Gradient calculation for anisotropic FWI

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Summary

The 3-D TTI medium can be characterized as 5 independent elastic moduli in constitutive coordinate system, density, tilt and azimuth angle at each spatial point. To calculate the gradient of each independent parameter involves the synthetic and adjoint data, as well as the derivatives of elastic moduli with respect to the independent parameters of the model. In this paper, the synthetic and adjoint wavefields are simulated with a staggered-grid finite-difference algorithm in TTI media. Numerical examples of the gradient calculation are illustrated for a three-layer TTI model. One of the issues is that the synthetic data at each time step should be stored on the disk so as to perform cross-correlation with the adjoint wavefields to generate the gradient for FWI in time domain. The huge dataset storage during synthetic wavefield simulation and loading when calculating the gradient is highly memory cost and time consuming. The use of random boundary layer allows us to compute both the adjoint and synthetic wavefield simultaneously without the need of storing total synthetic data. In this paper, cubic grains are implemented as random boundary layers. The randomized elastic moduli instead of velocities are thus added in elastic wave equations in TTI medium. Gradients of Thomsen parameters and tilt angle using random layers are finally illustrated as a comparison.

Introduction

The parameter characterization of tilted transversely isotropic (TTI) media using elastic moduli, tilt angle, polar angle as well as density is of great importance. Seismic full-waveform inversion (FWI) (Tarantola, 1984; Gauthier, 1986; Mora, 1987) is capable of providing formation parameters with high spatial resolution. Over the past few decades, space and time domain FWI (Kolb et al., 1986; Mora, 1987; Bunks et al., 1995) has been paid an increasing interest since the initial work proposed by Lailly (1983) and Tarantola (1984). Recent development of FWI in elastic and anisotropic media (Lee et al., 2010; Kamath et al., 2013; Pan et al., 2016) makes it possible for the use of multicomponent reflection data. The gradient of a misfit function can be obtained based on forward simulation for current model and time-reversed simulation for adjoint source(Tromp et al., 2005). Liu and Tromp (2006) derived the gradients of the objective function for an elastic earth model. Kamath et al. (2013) calculated gradients of Thomsen parameters for 2D laterally heterogeneous VTI media. On the other hand, the conventional method of gradient calculation in time domain needs to record synthetic wavefield at each time step to correlate with the adjoint wavefield time reversely, which is highy memory cost. And the shabby input/output (I/O) of recorded data dramatically increases the total running time of the algorithm, which impedes the computational efficiency. Nguyen and McMechan (2014) discussed 5 alternative algorithms avoid storing source wavefield snapshots during forward and backward wavefield correlation in reverse time migration (RTM). Symes (2007) and Anderson et al. (2012) used an optimal checkpointing algorithm to minimize the total storage at the cost of increasing the computational complexity of the adjoint wavefield simulation. Clapp et al. (2009) proposes an alternate random boundary condition method that uses an increasingly random velocity region by replacing the conventional damped region.

In this paper, we first discuss the FWI scheme formulated in the time domain for a 3-D elastic medium, based on which, the gradients for elastic moduli and Thomsen parameters in TTI medium are obtained. A three-layer TTI media model is used for gradient calculation. Because of the huge dataset storage during synthetic wavefield simulation and loading when calculating the gradient, we suggest useing random boundary layer to compute both the adjoint and synthetic wavefield simultaneously without the necessity of storing total synthetic data. During synthetic model simulation, cubic grains are implemented as random boundary layers in this paper. The randomized elastic moduli instead of velocities are thus added in elastic wave equations in TTI medium. The synthetic waveforms with random boundary layers are finally illustrated.

Theory and Method

Given a general anisotropic model ρ and c_{ijkl}, in which ρ is the model density and c_{ijkl} is the elastic moduli, according to Liu and Tromp (2006), the *Fréchet* derivatives with respect to density and the elastic moduli are the 3-D waveform misfit kernels K_ρ and $K_{c_{ijkl}}$ that can be defined as

$$K_{c_{ijkl}} = -\sum_{s=1}^{S}\sum_{r=1}^{R}\int \frac{\partial \delta u_i}{\partial x_j}\frac{\partial G_k}{\partial x_l}dt \qquad (1)$$

in which, u and G are the forward and adjoint displacement wavefields. The gradient of each model parameter m_n can be obtained by chain rule as

$$K_{m_n} = -\sum_{ijkl} K_{c_{ijkl}} \frac{\partial c_{ijkl}}{\partial m_n}dt \qquad (2)$$

For 3-D TTI medium characterized by eight parameters at each spatial point $m_v = \{c_{11}, c_{13}, c_{33}, c_{44}, c_{66}, \rho, \theta, \varphi\}$, in which $c_{11}, c_{13}, c_{33}, c_{44}, c_{66}$ are independent elastic moduli in VTI media in constitutive coordinate system $[x, y, z]$; θ, φ are the spherical tilt angle and azimuthal phase angle. The auxiliary Cartesian coordinate system $[x', y', z']$ can thus be rotated by the tilt angle θ around z axis. Based on Bond's law (Winterstein, 1990), the relationship between the elastic moduli matrix C' in TTI and C in VTI is

$$C' = RCR^T \qquad (3)$$

with Bond transform matrix R (when rotating around y-axis and $\varphi = 0$) as

$$R = \begin{bmatrix} \cos^2\theta & 0 & \sin^2\theta & 0 & -\sin(2\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2\theta & 0 & \cos^2\theta & 0 & \sin(2\theta) & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ \sin\theta\cos\theta & 0 & -\sin\theta\cos\theta & 0 & \cos(2\theta) & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad (4)$$

Based on above equations, in 3D polar TTI media ($\varphi = 0$), considering a constant density, the gradients of 6 other independent elastic moduli can be calculated. Alternatively, the gradients of Thomsen parameters can also be obtained based on their relationship with the elastic moduli in VTI medium.

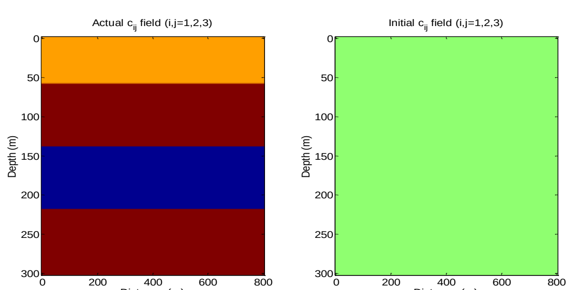

Figure 1. Cross-section of three-layered anisotropic model and its corresponding initial model.

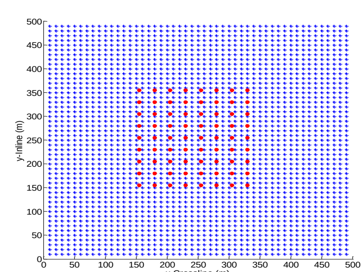

Figure 2. Source-receiver distribution geometry of the three-layered model.

Gradient calculation in a three-layer TTI media

In this example, the model is a three-layer anisotropic model with different elastic moduli for each layer, whose cross-sectional is shown in Figure (1), in which the upper formation is an isotropic layer with VP, VS and density of 3500 m/s, 2000 m/s and 2500 kg/m^3. Figure (2) shows the acquisition survey in XOY plane. The right picture of Figure (1) is the initial model. The positions of sources and receivers are denoted by red circles and blue stars. 72 sources at a depth of 25 m are excited simultaneously. The evenly spaced receiver arrays (the total number of receivers is 2304) are arranged within the same depth of the source. The 3-component residual data (adjoint source) can be acquired by calculating the difference between the observed data in the presence of TTI layers and the background modeled data, shown in Figure (3). The gradients of Thomsen parameters can be acquired, shown in Figure (4). And the gradients with respect to elastic moduli can also obtained, which will not be illustrated here.

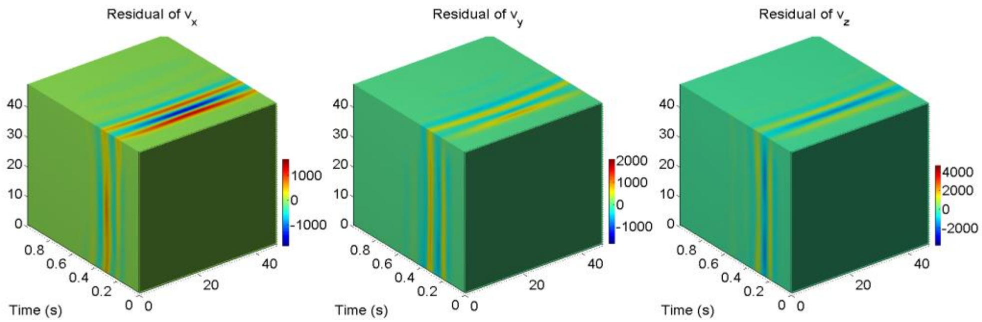

FIG. 3. The residual data for the three component velocity field.

Random boundary condition

For wavefield simulation using finite difference scheme, the synthetic data at each time step should be stored on the disk so as to perform cross-correlation with the adjoint wavefields to generate the gradient for FWI in time domian. The huge dataset storage during synthetic wavefield simulation and loading when calculating the gradient is highly memory cost and time consuming. Clapp et al. (2009) proposes a random boundary condition method that uses an increasingly random velocity region by replacing the conventional damped region. Shen and Clapp (2015) applied the random boundary condition into gradient calculation for FWI. In this paper, cubic grains with 5 m side length (the same as the FD space interval) are implemented as random boundary layers. The randomized elastic moduli instead of velocities are thus added in elastic wave equations in TTI medium.

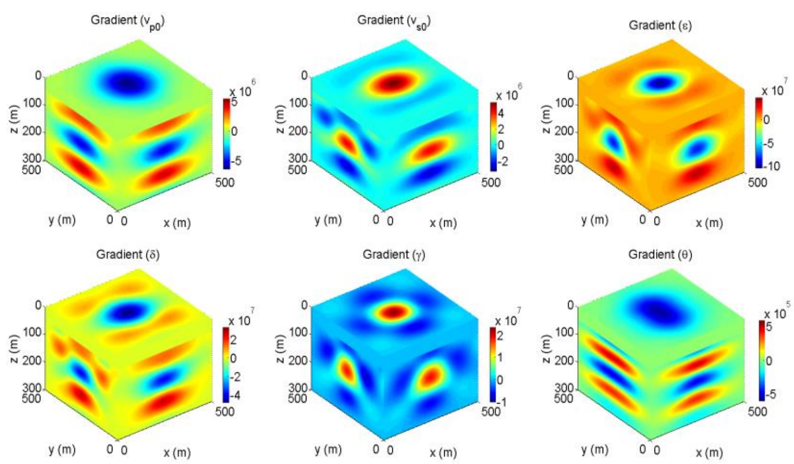

FIG. 4. Gradients of the objective function with respect to Thomsen parameters and tilt angle.

Figure 5 (Left) shows the random parameter cubic grains with 5 m side length outside the computational areas. Instead of adding a random term that relates to the distance within boundary(Clapp et al., 2009), an exponential attenuation term r is multiplied with the elastic moduli to guarantee no boundary reflections are generated in random layers. The attenuation term r can be described as $r=\left(rand\cdot\frac{d}{AB}\right)^n$, where, *rand* denotes random number within the interval of [0,1]; AB is the number of boundary layers (AB = 30 in this paper); d is the distance within boundary and n is the attenuation index. Figure 6 (Right) shows the elastic modulus c11 in a three-layer model when the random boundary layers are applied (n=4). The randomized elastic moduli are thus used in synthetic data forward simulation in FWI.

Conclusions

In 3-D TTI medium, the model can be characterized by the constitutive elastic moduli, density, tilt angle as well as the phase angle. In this paper, we discussed gradient computation in regard to both the elastic moduli and Thomsen parameters for elastic multicomponent wavefields from 3D TTI media. Based on the adjoint state method, the synthetic and adjoint wavefields are simulated with a staggered-grid finite-difference algorithm in anisotropic media. Numerical examples of the gradients calculation in a three-layer anisotropic media are illustrated. One of the issues is that the synthetic data at each time step should be stored on the disk so as to perform cross-correlation with the adjoint wavefields to generate the gradient for FWI in time domain. The huge dataset storage during synthetic wavefield simulation and loading when calculating the gradient is highly memory cost and time consuming. In this paper, the random boundary layer is used to compute both the adjoint and synthetic wavefield simultaneously without the necessity of storing total synthetic data. The randomized elastic moduli instead of velocities are thus added in elastic wave equations in TTI medium. Gradients of Thomsen parameters and tilt angle using random layers are finally illustrated.

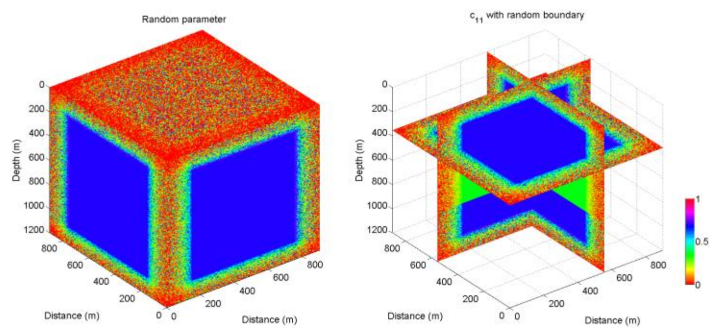

FIG. 5. Random parameter in 3D volume (Left) and randomized elastic modulus c_{11} (Right).

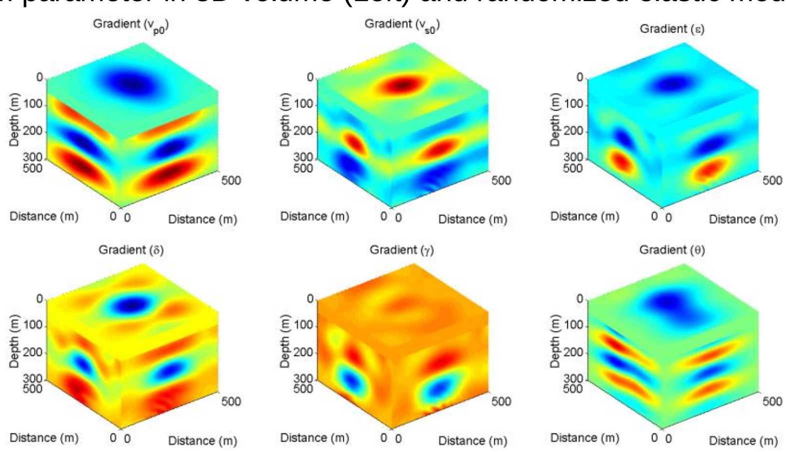

FIG. 6. Gradients of Thomsen parameters and tilt angle using random layers.

Acknowledgements

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References

Anderson, J. E., Tan, L., and Wang, D., 2012, Time-reversal checkpointing methods for rtm and fwi: Geophysics, **77**, No. 4, S93–S103.

Bunks, C., Saleck, F. M., Zaleski, S., and Chavent, G., 1995, Multiscale seismic waveform inversion: Geophysics, **60**, No. 5, 1457–1473.

Clapp, R. G. et al., 2009, Reverse time migration with random boundaries, in 2009 SEG Annual Meeting, Society of Exploration Geophysicists.

Gauthier, D., 1986, Morals by agreement: Oxford University Press on Demand.

Kamath, N., Tsvankin, I. et al., 2013, Gradient computation for elastic full-waveform inversion in 2d vti media, in 2013 SEG Annual Meeting, Society of Exploration Geophysicists.

Kolb, P., Canadas, G. et al., 1986, Least-squares inversion of prestack data: Simultaneous identification of density and velocity of stratified media, in 1986 SEG Annual Meeting, Society of Exploration Geophysicists.

Lee, H.-Y., Koo, J. M., Min, D.-J., Kwon, B.-D., and Yoo, H. S., 2010, Frequency-domain elastic full waveform inversion for vti media: Geophysical Journal International, 183, No. 2, 884–904.

Liu, Q., and Tromp, J., 2006, Finite-frequency kernels based on adjoint methods: Bulletin of the Seismological Society of America, **96**, No. 6, 2383–2397.

Mora, P., 1987, Nonlinear two-dimensional elastic inversion of multioffset seismic data: Geophysics, **52**, No. 9, 1211–1228.

Nguyen, B. D., and McMechan, G. A., 2014, Five ways to avoid storing source wavefield snapshots in 2d elastic prestack reverse time migration: Geophysics, **80**, No. 1, S1–S18.

W. Pan, K. Innanen, G. Margrave, M. Fehler, X. Fang, J. Li. Estimation of elastic constants for HTI media using Gauss-Newton and full-Newton multiparameter full-waveform inversion. Geophysics **81** (5), R275-R291.

Shen, X., and Clapp, R. G., 2015, Random boundary condition for memory-efficient waveform inversion gradient computation: Geophysics, **80**, No. 6, R351–R359.

Symes, W. W., 2007, Reverse time migration with optimal checkpointing: Geophysics, **72**, No. 5, SM213–SM221.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, No. 8, 1259–1266.

Tromp, J., Tape, C., and Liu, Q., 2005, Seismic tomography, adjoint methods, time reversal and bananadoughnut kernels: Geophysical Journal International, **160**, No. 1, 195–216.

Winterstein, D., 1990, Velocity anisotropy terminology for geophysicists: Geophysics, **55**, No. 8, 1070–1088.