SUMMARY

In the microseismic field, a complete characterization of events involves determining the positioning and focial mechanism properties, which requires an accurate seismic velocity model. Conversely, the construction of a velocity model with surface seismic data suffers from a lack of full illumination, a deficit which could in principle be mitigated by microseismic event ray paths. The velocity model and event characterization problems are in this sense strongly interconnected, each, if it were solved, supporting the other. Assuming an imperfect velocity model derived from surface seismic data as a starting point, and microseismic events as data, a full waveform inversion (FWI) methodology involving simultaneous updates in both source property model parameters and velocity model parameters can be formulated. The resulting gradient and Hessian quantities can be used to solve the inverse problem for all unknowns, or, equally importantly, to help quantify the cross-talk between velocity and source models, acquire acquisition, and act as a tool for appraisal. In this paper we formulate a basic scalar acoustic version of microseismic full waveform inversion (MFWI), set out the forms for the gradients, present an early-stage numerical analysis of MFWI updates, and comment on some important aspects of cross-talk between velocity model and source properties.

INTRODUCTION

Many (perhaps most) seismic inverse problems are in principle amenable to treatment with full waveform inversion methods (e.g., Virieux and Operto, 2009), but the feasibility of FWI formulations varies widely. Data limitations, acquisition parameters, number of unknowns, ray-path coverage, etc., determine the character of FWI implementations, and so any new candidate FWI formulation with a novel combination of these features must be freshly analyzed. A central problem in current seismic technology development is microseismic source characterization, i.e., the determination of the positioning, timing, source time signature, and moment tensor/focal mechanism of the microseismic event (Baig and Urbancic, 2010). Whether full waveform methods can be formulated. The resulting gradient and Hessian quantities can be used to solve the inverse problem for all unknowns, or, equally importantly, to help quantify the cross-talk between velocity and source models, optimize acquisition, and act as a tool for appraisal. In this paper we formulate a basic scalar acoustic version of microseismic full waveform inversion (MFWI), set out the forms for the gradients, present an early-stage numerical analysis of MFWI updates, and comment on some important aspects of cross-talk between velocity model and source properties.

The problem has been formulated and discussed several times in the literature. One approach is outlined by Kaderli et al. (2015), involving an acoustic, gradient-based FWI that inverts for the source characteristics alone, with a fixed velocity model. Huang et al. (2017) formulate a similar approach, using FWI to invert for the source location on field microseismic data. Likewise, Sethi and Shekar (2017) formulate an FWI scheme for determining the source location and waveform shape, and explore regularization using sparsity constraints. Michel and Tsvankin (2017) invert for both source location and velocity model in a VTI medium, but with the source location update and velocity model update treated sequentially, not simultaneously. These approaches which separate the source and velocity model characteristics make the implicit (and sensible) assumption that simultaneous errors in velocity model and source parameters will be difficult to reduce. We deliberately set up a simultaneous multiparameter FWI scheme, with the goal being to induce cross-talk and analyze its character in detail.

In this paper we formulate a simple 2D time domain scalar acoustic MFWI framework permitting updates in P-wave velocity and source position parameters either independently or simultaneously. Source numbers, positions, timing and frequency content can be varied, as can the velocity model. In the context of this model we examine:

1. Basic character of the source positioning gradient;
2. Energy distribution in the velocity model of kernels excited by sources at depth; and
3. Leakage of source position information into velocity model updates.

We then comment on next steps.

FORMULATION AND ALGORITHM

Formulation

We base our MFWI formulation on the objective function

\[ \phi = \sum_{s_i, r_i} \frac{1}{2} \int \left( d(r, s, t) - p(r, s, t | s_i, s_j) \right)^2 dt, \]

where \( r \) is the receiver position, \( s \) is the source position, \( s_i \) is the squared-slowness velocity model, \( s_j \) is the source distribution, \( d(r, s, t) \) are the observed data and \( p(r, s, t | s_i, s_j) \) is the modelled data; \( p \) is assumed to satisfy

\[ \left( \nabla^2 - s_i(t) \frac{\partial^2}{\partial t^2} \right) p(r, s, t, t' | s_i, s_j) = s_j(r, s, t'). \]

The Newton system

\[ \begin{bmatrix} \frac{\partial \phi}{\partial s_i} \\ \frac{\partial \phi}{\partial s_j} \end{bmatrix} = - \begin{bmatrix} H_{ss} & H_{sp} \\ H_{ps} & H_{pp} \end{bmatrix}^{-1} \begin{bmatrix} g_e \\ g_s \end{bmatrix} \]

is formulated, where

\[ g_e = \frac{\partial \phi}{\partial s_i} \quad \text{and} \quad g_s = \frac{\partial \phi}{\partial s_j} \]

are the gradients with respect to vectors \( s_i \) and \( s_j \) containing discretized realizations of the velocity model and sources, and the Hessian consists of block matrices of second order partial derivatives of \( \phi \); the
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The system is solved for the velocity model updates \( \delta s \) and the source updates \( \delta s_c \), and the process is repeated. The gradients are

\[
g_s(r) = - \sum_{t_p,t_c} \int dt \hat{\delta} p(t_p,r,s_c) g(t_p,r,t-t^*|s,s_c). \tag{5}
\]

where \( \hat{\delta} p(t_p,r,s_c) \) are the residuals, and \( t^* \) is the origin time of the source. Similarly, the velocity-model gradient is

\[
g_v(r) = - \sum_{t_p,t_c} \int dt \hat{\delta} p(t_p,r,s_c) \times \int dt' \int dt'' g(t_p,r,t-t''|s_c,s) \times \frac{\partial^2 g(t_p,r,t'|s_0,s_0)}{\partial t'^2} s_0(r,t') \tag{6}
\]

A 2D python implementation of time domain scalar acoustic FWI (Almuter, 2017), implementing equation (6) and updating with a backtracking line search algorithm, has been adapted to add the source gradient term. Currently the Hessian operator is not included in the algorithm, i.e., we set \( H = I \) in equation (3), though it will likely play an important role in management of cross-talk between source and velocity model.

**Algorithm and source position gradient example**

**Example 1:** Source updates leaking into the velocity model

The MFWI update is clear, however we have found empirically that updating by simply taking the maximum of the gradient as the next source position tends to converge towards the best result in our formulation.

The iterations and updates continue until stopping criteria are met for both the source term and velocity model term. For the source term, the stopping criterion is based on the distance to the true source location. If the updated source location is within \( \sqrt{3} \) grid positions of the true position, the source term is considered to be converged. The stopping criteria for the velocity model term is based on the line search comparing the gradients before and after the update. If there is no improvement with the update, the velocity model term is considered to have converged.

**CHARACTERISTICS OF MFWI UPDATES**

In MFWI we update the source location and the velocity model. Cross-talk between the two occurs when the gradients ascribe residual energy which is due to the source location to the velocity model, and vice versa. Some types of cross-talk are suppressed by incorporating second order or curvature information from the objective function, via the Hessian, especially the off-diagonal elements of the Hessian. Gradient-based updates, such as the ones we are analyzing in this paper, are however exposed to cross-talk. Inducing cross-talk is deliberate in our current study, in order to understand the ambiguities between source and velocity model characteristics; our intent is to use these results to guide the choice of optimization, degree and type of Hessian information to incorporate, and regularization and prior information to impose.

**Example 1:** Source updates leaking into the velocity model

Consider a homogeneous background medium, with sources in a horizontal line in the middle, and receivers along the surface. This sort...
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Figure 3: Results of MFWI after 5 iterations for example 1. Panel (a) shows the velocity model, with the velocity in the colorbar, and the source locations indicated by the circles. Panel (b) shows the source-term gradient for the fifth source at the fifth iteration. The correct location is indicated by the red dot, and the location at that iteration is indicated by the green dot.

Figure 4: Results of MFWI after 5 iterations for example 2. Panel (a) shows the velocity model, with the velocity in the colorbar, and the source locations indicated by the circles. Panel (b) shows the source-term gradient for the source at the fifth iteration. The correct location is indicated by the red dot, and the location at that iteration is indicated by the green dot.

of geometry is closest to the microseismic set-up, where there are surface sensors, and microseismic events occurring at the depth of the reservoir. Source number 5 in the middle has a starting position that deviates from the true position. The initial velocity model is equal to the actual velocity model. So, in this case MFWI needs only to correct the position of the deviated source. This set-up facilitates one aspect of the study of cross-talk, specifically, leakage into the velocity model caused by the source positioning error. Figure 2 shows the geometry, and Figure 3 shows the velocity model after 5 iterations, as well as the source-term gradient at that iteration. All of the sources were Ricker wavelets, each with a central frequency of 10Hz.

In Figure 3a and b, we can see the velocity model gradient and source term gradient after 5 iterations, along with the source locations overlain (green meaning current updated source position from the maximum of the update). The source location converges to a position very close to the correct location at the end of that iteration. However, in the process the inverted velocity model has accumulated small amplitude, but non-negligible, artificial structure resembling source and receiver sampling and aperture artifacts. At the location of the fifth source, there is a velocity anomaly that resembles a dipole.

In order to test if this dipole-like velocity artifact was a function of the geometry, the same test was repeated with receivers on the top and bottom of the model, but not on the sides, and then again with receivers on all four sides. We find that more artifacts appear in the presence of greater limitations in receiver geometry, but, the overall character of the velocity and source positioning solutions stay consistent.

We conclude that with surface-only receiver coverage, the MFWI tends to converge towards useful source positioning solutions, but if velocity model updates are sought simultaneously artifacts tend to accrue.

Example 2: Poor source coverage

The previous example demonstrated the effects of having limited receiver coverage, but there are also tradeoffs with having insufficient source coverage. To study an extreme case, consider a homogeneous background model and one source at the position of source 5 in Figure 2. Let this source have a starting position 150 meters above the true position. Since we saw that there are few changes with a change in geometry, we use a surface only receiver geometry.

Figure 4 shows the results after five iterations. The source converges at the correct location, but there is a clear artifact in the velocity that appears to start from the original starting position and move downward toward the true position. As with the previous examples, the behaviour is that of a dipole, but in this case the difference between the peak and trough is significantly larger. Therefore, with limited source coverage, the size and magnitude of the cross-talk increase substantially.

Example 3: Source term cross-talk

Having characterized the cross-talk on the velocity term due to the source term, we can now do the opposite by studying the cross-talk on the source term due to the velocity. In this case, we choose a surface
receiver geometry with one source in the centre of the model. The starting position of the source is equal to the true position, so there should be no update to the source position. However, we introduce a change in the velocity model that is not seen in the starting velocity model in the form of a circular anomaly of radius of 75 meters, and a velocity of 1800 m/s. After five iterations, Figure 5 shows the inverted velocity and source term gradient in that case. With only one source, the velocity anomaly cannot be recovered, and we see a velocity update that resembles the results in Example 2. However, in Figure 5b we can see that the source term gradient now has a circular anomaly that is the same shape and is in the same position as the velocity anomaly. Therefore, the cross-talk in this case expresses itself by introducing anomalies to the source-term gradient. Though this did not change the inverted source location in this case, a more significant or complex velocity anomaly could.

DISCUSSION

Based on the previous examples, we can see that the number of sources and source coverage is more important than the receiver geometry. Considering that most microseismic surveys have a limited number of receivers, this is an encouraging result. Often, there are thousands of microseismic events recorded during a hydraulic fracturing completion, so conceivably there would be sufficient illumination to avoid significant artifacts.

With a clearer picture of the exact nature of the cross-talk between the two parameters, it is possible to come up with a regularization to help remedy the situation. The necessary modification would penalize the addition of sharply contrasting, dipole-like features to the velocity model update. Future work involves exploring adding smoothness constraints to the velocity model.

Future work also involves changing the line search for the velocity model term to a more computationally efficient and robust implementation. The stopping criteria for both gradients are also worth amending. It would be valuable to define the stopping criteria for the source term gradient in terms of the character of the gradient itself, and not simply its maximum. In terms of the cross-talk, it would be valuable to incorporate the Hessian into this implementation and observe the effects of the off-diagonal terms, $\mathbf{H}_{cc}$ and $\mathbf{H}_{sc}$. Finally, with real microseismic events, each event has a different dominant frequency, and those frequencies tend to be higher than conventional seismic sources. Therefore, a proper MFWI implementation would properly adjust for varying frequency content to build up different wavelengths of the velocity model. Ultimately, the goal of this work is to develop into a moment tensor inversion scheme on the backdrop of an elastic model that can be applied to field data.

CONCLUSIONS

We have formulated a simultaneous, multi-parameter MFWI scheme to invert for both source location and velocity model updates from microseismic waveforms, and outlined a workflow for implementation of this method in an acoustic environment. Understanding the cross-talk between parameters in multiparameter FWI is an important and worthwhile endeavour. By showing different examples of cross-talk in our microseismic FWI implementation, we illustrate that the character of the cross-talk due to an erroneous source location on a velocity model has the character of a dipole. Similarly, we showed that the cross-talk on the source-term gradient due to an error in the velocity model mimics the structure of the velocity model. Further work on the choice of parameterization in an elastic environment is necessary, as well as a mechanism (be it through regularization or the Hessian) to suppress cross-talk.

ACKNOWLEDGEMENTS

We thank the sponsors of CREWES for their continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13. Nadine Igonin is also funded through the NSERC PGS-D and Earl D. and Reba C. Griffin Memorial SEG memorial scholarship.