Multiparameter inverse scattering: A preliminary computational approach

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CREWES

Introduction

- This study investigates the approach to imaging and inversion based upon the paper by Clayton and Stolt (1981) using inverse scattering methods.

- They discuss two cases, Imaging/inversion in a constant and variable background.

- Here we treat the 2D constant background case only.

- In this study:
  - Created synthetic data for 2D targets as input for the 2nd step,
  - Test a prototype inverse scattering algorithm using a single parameter only (velocity) as a check on the correctness and accuracy of this algorithm.

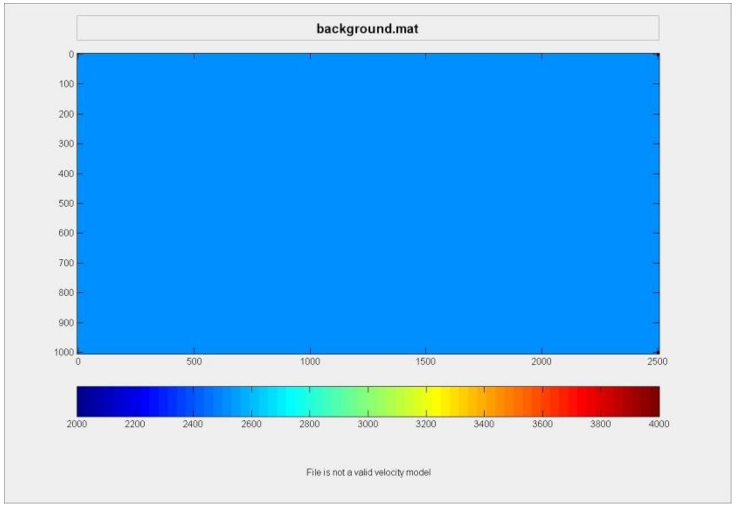
Forward Modeling

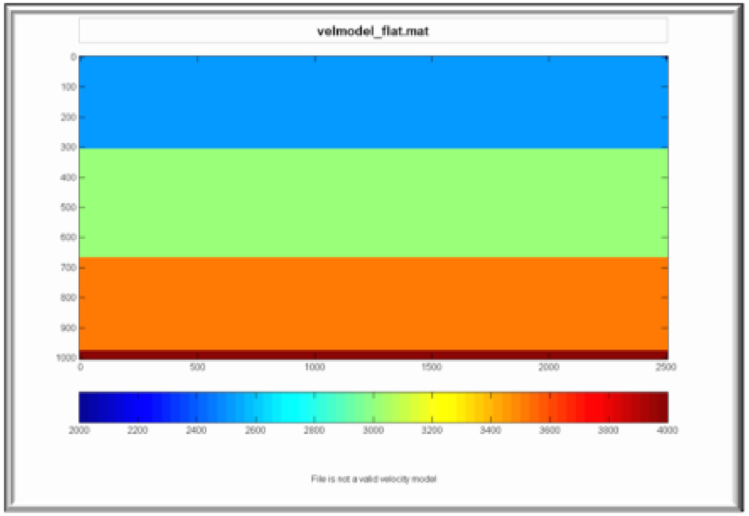
- Creation of a synthetic 2D velocity model using a gui based tool AFD_VELCREATE.

- Generate the scattered field:

(1) Forward model in the reference medium (constant velocity)

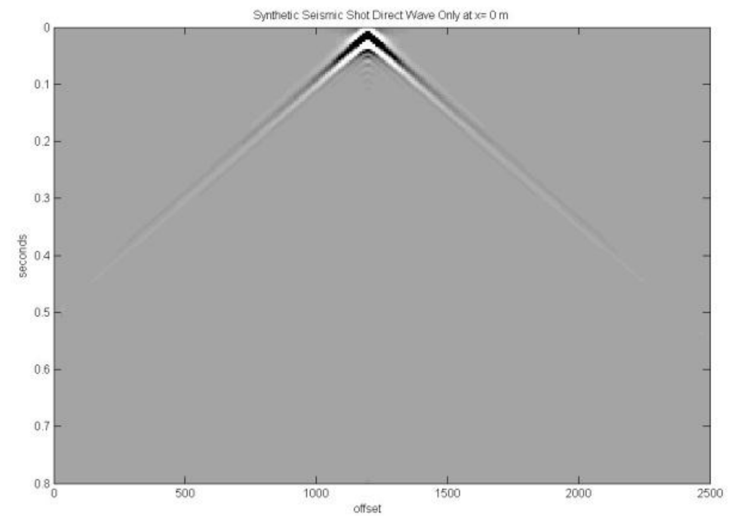
(2) Forward model in the layered medium using the CREWES FD routine AFD_SHOTREC to generate 2D shot records.

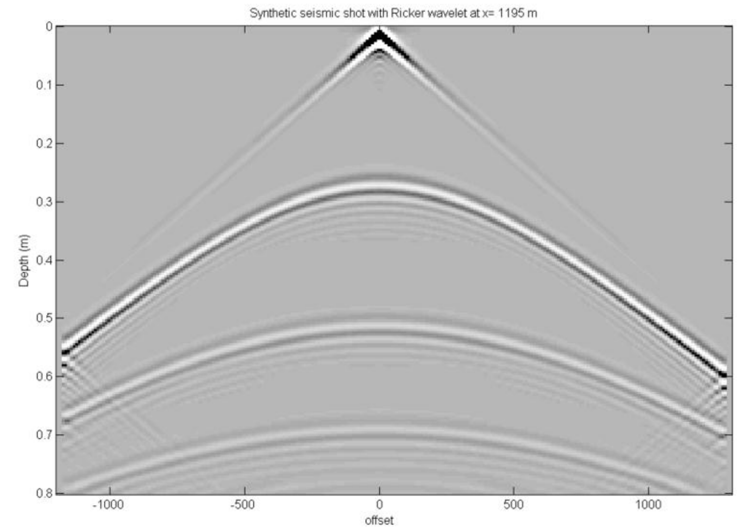




Forward Modeling

- Convolve with a Ricker wavelet.
- Generation of a background and model shotpoint gather from the background and model velocity profiles for each receiver.
- repeated shot gathers created for each receiver point along the desired seismic line with the desired receiver spacing.





Basic Scattering Theory

Lippmann-Schwinger Equation

$$G(\mathbf{r_g}|\mathbf{r_s};\omega) - G_r(\mathbf{r_g}|\mathbf{r_s};\omega) = \int_{V'} G_r(\mathbf{r_g}|\mathbf{r}';\omega)V(\mathbf{r}')G(\mathbf{r}'|\mathbf{r_s};\omega)\mathrm{d}V'$$

Inhomogeneous Helmholtz equation

$$\mathbf{LG} = \left(\frac{\omega^2}{K} + \nabla\cdot\frac{1}{\rho}\nabla\right)\mathbf{G} = -\delta(\mathbf{r}' - \mathbf{r_s}),$$

$$\mathbf{L}_r\mathbf{G}_r = \left(\frac{\omega^2}{K_r} + \nabla\cdot\frac{1}{\rho_r}\nabla\right)\mathbf{G}_r = -\delta(\mathbf{r}' - \mathbf{r_s}),$$

V is called the scattering potential

$$\mathbf{V} = \mathbf{L} - \mathbf{L_r} = \left(\frac{\omega^2}{K} - \frac{\omega^2}{K_r}\right) + \nabla\cdot\left(\frac{1}{\rho} - \frac{1}{\rho_r}\right)\nabla = \frac{\omega^2}{K_r}a_K + \nabla\cdot\frac{a_\rho}{\rho_r}\nabla,$$

where a_K, a_ρ are

$$a_K = \left(\frac{K_r}{K(\mathbf{r}')} - 1\right),\ \ a_\rho = \left(\frac{\rho_r}{\rho(\mathbf{r}')} - 1\right),$$

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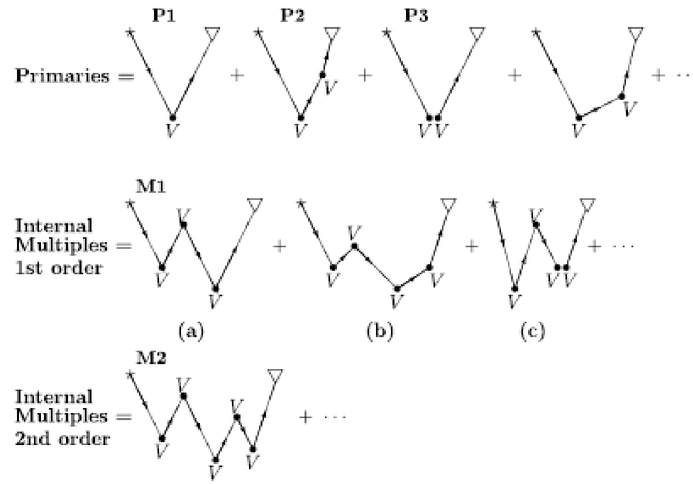
Basic Scattering Theory

Born-Neumann Series

$$G = G_r + G_r V G_r + G_r V G_r V G_r + ...,$$



We define the scattered field as:

$$\Psi_{scattered}(\mathbf{r}) = \mathbf{G} - \mathbf{G_r} \rightarrow data$$

Born Approximation: $|\psi_{scattered}| << |\psi_{inc}|$

$$\psi_{scattered} \approx \int_V G_r(\mathbf{r}|\mathbf{r}')V(\mathbf{r}')\Big[\psi_{inc}(\mathbf{r}') + (\psi_{scattered}(\mathbf{r}') \rightarrow 0)\Big]\mathrm{d}V'$$

In operator form the linearized series becomes

$$G = G_r + G_r V G_r,$$

Inversion

$$D = (G - G_r)S(\omega) = (G_r V G_r)S(\omega).$$

$$D'(k_m, k_h, k_z) = \frac{-1}{\rho_r}\frac{D(k_m, k_h, \omega)}{S(\omega) + \epsilon},$$

Deconvolve the source wavelet from data.

Fourier transform from x_g, x_s coordinates to source-receiver wavenumbers k_g, k_s.

$$D(k_g, k_s, \omega) = \frac{1}{2\pi}\int \mathrm{d}x_g \int \mathrm{d}x_s e^{-ik_g x_g} D(x_g, x_s, \omega) e^{ik_s x_s},$$

$$= \int \mathrm{d}x' \int \mathrm{d}z' G_r^+(k_g, 0|x', z'; \omega) V(x', z'; \omega) G_r^+(x', z'|k_s, 0|; \omega) S(\omega),$$

$$k_m = k_g - k_s \;, k_h = k_g + k_s \;, x_m = \frac{x_g + x_s}{2},\;\; x_h = \frac{x_g - x_s}{2},$$

Inversion is driven by mapping the spectrum of the data to the spectrum of the model.

$$\boxed{D(k_m, k_z, k_h) = \frac{-\rho_r S(\omega)}{C(k_m, k_z, k_h)}\left[\frac{\omega^2}{\nu_r^2} a_K(k_m, k_z) + F(k_m, k_z, k_h) a_\rho(k_m, k_z)\right]}$$

Inversion

When transformation variables are substituted in and simplified we obtain the system of equations which need to be inverted.

$$D(k_m, k_z, k_h) = -\rho_r \left[\sum_{i=1}^{2} A_i(k_m, k_h, k_z) a_i(k_m, k_z) \right] S(\omega),$$

$$A_1 = (k_m, k_h, k_z) = \frac{1}{4} \frac{(k_z^2 + k_h^2)(k_z^2 + k_m^2)}{k_z^4 - k_m^2 k_h^2},\ A_2 = (k_m, k_h, k_z) = \frac{1}{4} \frac{(k_z^2 - k_h^2)(k_z^2 + k_m^2)}{k_z^4 - k_m^2 k_h^2},$$

After the deconvolution stage we are left with

$$D'(k_m, k_z, k_h) = \left[\sum_{i=1}^{2} A_i(k_m, k_h, k_z) a_i(k_m, k_z) \right],$$

of which we need to determine the $a_i(k_m, k_z)$ through perhaps a least squares method.

Synthetic Examples

- Forward modelling:
  - x line length of 2500m , Max depth of 1000m (t=0.8 sec),
  - source/receiver spacing of 10m, sampling time of 4ms.
  - F-D calculations: time step is 0.5ms computation grid of 10mx10m.
  - A band limited Ricker wavelet at 30Hz and 0.1 seconds was also added.
  - Scalar(1 parameter acoustic) model.

- Pre-processing:
  - Deconvolution (already know the type of wavelet).
  - Subtraction of direct wave from input dataset.

- Inversion:
  - Direct Fourier transform of the dataset.
  - Fill in model spectra on a regular k_m, k_z grid
  - Inverse Fourier transform back into the image space x,z.

Four layer horizontal model

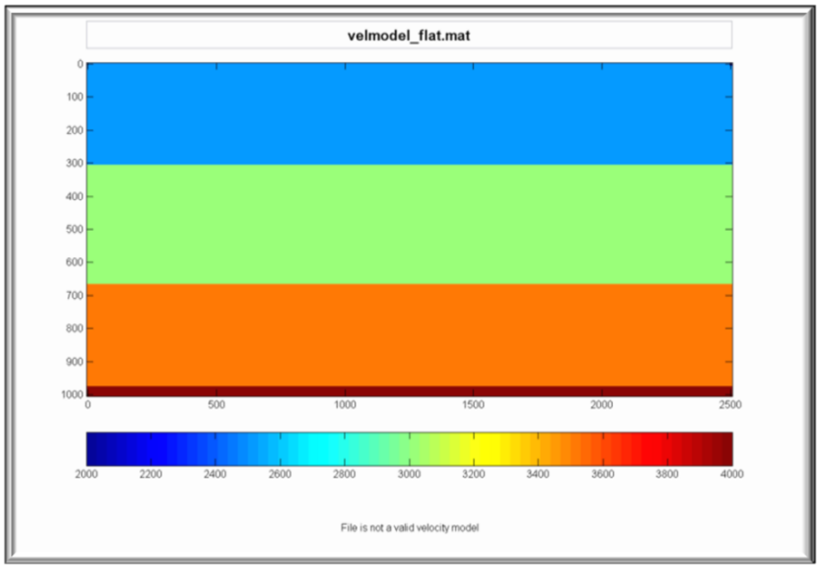


Figure 1: Four Layer Horizontal Velocity Model

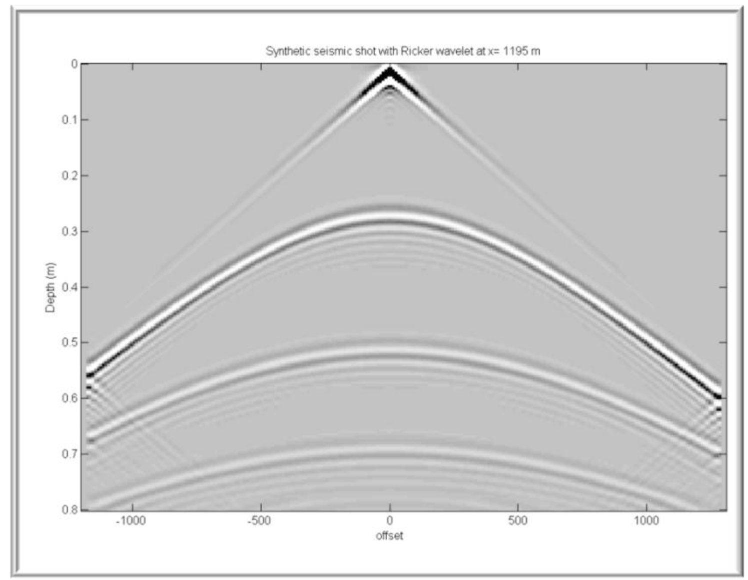
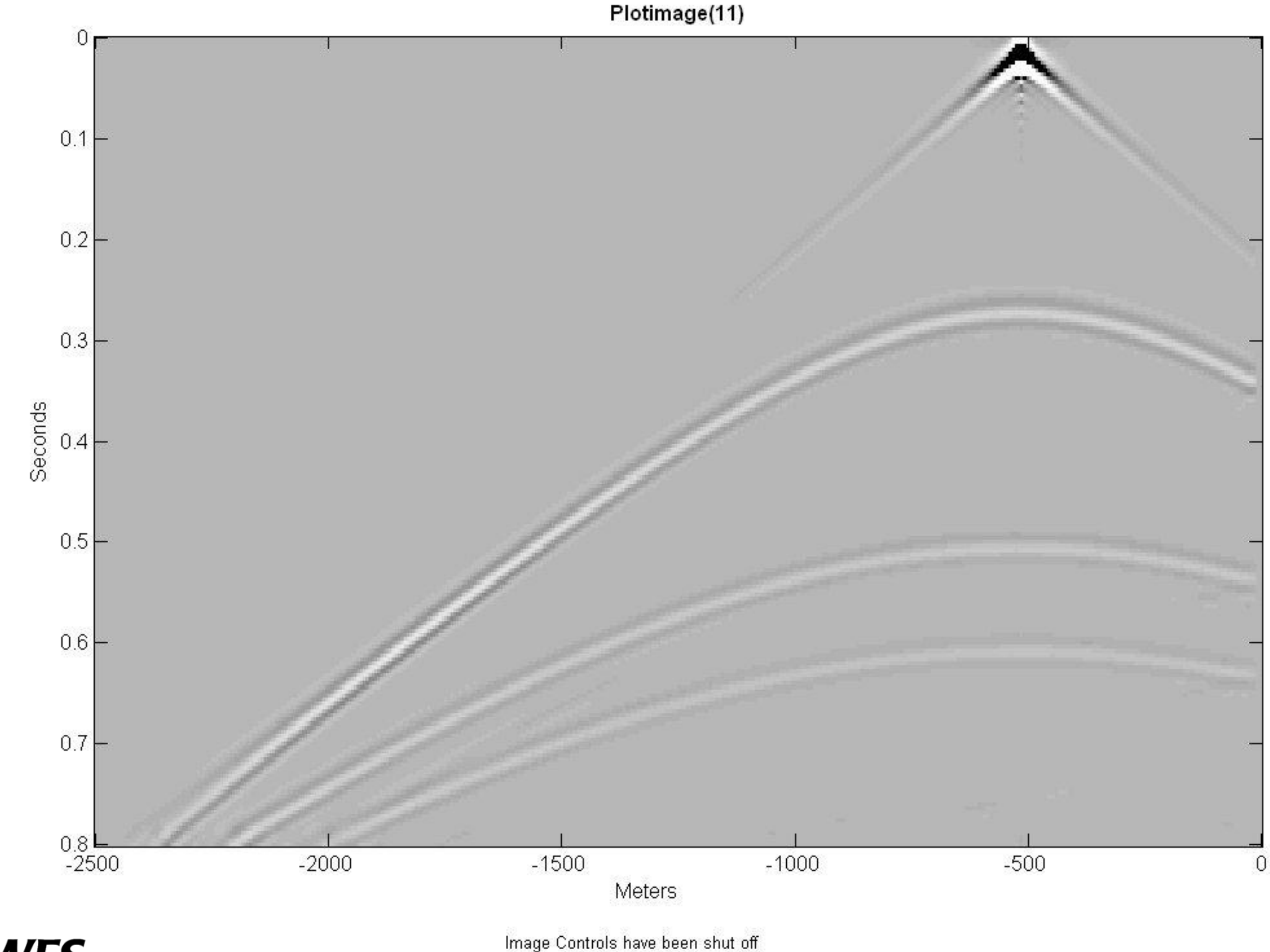


Figure 2: Four Layer Shot Profile w Ricker wavelet

Four layer horizontal model



Four layer horizontal model

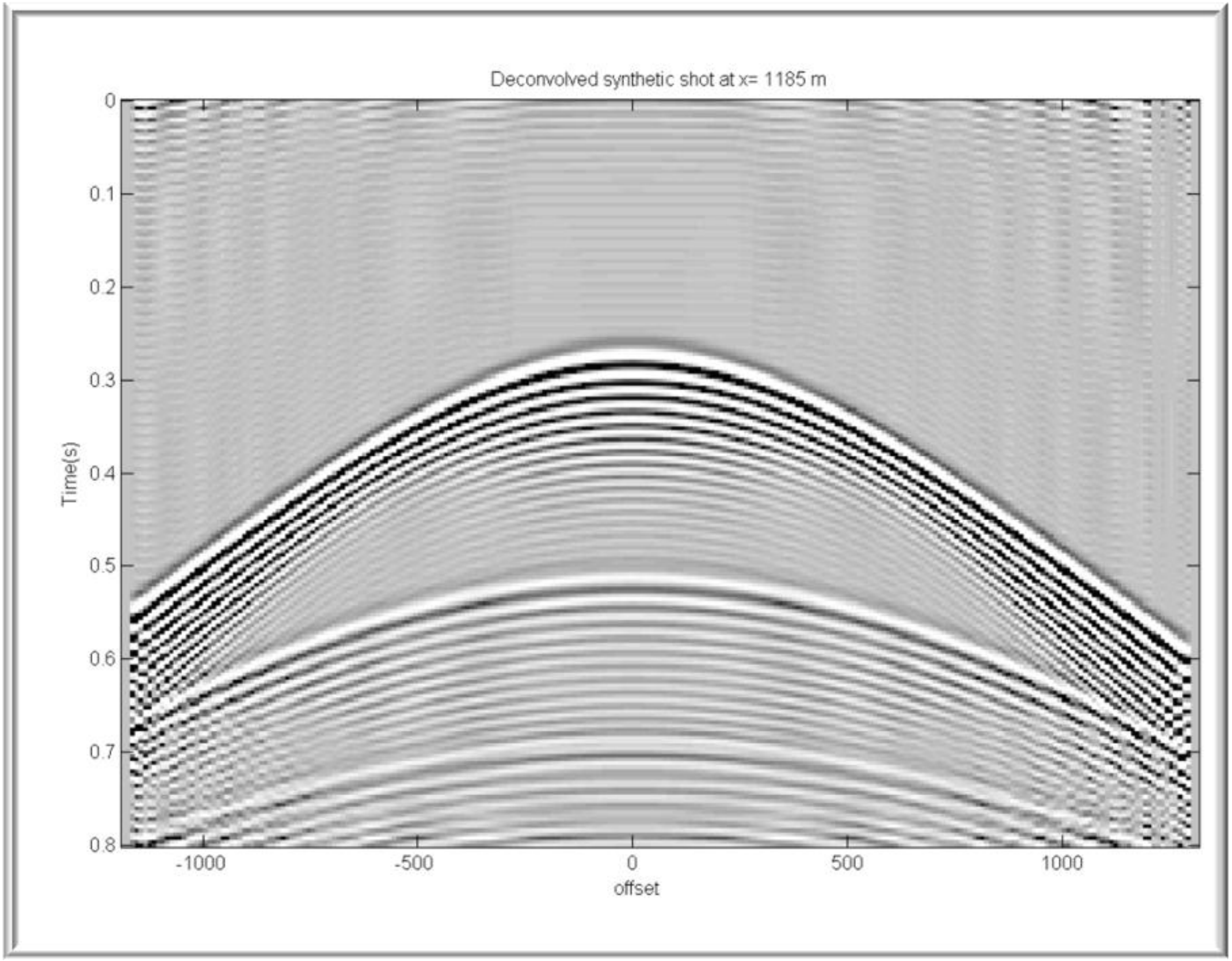


Figure 3: Four Layer, Deconvolved w/o Direct Wave

Four layer horizontal model

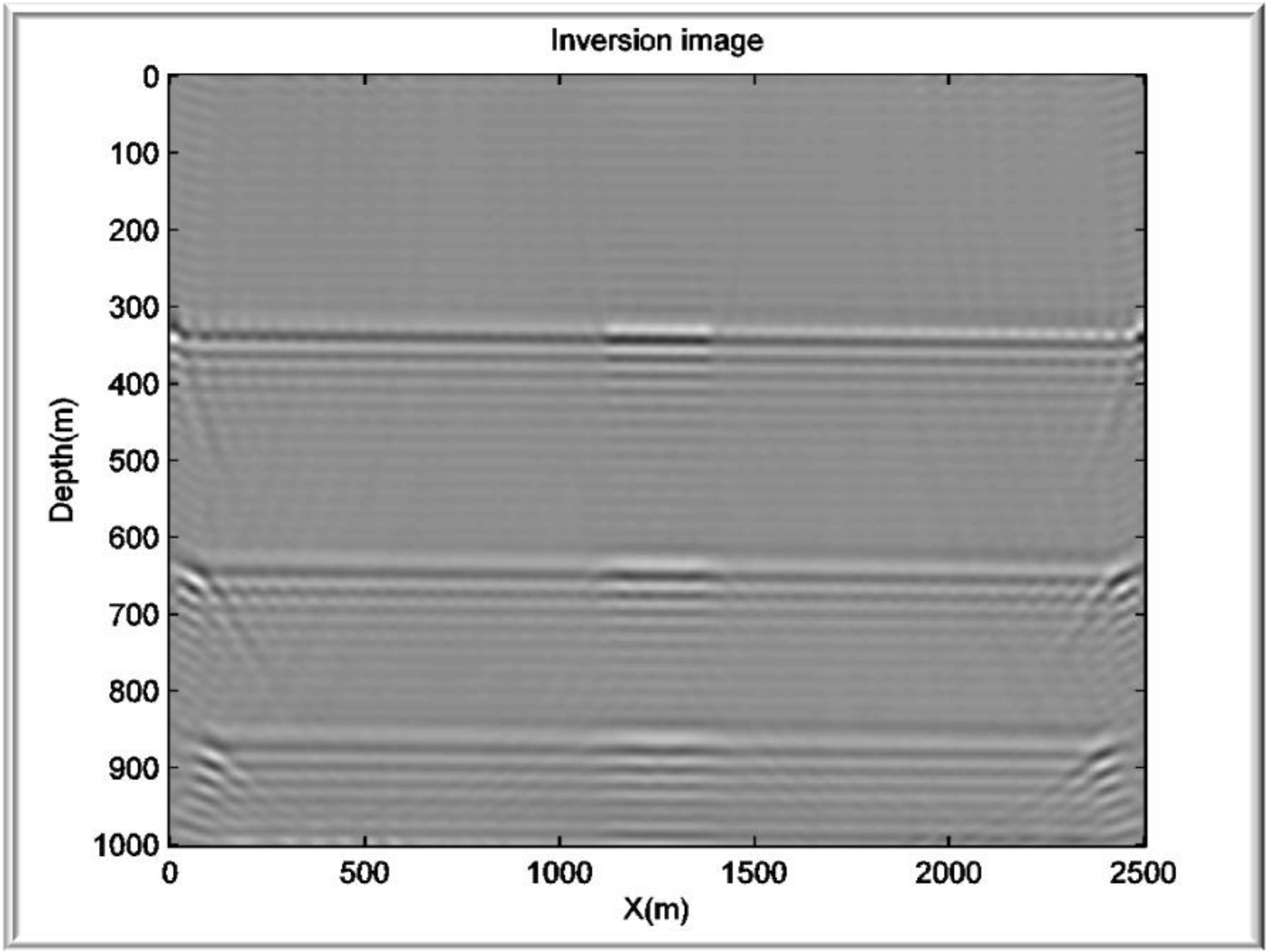


Figure 5: Four layer Horizontal Model, Migrated Image

Shallow low velocity model

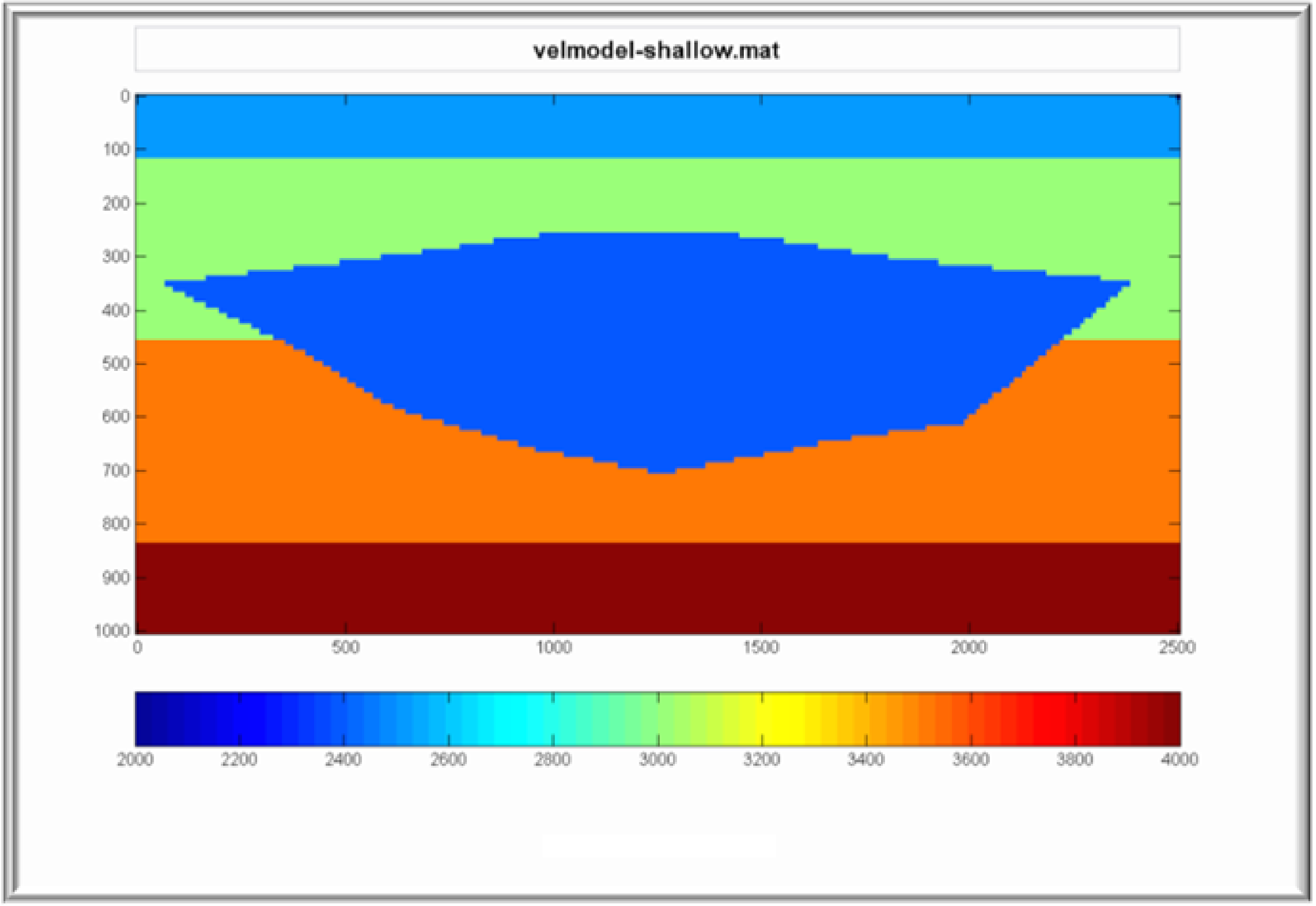


Figure 6: Shallow Lens Velocity Model

Shallow low velocity model

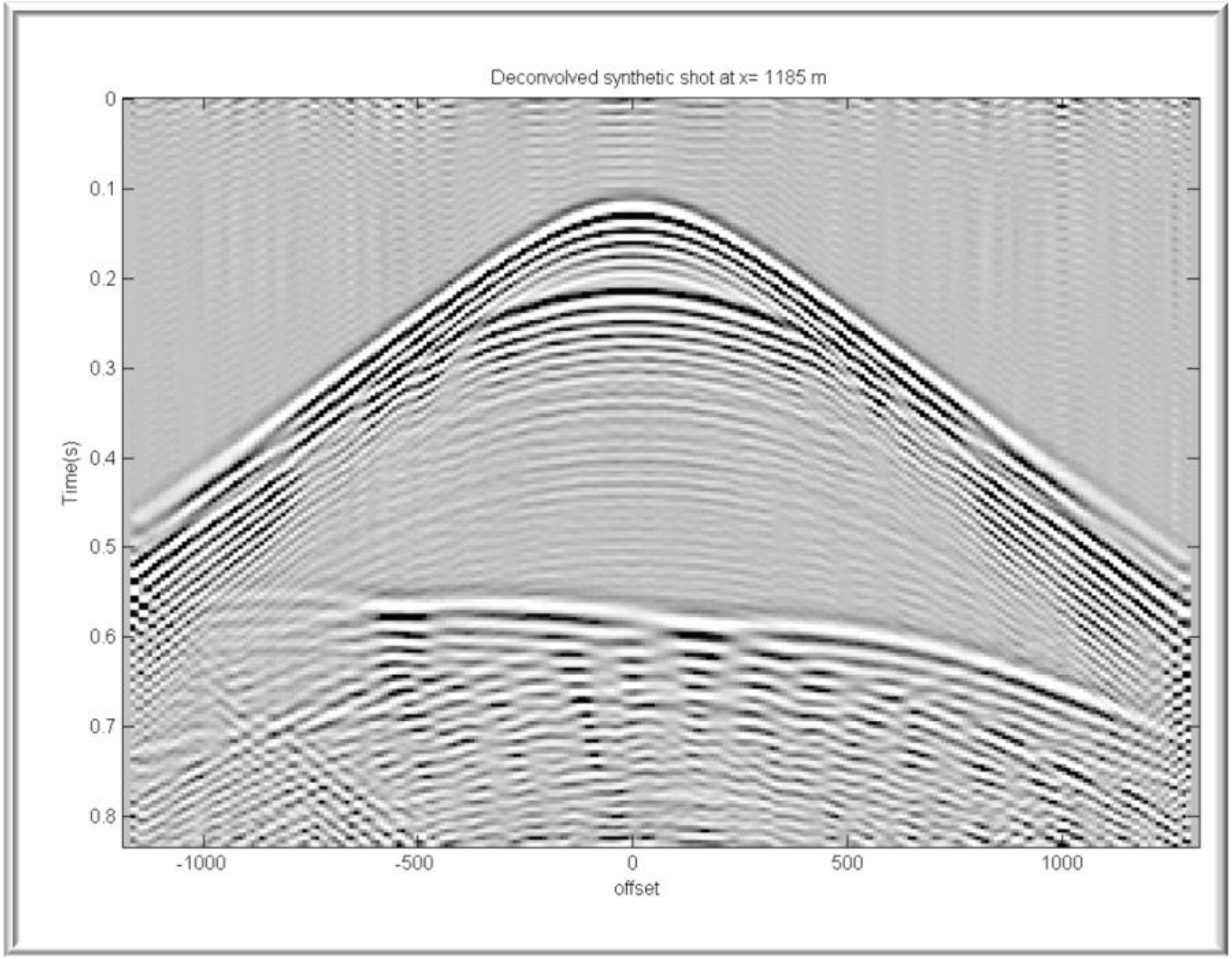


Figure 8: Shallow Lens, Deconvolved w/o Direct Wave

Shallow low velocity model

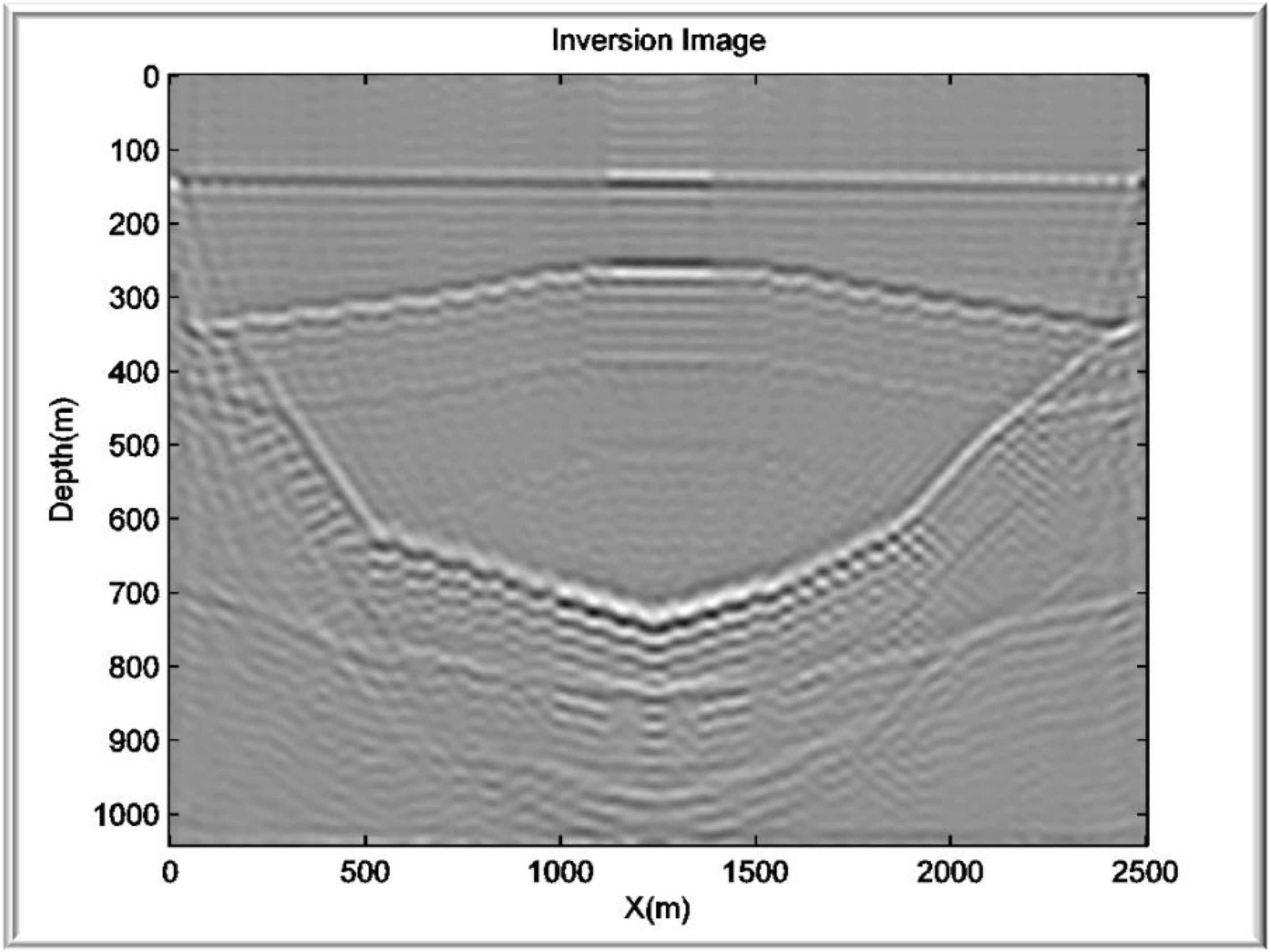


Figure 10: Shallow, Low Velocity Model, Migrated Image

Anticline velocity model

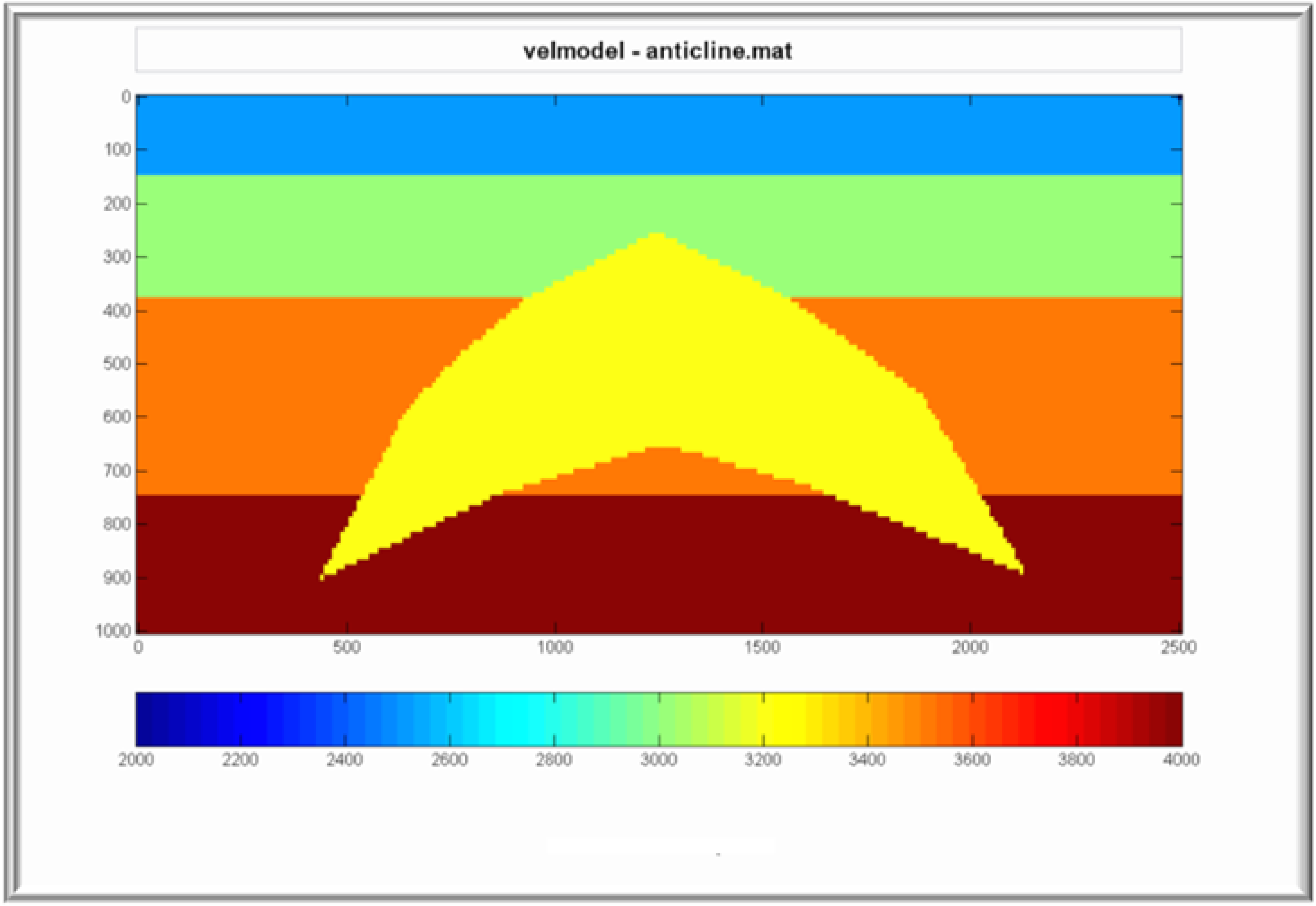


Figure 11: Anticline Velocity Model

Anticline velocity model

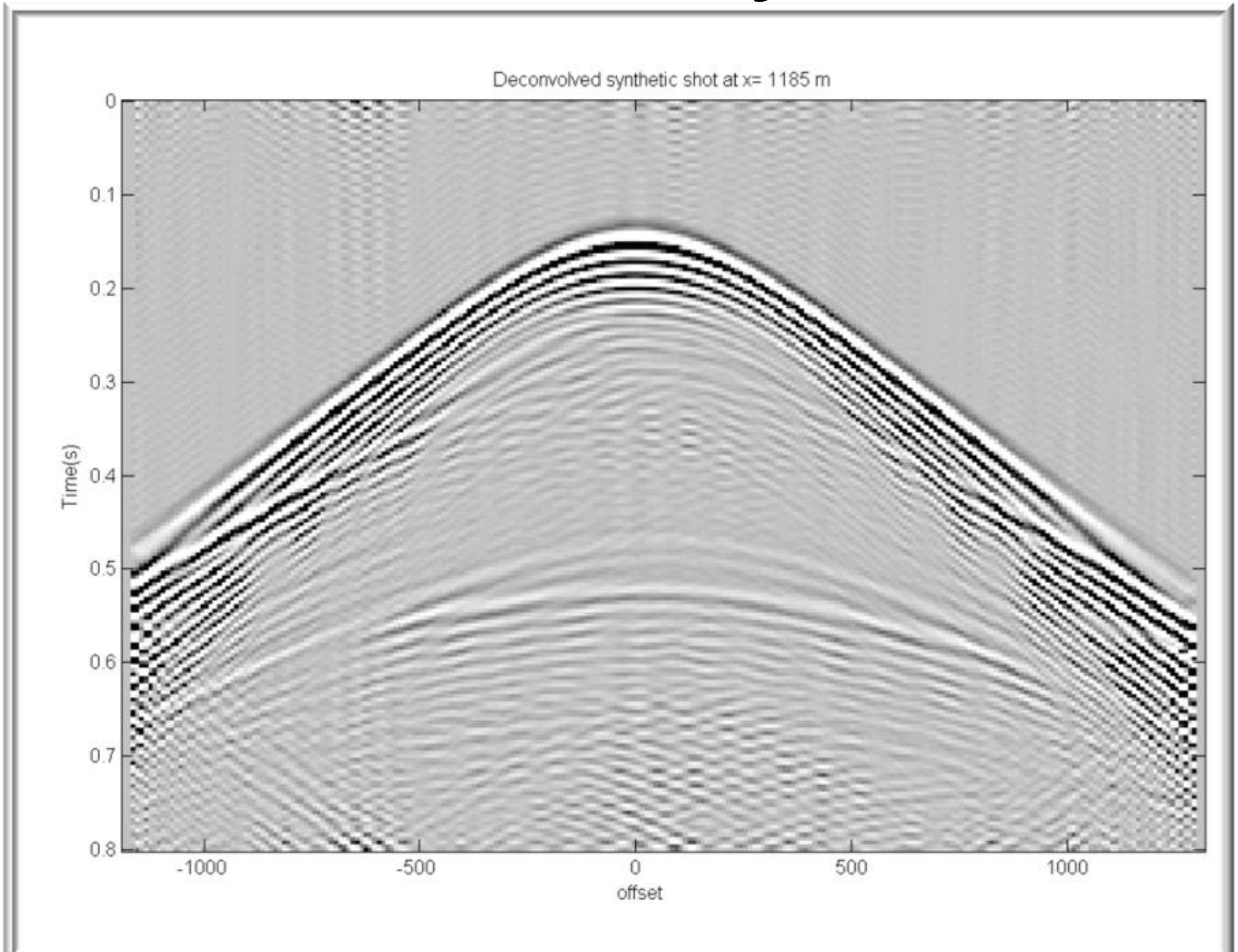


Figure 13: Anticline Deconvolved w/o Direct Wave

Anticline velocity model

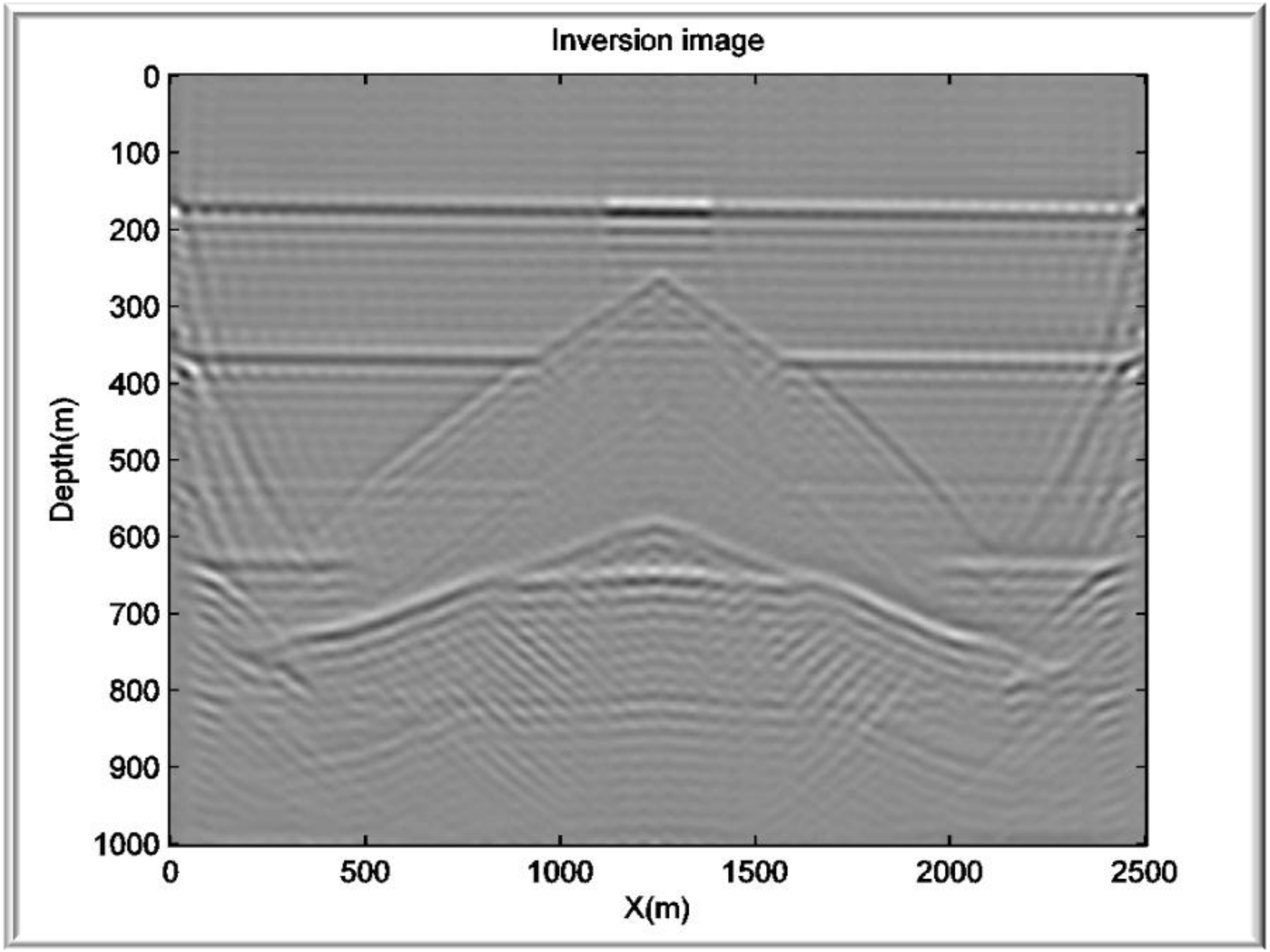


Figure 15: Anticline Velocity Model, Migrated Image

Conclusion

- Even with the relatively straightforward algorithm for a constant background, we are able to image the models to a fairly high degree of accuracy.
- Our simulations have not taken into account real world effects such as multiples and anisotropic and anelastic media which are variable background effects.
- We need to verify that the amplitudes of the reflections found in these results are true amplitudes.
- Next step incorporate material parameters such as bulk modulus, variable density into the forward model(true two parameter mode).
- test the accuracy of the inversions in producing the correct and physically realistic answers.

Acknowledgements

- Kris Innanen for suggesting this as a “short” summer project.
- Also for lending me his GPR code based on the Clayton and Stolt paper as a template.
- Heather Lloyd for pointing out those nice FD tools I initially used
- Plus pointing out the sources of my F-D errors to this F-D noob!

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Appendix A: Inversion

The complete derivation from the Clayton and Stolt paper.

Appendix A: Inversion

$$D = (G - G_r)S(\omega) = (G_r V G)S(\omega).$$

$$D'(k_m, k_h, k_z) = \frac{-1}{\rho_r}\frac{D(k_m, k_h, \omega)}{S(\omega) + \epsilon},$$

Deconvolve the source wavelet from data.

The first step: Fourier transform from x_g, x_s coordinates to source-receiver wavenumbers k_g, k_s.

$$D(k_g, k_s, \omega) = \frac{1}{2\pi}\int \mathrm{d}x_g \int \mathrm{d}x_s e^{-ik_g x_g} D(x_g, x_s, \omega) e^{ik_s x_s},$$

$$= \int \mathrm{d}x' \int \mathrm{d}z' G_r^+(k_g, 0|x', z'; \omega) V(x', z'; \omega) G_r^+(x', z'|k_s, 0|; \omega) S(\omega),$$

with the Green's operators for a 2D constant background are given by

$$G_r^+(k_g, 0|x', z'; \omega) = \frac{i\rho_r}{\sqrt{2\pi}}\frac{e^{-i(k_g x' - q_g|z'|)}}{2q_g}, \;\; G_r^+(x', z'|k_s, 0; \omega) = \frac{i\rho_r}{\sqrt{2\pi}}\frac{e^{i(k_s x' + q_s|z'|)}}{2q_s},$$

where

$$q_g = \frac{\omega}{\nu_r}\sqrt{1 - \frac{\nu_r^2 k_g^2}{\omega^2}}, \;\; q_s = \frac{\omega}{\nu_r}\sqrt{1 - \frac{\nu_r^2 k_s^2}{\omega^2}},$$

Appendix A: Inversion

$$\mathbf{V} = \frac{\omega^2}{K_r} a_K + \nabla \cdot \frac{a_\rho}{\rho_r} \nabla, \qquad a_K = \left(\frac{K_r}{K(\mathbf{r}')} - 1 \right), \;\; a_\rho = \left(\frac{\rho_r}{\rho(\mathbf{r}')} - 1 \right),$$

using the definitions for a_K and a_ρ then integration by parts we get

$$D(k_g, k_s, \omega) = \frac{-\rho_r^2}{2\pi} \int \mathrm{d}x' \int \mathrm{d}z' \, \frac{e^{-i[(k_g - k_s)x' - (q_g + g_s)z')]}}{4 q_g q_s} \times$$
$$\left[\frac{\omega^2}{\nu_r^2} a_K(x', z') + (q_g q_s - k_g k_s) a_\rho(x', z') \right] S(\omega),$$

The above equation is of the same form as a double Fourier transform in the x' and z' variables if we do some rearranging the result of the evaluation yields

$$D(k_g, k_s, \omega) = \frac{-\rho_r^2 S(\omega)}{4 q_g q_s} \left[\frac{\omega^2}{\nu_r^2} a_K(k_g - k_s, -q_g - q_s) \right.$$
$$\left. + (q_g q_s - k_g k_s) a_\rho(k_g - k_s, -q_g - q_s) \right],$$

Appendix A: Inversion

Change of Coordinates to ω, q_g, q_s space

To solve for a_K and a_ρ change to midpoint/offset coordinates from the source/receiver system. We have the midpoint wavenumber $k_m = k_g - k_s$, the half offset wavenumber $k_h = k_g + k_s$ which in x,z domain corresponds to

$$x_m = \frac{x_g + x_s}{2}, \;\; x_h = \frac{x_g - x_s}{2},$$

and a new independent variable, Solving for ω, q_g and q_s we have the expressions

$$k_z = -q_g - q_s = -\frac{\omega}{\nu_r}\sqrt{1 - \frac{\nu_r^2 k_g^2}{\omega^2}} - \frac{\omega}{\nu_r}\sqrt{1 - \frac{\nu_r^2 k_s^2}{\omega^2}},$$

$$\omega(k_m, k_h, k_z) = -\frac{\nu_r k_z}{2}\sqrt{(1 + \frac{k_m^2}{k_z^2})(1 + \frac{k_h^2}{k_z^2})},$$

$$q_g(k_m, k_h, k_z) = -\frac{k_z}{2}(1 - \frac{k_m k_h}{k_z^2}), \;\; q_s(k_m, k_h, k_z) = -\frac{k_z}{2}(1 + \frac{k_m k_h}{k_z^2}),$$

$$k_m = k_g - k_s \;\; , k_h = k_g + k_s \;\; , x_m = \frac{x_g + x_s}{2}, \;\; x_h = \frac{x_g - x_s}{2},$$

Appendix A: Inversion

Now that we have expressions for ω, q_g, q_s the Direct Fourier transform computed in these coordinates and data can be transformed back after the inversion.

$$D(k_m,k_z,k_h) = \frac{-\rho_r^2 S(\omega)}{4q_g q_s}\left[\frac{\omega^2}{\nu_r^2}a_K(k_m,k_z) + (q_g q_s - k_g k_s)a_\rho(k_m,k_z)\right],$$

$$D(k_m,k_z,k_h) = \frac{-\rho_r S(\omega)}{C(k_m,k_z,k_h)}\left[\frac{\omega^2}{\nu_r^2}a_K(k_m,k_z) + F(k_m,k_z,k_h)a_\rho(k_m,k_z)\right],$$

When transformation variables are substituted in and simplified we obtain the system of equations which need to be inverted.

$$D(k_m,k_z,k_h) = -\rho_r\left[\sum_{i=1}^{2} A_i(k_m,k_h,k_z)a_i(k_m,k_z)\right]S(\omega),$$

$$A_1 = (k_m,k_h,k_z) = \frac{1}{4}\frac{(k_z^2+k_h^2)(k_z^2+k_m^2)}{k_z^4 - k_m^2 k_h^2},\ A_2 = (k_m,k_h,k_z) = \frac{1}{4}\frac{(k_z^2-k_h^2)(k_z^2+k_m^2)}{k_z^4 - k_m^2 k_h^2},$$

After the deconvolution stage we are left with

$$D'(k_m,k_z,k_h) = \left[\sum_{i=1}^{2} A_i(k_m,k_h,k_z)a_i(k_m,k_z)\right],$$

of which we need to determine the $a_i(k_m,k_z)$ through perhaps a least squares method.