figure 8: Common p_s gathers extracted at p_s values of -0.3, 0.0, and 0.3.

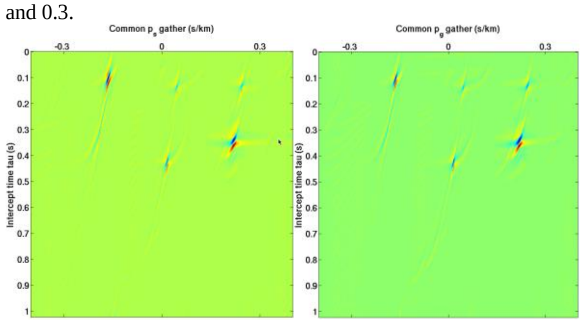

Figure 9: Common p_s and p_g gathers extracted from the input at -0.3, 0, and 0.3, respectively. Left: common p_s gathers. Right: common p_g gathers.

The data volume is adjusted by multiplying with $-2iq_s$ to produce the correctly scaled input for the ISS algorithm. We extract common p_s and p_g gathers from the input, examples of which are plotted in Figure 9.

The internal multiple prediction is then calculated for 128 shot gathers in the double plane wave domain, and an inverse τ-p_s-p_g transform is applied to transform the prediction into source-receiver coordinates. Example results, corresponding to three common shot gathers from the predicted volume at same locations as those of the input in Figure 7, are shown in Figure 10. In Figure 10, the predictions of the first-order internal multiple in the three shot gathers are seen to correctly capture the 2D times and trajectories of the multiples identified in the data, as well as the relative amplitudes (e.g., the brightening visible at the right edge of the 310m positioned shot gather).

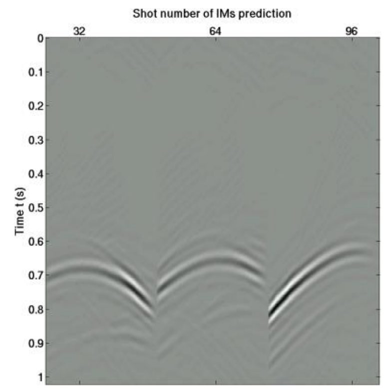

Figure 10: Common shot gathers extracted from the prediction, at shot locations 310m, 630m, and 950m.

Conclusion

Internal multiples propagating in unknown media and caused by unknown or imperfectly known generators can be predicted with the ISS algorithm in an automatic manner, due to its fully data-driven character. Implementation domain and parameter selection are important aspects of successful application of this method. We present an implementation of the internal multiple prediction in the double (or coupled) plane wave domain, which requires a tailoring of the prediction algorithm and a high fidelity τ-p_s-p_g transform. In 1.5D we have argued that this domain naturally mitigates artifacts and involves a compact representation of input data events, which in turn leads to an easier parameter selection problem. This provides incentive to implement and numerically test the 2D coupled plane wave prediction, which has not been reported elsewhere. The input in this domain can moreover be treated as a sparse matrix in the processing of prediction, to some extent, which will allow the computational burden to be reduced. This is a matter of ongoing research.

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