

# Frequency dependent attenuation and dispersion in patchysaturated porous rocks

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## Summary

From seismic wave equations in modified patchy-saturated model established on the basis of White model, we derived the formulas of reciprocal quality factors and velocities of the two kinds of P-waves and analyzed the seismic attenuation and velocity dispersion of the fast P-wave in modified patchy-saturated rocks within seismic band. Owing to the importance of porosity, permeability and fluid saturation, we also studied and analyzed the effects of these three factors on seismic attenuation and velocity dispersion of fast P-wave in patchy-saturated rocks within the seismic attenuation and velocity dispersion of fast P-wave in patchy-saturated rocks within the seismic band.

## Introduction

Attenuation and dispersion often occur as seismic waves propagate in underground media, especially in oil and gas reservoirs (Chapman et al., 2006; Quintal et al., 2011; Yan et al., 2014). A good understanding of seismic attenuation and velocity dispersion is of great importance to seismic interpretation and inversion and also is helpful to infer the fluid type and property. However, the physical mechanisms responsible for such high attenuation and dispersion in the seismic band are currently not fully understood. In seismic exploration, we are most interested in the wave-induced fluid flow mechanism. At present, numerous models for seismic attenuation and velocity dispersion from wave-induced flow have been developed with varying degrees of rigor and complexity (Müller et al., 2010). These models can be categorized roughly into three groups: the Biot model (Biot, 1956), the BISQ model and the patchy-saturated model. The predicted attenuation based on the Biot model can only be applied for the ultrasonic band and was much smaller than that measured in the seismic frequency range(1-100Hz) (Berryman, 1988). Dvorkin and Nur (1993) proposed the Biot-Squirt (BISQ) model that combined both mechanisms based on one-dimensional isotropic pores. Unfortunately, the seismic attenuation predicted by BISQ model only applies to the ultrasonic frequency range (Dvorkin et al., 1994; Dvorkin et al., 1995). Furthermore, it cannot describe the relative relation of the seismic amplitude between P-waves of the first and second kind (Bordakov, 1999).

Wave induced fluid flow can also be caused by pressure gradients between areas of the rock which are much larger than the typical pore size but much smaller than the seismic wavelength. This kind of fluid flow is called mesoscopic flow. Mesoscale flow was modeled by White (1975) and White et al. (1975). Since the White model, which is also known as the patchy-saturated model, was established, many scientists studied seismic attenuation and velocity dispersion in the patchy saturated model (Dutta and Ode, 1979a, 1979b; Lopatnikov and Gurevich, 1988; Yin et al., 1992; Gurevich and Makarynska, 2012; Quintal, 2012; Tisato and Quintal, 2014; Hu et al., 2014; Yao et al., 2015; Pimienta et al., 2015; Spencer and Shine, 2016). Pride et al. (2004) illustrated that the microscale squirt flow mechanism describes attenuation in the low frequency range. Johnson (2001) modified the White model and theoretically analyzed the wave attenuation characteristics in patchy-saturated rocks. However, Johnson didn't propose a new seismic wave equation for patchy-saturated model and established the corresponding seismic wave equations for partially saturated porous rocks.

This paper is a sequel to our previous work (Zhang and He, 2015). In this paper, we obtain the reciprocal quality factor  $Q^{-1}$  through solving the seismic wave equations to study the frequency dependent attenuation of the patchy-saturated model. Moreover, we study the effects of porosity, permeability and fluid saturation on seismic attenuation and velocity dispersion in patchy-saturated rocks.

#### Theory and/or Method

The modified patchy-saturated model we established before is shown in FIG. 1. This model was proposed according to the White model which suggested that some regions were fully saturated with water and others were fully saturated with gas. FIG. 1a shows the rock skeleton and two kinds of fluids

where the dots represent for fluid 1 and dashes represent for fluid 2. For convenience, we rearrange the model to be as in FIG. 1b, which consists of two concentric spheres with radii  $R_a$ and  $R_b$ . The volume of the inner sphere, which is saturated with fluid 2 (gas pocket in the White model), is the total space of pores filled with fluid 2 in FIG. 1a. The outer space represents the rock skeleton and the pores filled with fluid 1. Besides White's assumption (White, 1975), we also assumed that there is no movement between fluid 1 and the skeleton but there is relative movement between fluid 2 and the skeleton. On the basis of the above, the dilatational wave equations in modified patchy-saturated model were established as follows:



FIG.1. Modified patchy-saturated model for porous media

$$\rho \frac{\partial^2 \theta}{\partial t^2} + \rho_{f_2} \frac{\partial^2 \varepsilon}{\partial t^2} = H \nabla^2 \theta + 2\gamma D \nabla^2 \varepsilon, \tag{1}$$

$$\rho_{f_2} \frac{\partial^2 \theta}{\partial t^2} + m \frac{\partial^2 \varepsilon}{\partial t^2} = 2\gamma D \nabla^2 \theta + 2D \nabla^2 \varepsilon - \frac{\eta_2}{\kappa} \frac{\partial \varepsilon}{\partial t},$$
(2)

with  $\rho = (1 - \phi)\rho_s + \phi S_1 \rho_{f_1} + \phi S_2 \rho_{f_2}$  and  $S_1 + S_2 = 1$ .

In the above equations,  $\rho$  is the total mass of the patchy-saturated porous rock per unit volume, and  $\rho_s$  is the mass density of solid grains;  $S_1$ ,  $\rho_{f1}$  and  $\eta_1$  (for later use) are the saturation, density and viscosity of fluid 1, respectively;  $S_2$ ,  $\rho_{f2}$  and  $\eta_2$  are the saturation, density and viscosity coefficient of fluid 2, respectively;  $\theta$  and  $\varepsilon$  are the volume strain of the "solid" (frame rock containing fluid 1) and that of fluid 2 relative to "solid", respectively;  $\phi$  and  $\kappa$  are the porosity and permeability of the rock, respectively; H is the plane-wave modulus of partially-saturated rock,  $H = K + 4/3 \mu$ , K and  $\mu$  are bulk and shear moduli of the patchy-saturated rock, respectively;  $\gamma$  and D are elastic coefficients which are related to parameters  $\phi$ ,  $S_1$ ,  $S_2$ ,  $K_{f1}$ ,  $K_{f2}$  and  $K_s$  (Zhang and He, 2015).  $K_{f1}$ ,  $K_{f2}$  and  $K_s$  are the bulk moduli of fluid 1, fluid 2 and solid grain, respectively; m is a parameter with the dimension of density and can be computed through  $m = s \rho_{f_2}/\phi S_2$ , where s is a structure constant depending on the pore structure and orientation; t is time.

The bulk modulus K of the patchy-saturated rock can be computed by the following formula Johnson(2001) established:

$$K(\omega) = K_{BGH} - \frac{K_{BGH} - K_{BGW}}{1 - \zeta + \zeta \sqrt{1 - i\omega\tau / \zeta^2}}.$$
(3)

For the expressions of  $K_{BGH}$ ,  $K_{BGW}$ ,  $\zeta$  and  $\tau$  see our previous work (Zhang and He, 2015).  $\omega$  is angular frequency of the seismic waves.

Solving wave equations (1) and (2), we can obtain the reciprocal quality factor and velocities:

$$v_{1,2} = \frac{\omega}{k_{1,2}} = \frac{\omega}{\operatorname{Re}(\sqrt{k_{1,2}'})}, \quad Q_{1,2}^{-1} = \frac{v_{1,2}\alpha_{1,2}}{\pi f} = \frac{2\alpha_{1,2}}{k_{1,2}} = \frac{2\operatorname{Im}(\sqrt{k_{1,2}'})}{\operatorname{Re}(\sqrt{k_{1,2}'})},$$
(4)

with

$$A = 4\gamma^{2}D^{2} - 2DH, \quad B = 2D\rho\omega^{2} - 4\gamma D\rho_{f_{2}}\omega^{2} + m\omega^{2}H + i\frac{\eta_{2}}{\kappa}\omega H, \quad C = \rho_{f_{2}}^{2}\omega^{4} - m\rho\omega^{4} - i\frac{\eta_{2}}{\kappa}\rho\omega^{3},$$
$$k_{1,2}^{'} = \frac{-B \pm \sqrt{B^{2} - 4AC}}{2A}, \quad k_{1,2} = \operatorname{Re}(\sqrt{k_{1,2}}), \quad \alpha_{1,2} = \operatorname{Im}(\sqrt{k_{1,2}}),$$

where  $k_{1,2}$  and  $\alpha_{1,2}$  represent the wavenumber and attenuation coefficient of two kinds of P-waves, respectively;  $v_{1,2}$  and  $Q_{1,2}^{-1}$  represent the velocity and the reciprocal quality factor of two kinds of P-waves, respectively. The symbol "Re" and "Im" represent real part and imaginary part of a complex number, respectively.

#### **Examples**

FIG. 2 shows that the reciprocal quality factor and velocity of the fast P-wave change with frequency when the rock has different porosity, permeability and gas saturation. The rock parameters we used are in Table1. The parameters we varied are porosity  $\phi$ , permeability  $\kappa$  and saturation  $S_1$  and  $S_2$ .

| frame<br>parameters | values                        | fluid<br>parameters           | values                          |
|---------------------|-------------------------------|-------------------------------|---------------------------------|
| $\phi$              | 0.3                           | $S_1$                         | 0.5                             |
| $K_s$               | 38×10 <sup>9</sup> Pa         | $S_2$                         | 0.5                             |
| μ                   | 14.61×10 <sup>9</sup> Pa      | $K_{f_1}$                     | 2.25×10 <sup>9</sup> Pa         |
| K                   | $1.0 \times 10^{-13} m^2$     | $K_{f_2}$                     | $1.0{\times}10^5$ Pa            |
| $ ho_{s}$           | <b>2650</b> $kg \cdot m^{-3}$ | $oldsymbol{ ho}_{f_1}$        | 1000 $kg \cdot m^{-3}$          |
| $R_a$               | 7.937cm                       | $oldsymbol{ ho}_{f_2}$        | <b>78</b> $kg \cdot m^{-3}$     |
| $R_{b}$             | 10cm                          | $\eta_{\scriptscriptstyle 1}$ | $1.0 \times 10^{-3} Pa \cdot s$ |
| S                   | 2                             | $\eta_2$                      | $1.0 \times 10^{-5} Pa \cdot s$ |

Table 1. Elastic parameters of the patchy-saturated model

From FIG. 2a, we can see that the fast P-wave exhibits large attenuation in patchy-saturated rocks within the seismic frequency band and the attenuation becomes higher with increasing frequency and porosity, because the amount of fluid increases with increasing porosity at the same degree of saturation. From FIG. 2b, we can see that the peaks of the reciprocal quality factor move to high frequency with increasing

permeability. Furthermore, at low frequencies, the attenuation increases with decreasing permeability. The reason is possibly that there is low fluid mobility when the permeability is low and thus viscosity of the fluid is relatively high and leads to higher attenuation. FIG. 2c shows that the attenuation increases with decreasing gas saturation, which is consistent with the results that Nie et al. (2012) achieved through studying wave attenuation in the BISQ model, and Deng et al. (2012) obtained through studying wave attenuation in a periodic layered patchy-saturated model. Kuteynikova et al. (2014) also concluded that the attenuation of the fast P-wave with water saturation of 90% is greater than that with water saturation of 83.6% through numerical modeling and laboratory measurement of seismic attenuation in partially saturated rock.

FIG. 2d shows that velocity dispersion of the fast P-wave becomes obvious with increasing porosity. When the rock porosity is 10%, there is very little velocity dispersion, because the amount of fluid is too small. Velocity dispersion is obvious when the rock permeability is high, whereas it only appears in the low frequency band when the rock permeability is low (FIG. 2e). Therefore, low frequency is more important than high frequency for detecting oil and gas reservoirs. FIG. 2f shows that the velocity dispersion increases with increasing water saturation, which also agrees with Deng et al. (2012).



FIG. 2. Attenuation and dispersion of the fast P-wave with different porosity, permeability and gas saturation in patchy-saturated rocks: (a) Reciprocal quality factor versus frequency with different porosity. (b) Reciprocal quality factor versus frequency with different permeability. (c) Reciprocal quality factor versus frequency with different gas saturation. (d) Velocity versus frequency with different porosity. (e) Velocity versus frequency with different permeability. (f) Velocity versus frequency with different permeability. (a) Velocity versus frequency with different permeability. (b) Velocity versus frequency with different permeability. (c) Neuropersus frequency with different permeability. (c) Velocity versus frequency versus frequency with different permeability. (c) Velocity versus frequency versus frequency with different permeability. (c) Velocity versus frequency versus frequency with different permeability. (c) Velocity versus frequency versus fre

### Conclusions

We derived the reciprocal quality factor and velocity versus frequency from the seismic equations in a modified patchy-saturated model and studied the seismic attenuation and velocity dispersion of the fast P-waves in patchy-saturated rocks. By studying the effects of porosity, permeability and fluid saturation on seismic attenuation and velocity dispersion, we obtained: First, seismic attenuation increases with increasing porosity and frequency and decreasing gas saturation within the seismic band. Attenuation peaks move to high frequencies as rock permeability increases. Moreover, at low frequencies (below about 10Hz), attenuation increases with decreasing permeability. Second, velocity dispersion becomes severe as the porosity goes up and the gas saturation goes down. It is obvious only at low frequencies when the permeability is low.

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