Finite-difference modeling comparison between a fractured HTI model and its equivalent model
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Summary
In this study, we compared a 3D finite-difference elastic modeling of a periodically layered isotropic vertically fractured HTI model with its 3D finite-difference equivalent modeling obtained from linear slip theory to verify the suitability of these two modeling approaches for anisotropic studies. We focused on P-wave and PS-wave reflections from the top and bottom of the fractured HTI medium. Although, geophysicists often prefer to use homogeneous equivalent models for various seismic modeling and imaging tasks, there are however some benefits of using isotropic heterogeneous models over homogeneous equivalent models. We show that the modeling using equivalent medium theory predicts strong interbed multiples and multimodes which are attenuated in the heterogeneous isotropic vertically fractured model. This is because a heterogeneous medium will cause irregular scattering of multiples and multimode events, thus diminishing these events. We infer that in some circumstances modeling using heterogeneous elastic model might be of higher processing and imaging value than with equivalent media.

Introduction
There are various advantages of 3C data, but one of the most important is the use of P and S waves for reservoir characterization – for lithology identification and fracture analysis (Qian et al., 2007; Bale et. al. 2009; Mahmoudian, et. al. 2013; Al Dulaian and Margrave 2016). In this paper, we will compare seismic AVO/AVAZ responses obtained from a finite-difference numerical modeling of (a) HTI model obtained from periodically layered isotropic vertical fractures and (b) HTI model obtained from Schoenberg and Muir (1989) equivalent model theory

The linear slip theory
Based on the continuum hypothesis and the linear slip assumption, fractures are treated as infinitely thin compliant layers or as planes of weakness with non-welded or imperfectly welded interface. This requires that only traction be continuous and displacement field need not be continuous. Proposed by Schoenberg (1980), Schoenberg and Muir (1989) extended the Backus average approach to develop a matrix formalism for the calculation of the average stiffnesses (and compliances) of fractured rock and then achieves a description for the composition and decomposition of the effective anisotropic medium. Carcione et al., 2012 conducted a numerical simulation to show that the Schoenberg-Muir theory is valid from the kinematic (traveltimes) and dynamic (amplitudes) viewpoints for a small crack aspect ratio or fracture opening, very long flat parallel fractures and thin layered media. They conclude that a fracture as an infinitely extended weakness plane is an element for assembling a fractured medium from a fracture and a host medium. A fractured medium can be separated into a fracture and a host medium. In this view, a vertically fractured HTI medium consists of a set of vertical linear slip interfaces embedded in an isotropic homogeneous host medium

Method
We used the SINTEF TIGER finite-difference elastic and anisotropic modeling codes to generate the seismic response from two similar HTI models: (1) a periodic isotropic vertically fractured HTI elastic model (hereafter simply referred to as *the elastic model*) and (2) an anisotropic homogeneous equivalent medium obtained from Schoenberg and Muir linear slip method (hereafter simply referred to as *the equivalent model*). The model is a 400m thick HTI layer sandwiched between isotropic half-spaces with thicknesses of 1000m and 600m respectively. Fracture spacing is 20m and fracture strike is 0 degrees in the azimuthal plane. Maximum offset range is 2km. The source, an explosive P-source with a 15 Hz Ricker spectrum, is located at the center of the model, at depth 40m. This gives full azimuthal coverage through all 360 degrees. 3C receivers were placed on each of the grid points and buried at source depth. 3D shot records were generated for both the elastic and the equivalent model. Modeling was done with a grid size of 201x201x101 and equal physical grid spacing \(dx = dy = dz = 20m\). We applied a 2D linear interpolation at every time-slice and translate the dataset from its acquisition domain to the offset-azimuth domain \(Z_{t,r,p}, R_{t,r,p} \) and \(T_{t,r,p} \). Figure 1 shows the geometry of our model and figure 2 shows a brief workflow. The parameters of equivalent model and elastic model are shown in Table 1 below.

<table>
<thead>
<tr>
<th>Table 1: Parameters of model used in modeling.</th>
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<tr>
<td>Periodic HTI model ((V_p ) and (V_s ) in km/s, (\rho ) in g/cm³)</td>
</tr>
<tr>
<td>Top layer</td>
</tr>
<tr>
<td>HTI layer</td>
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<tr>
<td></td>
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<tr>
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<tr>
<td>Base</td>
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Modeling in elastic and equivalent HTI medium

Figure 1. Three-layer model and survey geometry of shot and receivers showing fracture strike direction. The diagram on the right shows the plan view of constant offset azimuthal scans and the constant azimuth radial scans.

Figure 2. Workflow

Results

The vertical (Z), radial (R), and transverse (T) components datasets shown in figure 3 (a-c) are the constant azimuth radial scans along azimuths 0, 15, 60, 90, 105, and 150 degrees to the fracture strike with the fracture strike at 0 degrees. The finite-difference modeling predicts very similar arrival times to raytracing shown in figure 3 as solid red line (denoting PP arrival), dashed red line for PPPP arrival, solid blue line for PS arrival and dashed blue line for PPSS arrival. The left panel of figure 3 is the FD elastic modeling with isotropic HTI fractures; the right panel is the FD anisotropic equivalent modeling with Schoenberg’s linear slip effective model. Both elastic and equivalent modeling gave correct moveout behavior, however, the elastic modeling at later arrival times show fewer events than the equivalent modeling due likely to the fact that multimodes propagate less coherently in the elastic model due to the heterogeneity of the HTI layer.

Constant-radius azimuthal scans

Figure 4 (a-c) shows the common offset azimuthal scans of the vertical, radial and transverse datasets at offset of 1.6km for elastic modeling (left panels) and equivalent modeling (right panels). The signatures of shear-wave splitting are most clearly diagnosed from the polarity changes recorded along the principal axes on the transverse component $T_{r,\phi}$ alone. On the $Z_{r,\phi}$ (figure 4a) and $R_{r,\phi}$ azimuthal scans (figure 4b), the PS1 and PS2 waves overlapped and there is a need to further separate them into PS1 and PS2 domains. There is a strong multimode event appearing at 1.3 seconds in both modeling. The lateral heterogeneity in the periodic vertical fractured media (the left panels) justify the greater attenuation of multimodes in the elastic modeling (left panel) than in the equivalent modeling (right panel).

Figure 3a: Z component dataset at fixed azimuths. Elastic modeling is on the left and the equivalent model on the right.

(3b) Like 3a but the R (radial) components.

(3c) Like 3a but the transverse T components

Figure 4a. Z-azimuthal scan: elastic modeling (left) and equivalent modeling (right) at source-receiver offset of 1.6km. The red and blue line indicates PP and PS primary arrivals from HTI interfaces. The sinusoidal appearance of P waves shows fracture-induced PP azimuthal anisotropy whose imprint grows at increasing offset. There is a multimode event appearing at 1.3 seconds in both models which appear stronger in the equivalent modeling.
Modeling in elastic and equivalent HTI medium

Figure 4b. R-azimuthal scan: elastic modeling (left) and equivalent modeling (right), showing overlap of PS1 and PS2 mode. The sinusoidal appearance at time 1 seconds is the side effect of the overlap between S1 and S2 modes with a time delay between them. Same as at time 1.07 seconds.

Figure 4c. T-azimuthal scan: elastic modeling (left) and equivalent modeling (right), showing clear separation of S1 and S2 modes indicating shear wave birefringence. The polarity of both the fast and slow shear waves reverses across the symmetry planes. The green and orange arrows represent the isotropy and symmetry respectively.

**AVO/AVAZ studies**

The PP- Ruger amplitude modeling (panel 3 of figure 5) obtained from the equivalent model parameters shows strong coherent AVO/AVAZ behaviour and is a close comparison to the finite-difference elastic PP modeling result (panel 1 of figure 5) modeled from using vertical isotopic fractures and it is less noisy and has stronger azimuthal response than the equivalent PP modeling result (panel 2 of figure 5). We can infer that P-wave fracture-induced seismic anisotropy is stronger in heterogeneous isotropic fractured models than in homogenous equivalent models. A close study of the PP elastic and equivalent modeling (see panel 4 and 5 of figure 5) show that PS-wave AVOAZ response from equivalent modeling (panel 5) at offset/depth ratio less than 1 is comparably like the elastic modeling response of the isotropic HTI model (panel 4). The PS- AVAZ behavior at far offset however, have the same AVOAZ pattern but slightly stronger in the left panel than the right panel; the PS- elastic modeling result shows that shear wave splitting in the transverse mode are more clearly distinct at principal orientations than in the equivalent modeling. We also observed that both elastic and equivalent modeling of the converted-wave responses show stronger and earlier onset of anisotropy that spans a wider offset than their PP counterparts.

**PP and PS azimuthal amplitude anisotropy**

Given P and S wave azimuthal amplitudes at 0.4, 1, and 1.6km indicated by the blue, red and yellow lines (Figure 6), HTI model symmetry cause P and S wave amplitudes to deviate from circles to ellipses or nearly elliptical because amplitude varies with azimuth of propagation. The major axis of the PP reflection amplitude elliptical signature (row 1 of figure 6) points in the direction of fracture strike while the major axis of the PS wave-type R-mode reflection signature (row 2 of figure 6) is normal to the strike. We also see that the Ruger’s PP amplitude modeling and the PP elastic modeling show close resemblance and have stronger anisotropy than the PP equivalent modeling.

**PP and PS azimuthal interval traveltime anisotropy**

Figure 7 shows the traveltine difference in reflection arrivals between the top and bottom of the HTI layer. P-wave traveltime anisotropy in both elastic and equivalent models are not very evident at small offset but become evident at very large offset as indicated by the yellow ellipse. The modeling results also show that R-component azimuthal interval-traveltimes have weak elliptical distribution both in the isotropic fractured HTI models and its equivalent models. We infer that, for this model, the PP and PS amplitude anisotropy are more diagnostic of fracture orientation than their interval travelttime counterparts.

Figure 5. 2-D PP and PS- waves AVOAZ panels recorded from the top of the HTI reflector.
Modeling in elastic and equivalent HTI medium

Figure 6. Azimuthal Amplitude offset dependent anisotropy from top of HTI layer. The blue, red, and yellow lines are the azimuthal anisotropy at radii of 0.4, 1, and 1.6km from the source respectively.

Figure 7. Azimuthal interval-travel time offset dependent anisotropy between top and base of HTI layer. The blue, red, and yellow lines are the azimuthal anisotropy at radii of 0.4, 1, and 1.6km from the source respectively.

Conclusions

We have successfully carried out a 3D numerical modeling comparison between a vertically fractured isotropic heterogeneous HTI model and its homogeneous equivalent model computed from Schoenberg and Muir’s linear slip theory with the goal of understanding the benefits of using either of these model types in understanding seismic anisotropic response and noise challenges in multicomponent data processing. We observed that both models give the same moveout but the strength of AVO/AVAZ differs slightly between the two models. The homogeneous equivalent modeling is susceptible to strong multiple and multimode interferences at later times which are largely attenuated in the vertically fractured isotropic HTI models due to irregular scattering effect caused by the fractured medium heterogeneity. We infer that in some circumstances modeling using heterogeneous elastic models might be of higher imaging value than with equivalent media. Also, azimuthal anisotropy is weaker in the equivalent modeling than in the elastic modeling and the PS wave AVAZ analysis show stronger azimuthal anisotropy in the elastic model. In addition, PP and PS waves AVAZ analysis can be a better diagnostic tool for fracture orientation detection than interval-traveltime alone especially where overburden effect on anisotropy is very minimal.

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