Parameter cross-talk and leakage between spatially-separated unknowns in viscoelastic FWI

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SUMMARY

Elastic and attenuative effects play a major role in the determination of seismic wave amplitudes. Viscoelastic FWI has the potential to recover more information from measured data by accounting for these effects. A major obstacle to the effective use of viscoelastic FWI is inter-parameter cross-talk. This is typically characterized through the use of radiation patterns, but these are not well suited to viscoelastic FWI, because (1) there is significant potential for cross-talk between variables distant from one another in space, and (2) interpreting the effect of frequency and phase dependence in radiation patterns is not straightforward. We present and examine a numerical approach to assessing viscoelastic cross-talk. With it, we observe strong cross-talk both between velocity and Q variables, and into density for a variety of acquisition geometries. Of particular note is our characterization of the tendency for Q variables to leak into elastic variables from which they are spatially separated. This type of cross-talk is not easily characterized through the use of radiation patterns.

INTRODUCTION

Full waveform inversion (FWI) attempts to find the subsurface model best describing the full information content of a seismic experiment (Tarantola, 1984). While this goal cannot be completely achieved, techniques which come closer to it have greater potential to recover useful, accurate information about the subsurface. Much of the information constraining subsurface elastic properties resides in the amplitude and phase of the waveforms. Scalar-acoustic FWI lacks the wherewithal to use such information effectively, due to the neglect of elastic and attenuative effects. Elastic, viscoacoustic, and, less frequently, viscoelastic FWI approaches have been developed to address this deficiency. As in any multi-parameter FWI problem, inter-parameter trade-off, or 'cross-talk', is a major obstacle to the implementation of these approaches (e.g. Alkhalifah and Plessix, 2014; Pan et al., 2016). Cross-talk occurs when data residuals caused by an error in the estimate of one physical property are attributed to another, impeding convergence. Strategies exist for cross-talk reduction, but to design these effectively it is important to understand the cross-talk process; also, to determine which properties are leaking in to one another, and to what extent.

One tool for characterizing cross-talk is radiation pattern analysis (e.g. Tarantola, 1986; Moradi and Innanen, 2016; Oh and Alkhalifah, 2016). Radiation patterns express the change in an incident wave-field after interacting with a point scatterer, typically in an otherwise homogeneous medium. These patterns change for different choices of model perturbation. Greater cross-talk is typically experienced when inversion variables share similar radiation patterns in the measured part of the wavefield, because these variables have comparable effects on the measured data. Radiation patterns are typically not investigated for every variable, as the number of these is very large. Instead, a representative radiation pattern for each parameter type is investigated, with the scattering point set at a fixed location. Parameters with similar radiation patterns in a given angle range are predicted to experience cross-talk, and 'leak' into one another. In this approach one assumes that the radiation patterns at similar scattering angles represent similar data signatures, which in turn means assuming that the variables being confused are spatially coincident.

In elastic and anisotropic FWI, radiation pattern analysis has proved to be very useful. By choosing inversion parameters such that there is little overlap between different radiation patterns in the data, the extent of cross-talk in the inversion can be reduced (Virieux and Operto, 2009; Oh and Alkhalifah, 2016). Scattering patterns do not, however, completely characterize cross-talk (Pan et al., 2018).

Including attenuation in FWI complicates cross-talk in a way that may not be well managed through radiation pattern analysis (Pan et al., 2018). Cross-talk involving Q is likely to involve different parameters at different points in space. For instance, a density perturbation may cross-talk with a remote Q region obscuring it from the sources and receivers. This type of cross-talk is not easy to characterize with radiation patterns, as two different scattering angles for variables at different locations in model space may represent the same part of data space. Another complication is the frequency and phase dependence of these radiation patterns, which are key to distinguishing Q from velocity (Keating and Innanen, 2017), but whose role in reducing cross-talk is more difficult to discern from a radiation pattern alone.

Here we describe, and analyze with 2D frequency-domain simulations, an alternative approach to characterizing viscoelastic cross-talk, by comparing numerical FWI tests on simple models. This approach allows cross-talk between parameters to be understood as a function of either of or both incidence angle and/or frequency; it naturally accounts for the effect of iteration and spatial distribution on parameter resolution. If the models considered are suitably chosen, these results allow general conclusions to be drawn about the nature of cross-talk.

THEORY

To simulate viscoelastic wave propagation, we solve by the finite difference method the 2D viscoelastic system described by (Pratt, 1990):

$$\omega^2\rho u_x + \frac{\partial}{\partial x}\left[\tilde{\lambda}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)+2\tilde{\mu}\frac{\partial u_x}{\partial x}\right] + \frac{\partial}{\partial z}\left[\tilde{\mu}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)\right]+f=0 \tag{1}$$

and

$$\omega^2\rho u_z + \frac{\partial}{\partial z}\left[\tilde{\lambda}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)+2\tilde{\mu}\frac{\partial u_z}{\partial z}\right] + \frac{\partial}{\partial x}\left[\tilde{\mu}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)\right]+g=0, \tag{2}$$

where ω is the angular frequency, ρ is the density, u_x and u_z are, respectively, the horizontal and vertical displacements, f and g are their respective source terms, and $\tilde{\lambda}$ and $\tilde{\mu}$ are the complex, frequency dependent Lamé parameters. Assuming a Kolsky-Futterman model of attenuation (Kolsky, 1956; Futterman, 1962), these are defined in terms of ρ, the P and S wave speeds, v_P and v_S, and Q values Q_P and Q_S by

$$\tilde{\mu} = \left\{v_S\left[1+\frac{1}{Q_S}\left(\frac{1}{\pi}\log\frac{\omega}{\omega_0}+\frac{i}{2}\right)\right]\right\}^2\rho \tag{3}$$

and

$$\tilde{\lambda} = \left\{v_P\left[1+\frac{1}{Q_P}\left(\frac{1}{\pi}\log\frac{\omega}{\omega_0}+\frac{i}{2}\right)\right]\right\}^2\rho - 2\tilde{\mu}, \tag{4}$$

where ω_0 is a reference frequency.

We formulate FWI in terms of five parameters, selected for their simplicity: $\alpha_1\rho$, $\alpha_2 v_P^{-2}$, $\alpha_3 Q_P^{-1}$, $\alpha_4 v_S^{-2}$, and $\alpha_5 Q_S^{-1}$, where α_n are scale terms introduced to improve conditioning. The elastic problem having been thoroughly investigated, our focus is on cross-talk involving the Q variables. The Q^{-1} parameterization is convenient because of its limited numerical range (in comparison to a Q parameterization). We define our measure of cross-talk as follows. We first introduce three models: the true model which we attempt to reconstruct, $\mathbf{m}$, the initial estimate of the model, $\mathbf{m}_0$, and a model identical to $\mathbf{m}$ in four of five parameters, but equal to $\mathbf{m}_0$ in the nth, $\mathbf{m}_n$. We then consider two FWI experiments. The result of the first is given by

$$\hat{\mathbf{m}} = \mathrm{FWI}\big(\mathbf{D}(\mathbf{m})\big), \tag{5}$$

where $\mathrm{FWI}(\cdot)$ represents the full waveform inversion operator, and $\mathbf{D}$ are data measurements from a given model $\mathbf{m}$ and acquisition geometry. The model $\hat{\mathbf{m}}$ is a mixture of both desirable model updates and cross-talk terms, in which different parameters are confused. A second model $\hat{\mathbf{m}}_n$ corresponds to the case in which the initial model for one of the parameters, labelled n, is correct, i.e., equal to the actual synthetic model used to simulate the data:

$$\hat{\mathbf{m}}_n = \mathrm{FWI}\big(\mathbf{D}(\mathbf{m}_n)\big). \tag{6}$$

The difference $\Delta\hat{\mathbf{m}} = \hat{\mathbf{m}} - \hat{\mathbf{m}}_n$ is the part of $\hat{\mathbf{m}}$ attributed to the error in parameter n. The parts of $\Delta\hat{\mathbf{m}}$ which involve changes in any parameter other than n, therefore, represents cross-talk from n into that parameter. Evidently cross-talk from each model parameter into the others is calculated by applying FWI to six models: once to $\mathbf{m}$ to determine $\hat{\mathbf{m}}$, and once to $\mathbf{m}_n$ to determine $\hat{\mathbf{m}}_n$ for each of the five parameters n.

NUMERICAL ANALYSIS OF VISCOELASTIC CROSS-TALK

We investigate a simple model designed to allow relatively general conclusions to be drawn. An unknown region of low Q (≈ 20) obscures from the sources and receivers a smaller region containing unknown changes in the elastic parameters (each a 10% increase over the background model). The elastic region is a small circle at the center; the low Q region is a larger circle containing it (Figure 1). The differences in spatial distribution of the model errors is chosen to discriminate cross-talk between variables at one location from cross-talk between spatially separated variables. We investigate three acquisition geometries, each with sources and receivers evenly spaced along one or several edges of the model (Figure 1). In Type 1, sources and receivers are placed along the top of the model, and reflections are the main source of information. In Type 2, sources/receivers are placed on both the top and bottom of the model, simulating a transmission or cross-well geometry. In Type 3, to examine fundamental features of cross-talk sources/receivers are placed on all four sides. Explosive sources are considered. Ten frequency bands were inverted, each containing five evenly-spaced frequency values. The upper end of the bands increased with iteration, the first spanning 1-2Hz, and the last 1-20Hz. One steepest-descent iteration occurred at each frequency band.

The amplitudes of the model changes $\Delta\hat{\mathbf{m}} = \hat{\mathbf{m}} - \hat{\mathbf{m}}_0$ differ considerably for the different acquisitions. To allow for comparison, the cross-talk amplitudes were defined as fractions of the largest amplitude in the model change Δm calculated for that acquisition type. We also used the ratio between the maximum absolute value of the cross-talk and that of the model update as a coarse but effective quantifier. This measure was chosen due to its relative insensitivity to model geometry.

Figures 2 and 3 are cross-talk plots for leakage into v_P and Q_P respectively. These leak strongly into one another, as expected. Cross-talk into these parameters from all others is substantial in the Type 1 acquisition; cross-talk from v_S to v_P remains high in Types 2-3. In the Type 3 acquisition the cross-talk from v_P into Q_P is spatially similar to the v_P anomaly, while the cross-talk from Q_P into v_P does not match the Q_P anomaly. We point out that the leakage of v_P into Q_P occurs primarily for variables close to one another (i.e., near the v_P anomaly), resulting in a small circles, similar to the v_P anomaly, dominating in the second column of Figure 3. This type of cross-talk arises from velocity and Q at or near the same point being confused with one another.

Our key observation has to do with leakage between spatially remote model points. If cross-talk for the reverse case, i.e., from Q_P into v_P, was more or less reciprocal, the third column in Figure 2 would contain shapes resembling the Q_P anomaly (a large circle). This is present, but we also note substantial cross-talk similar in shape to the v_P anomaly. This implies that cross-talk between spatially distant variables is occurring, where the effects of missing Q obscuring a reflector are mistakenly interpreted as a smaller reflection contrast. Next, cross-talk into density is plotted in Figure 4; responses are generally large from all parameters in all three acquisition geometries. The Q to ρ cross-talk is notable because it is largely spatially coincident with the ρ anomaly, not the Q anomalies. As discussed above, this indicates cross-talk between spatially separated variables. Unlike the cross-talk from Q_P into v_P, there is almost no signature of the Q anomaly shape here that would indicate cross-talk between co-located variables. Stronger cross-talk into v_S and Q_S is noted in Figures 5 and 6 than was apparent occurring within the P-wave parameters. It is possible that the use of explosive sources aggravates S-wave parameter leakage. The cross-talk from Q_S into Q_P is limited (Figure 3), but cross-talk from Q_P into Q_S is very strong. This may arise from an ambiguity between attenuation before and after mode conversion.

DISCUSSION

The above examples are informative about the modes of cross-talk present in viscoelastic FWI; a key next step is to implement viscoelastic FWI based on this information to reduce cross-talk. Radiation patterns are often used in elastic FWI to guide strategies for cross-talk reduction based on scattering angles. In many of these approaches, data from angle ranges within which only one parameter has a significant radiation energy are used to update just these parameters. If such ranges do not exist, alternate parameterizations may be sought. This is difficult to apply when considering attenuation, because velocity and Q radiation patterns are not made distinct by scattering angle information (Keating and Innanen, 2017).

Further, because velocity and Q contribute similarly to the wave equation, differing only in phase and frequency (equations 3 and 4), no re-parameterization avoids the scattering angle independence. This means that any approach to reducing cross-talk should be based on the use of the information in the Hessian, wherein knowledge of frequency and phase differences can be used. Because of this necessity, the key consideration when considering a parameterization choice is not how to find a parameterization in which cross-talk can be avoided through choice of the data and parameter subsets considered, but rather to find the parameterization in which the cross-talk information in the Hessian is most easily extracted. The strategy for characterizing cross-talk we discuss here may be an useful approach for assessing the effectiveness of different parameterizations in achieving this goal.

CONCLUSIONS

Inter-parameter cross-talk in full waveform inversion is often characterized through the use of radiation patterns. These are poorly suited for viscoelastic FWI because of the significant potential for cross-talk between variables distant from one another in space, and the challenge of interpreting the frequency and phase dependence of radiation patterns on cross-talk. Simple numerical simulations offer an alternate approach for characterizing cross-talk which may be better suited to the viscoelastic problem. Tests using this approach suggest that cross-talk between velocity variables and the corresponding Q variables is quite strong, and occurs both between variables at the same location and those far apart. Cross-talk from Q into density seems to occur primarily between spatially separated variables. Cross-talk into v_S and Q_S is very strong when using explosive seismic sources.

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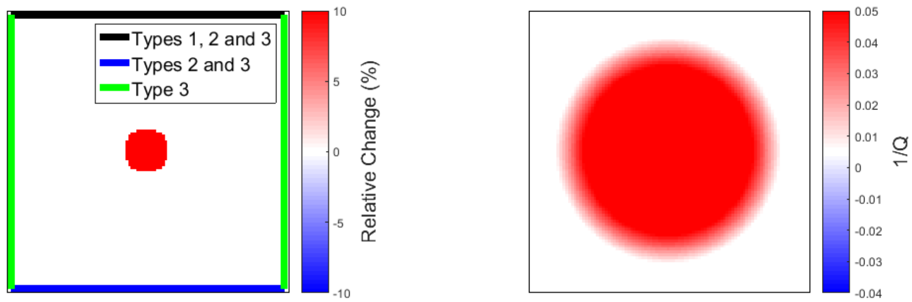

Figure 1: Change in elastic properties from background model in true model (left) and Q^{-1} for true model (right). In the left part of the figure, lines represent edges where sources and recievers are present for each acquisition type discussed.

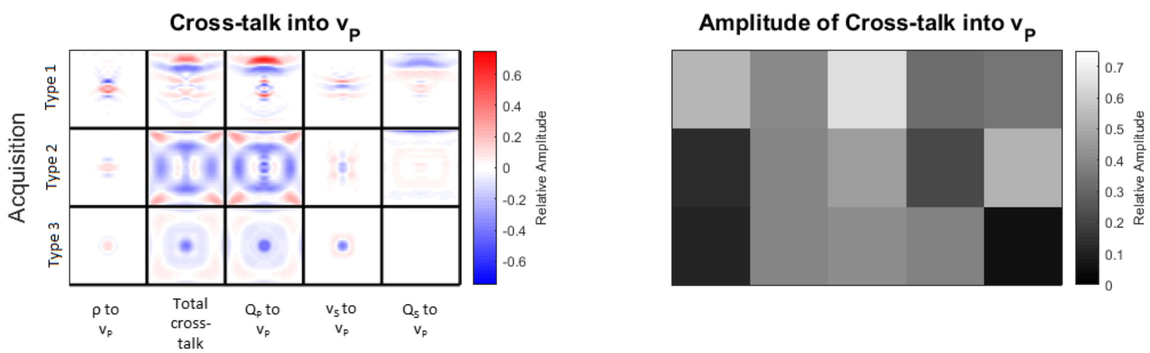

Figure 2: Numerically calculated cross-talk into v_P (left) and maximum value of cross-talk into v_P (right). Strong cross-talk from Q_P is evident for each acquisition geometry considered.

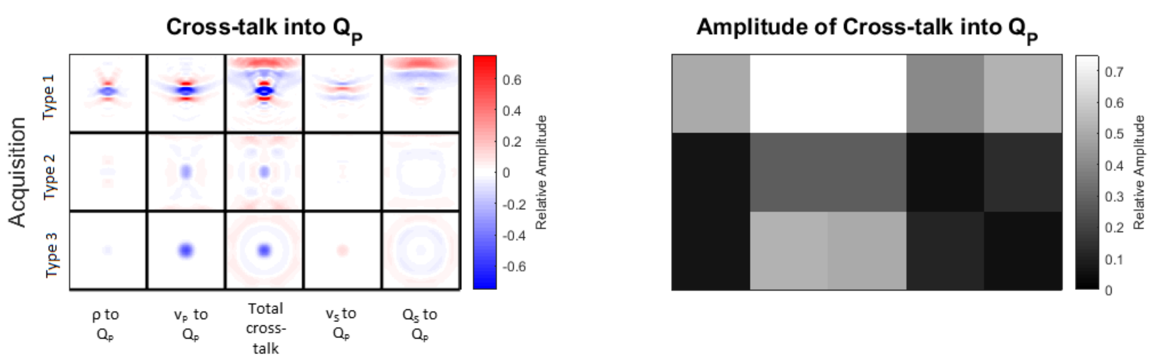

Figure 3: Numerically calculated cross-talk into Q_P (left) and maximum value of cross-talk into Q_P (right). Strong cross-talk from v_P is evident for each acquisition geometry considered.

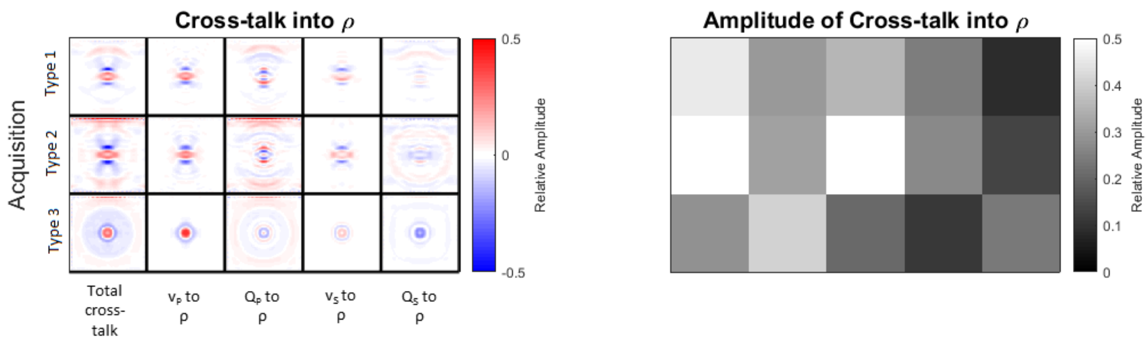

Figure 4: Numerically calculated cross-talk into ρ (left) and maximum value of cross-talk into ρ (right). Strong cross-talk from each other parameter is present in at least one model geometry. Cross-talk with Q variables takes place at the location of the density anomaly, not that of the low Q region. This may suggest cross-talk between spatially separated variables.

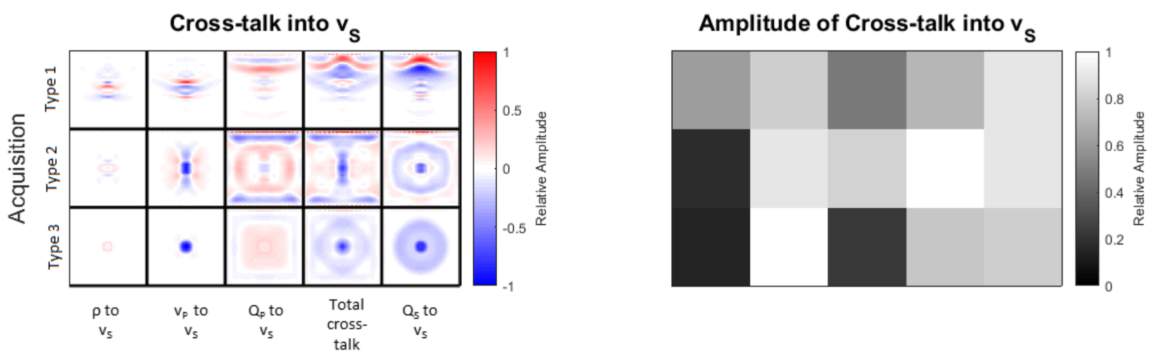

Figure 5: Numerically calculated cross-talk into v_S (left) and maximum value of cross-talk into v_S (right). Strong cross-talk with Q_S, v_P and Q_P is evident.

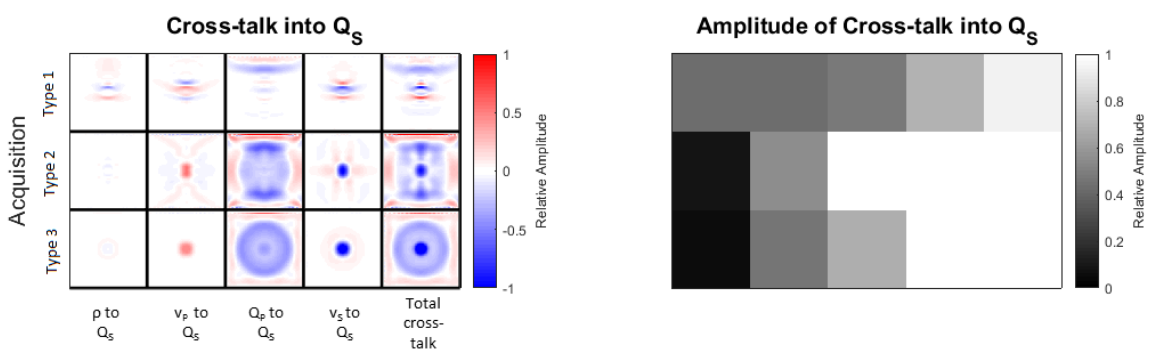

Figure 6: Numerically calculated cross-talk into Q_S (left) and maximum value of cross-talk into Q_S (right). Strong cross-talk with v_S, v_P and Q_P is evident.

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