Multiparameter inverse scattering: A preliminary computational approach

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Introduction

• This study investigates the approach to imaging and inversion based upon the paper by Clayton and Stolt (1981) using inverse scattering methods.

• They discuss two cases, Imaging/inversion in a constant and variable background.

• Here we treat the 2D constant background case only.

• In this study:
  o Created synthetic data for 2D targets as input for the 2\textsuperscript{nd} step,
  o Test a prototype inverse scattering algorithm using a single parameter only (velocity) as a check on the correctness and accuracy of this algorithm.
Forward Modeling

• Creation of a synthetic 2D velocity model using a gui based tool AFD_VELCREATE.

• Generate the scattered field:
  (1) Forward model in the reference medium (constant velocity)

(2) Forward model in the layered medium using the CREWES FD routine AFD_SHOTREC to generate 2D shot records.
Forward Modeling

• Convolve with a Ricker wavelet.

• Generation of a background and model shotpoint gather from the background and model velocity profiles for each receiver.

• repeated shot gathers created for each receiver point along the desired seismic line with the desired receiver spacing.
Basic Scattering Theory

Lippmann-Schwinger Equation

\[ G(\mathbf{r}_g | \mathbf{r}_s; \omega) - G_r(\mathbf{r}_g | \mathbf{r}_s; \omega) = \int_{V'} G_r(\mathbf{r}_g | \mathbf{r}'; \omega)V(\mathbf{r}')G(\mathbf{r}' | \mathbf{r}_s; \omega)\,dV' \]

Inhomogeneous Helmholtz equation

\[ \mathbf{L}G = \left( \frac{\omega^2}{K} + \nabla \cdot \frac{1}{\rho} \nabla \right) G = -\delta(\mathbf{r}' - \mathbf{r}_s), \]

\[ \mathbf{L}_r G_r = \left( \frac{\omega^2}{K_r} + \nabla \cdot \frac{1}{\rho_r} \nabla \right) G_r = -\delta(\mathbf{r}' - \mathbf{r}_s), \]

V is called the scattering potential

\[ \mathbf{V} = \mathbf{L} - \mathbf{L}_r = \left( \frac{\omega^2}{K} - \frac{\omega^2}{K_r} \right) + \nabla \cdot \left( \frac{1}{\rho} - \frac{1}{\rho_r} \right) \nabla = \frac{\omega^2}{K_r} a_K + \nabla \cdot \frac{a_{\rho}}{\rho_r} \nabla, \]

where \( a_K, a_{\rho} \) are

\[ a_K = \left( \frac{K_r}{K(\mathbf{r}') - 1} \right), \quad a_{\rho} = \left( \frac{\rho_r}{\rho(\mathbf{r}') - 1} \right), \]
Basic Scattering Theory

Born-Neumann Series

\[ G = G_r + G_r V G_r + G_r V G_r V G_r + \ldots, \]

We define the scattered field as:

\[ \Psi_{\text{scattered}}(r) = G - G_r \rightarrow \text{data} \]

Born Approximation: \[ |\psi_{\text{scattered}}| \ll |\psi_{\text{inc}}| \]

\[ \psi_{\text{scattered}} \approx \int_V G_r(r|r') V(r') \left[ \psi_{\text{inc}}(r') + (\psi_{\text{scattered}}(r') \to 0) \right] \text{d}V' \]

In operator form the linearized series becomes

\[ G = G_r + G_r V G_r, \]
Inversion

\[ D = (G - G_r)S(\omega) = (G_r V G_r)S(\omega). \]

\[ D'(k_m, k_h, k_z) = \frac{-1}{\rho_r} \frac{D(k_m, k_h, \omega)}{S(\omega) + \epsilon}, \]

Deconvolve the source wavelet from data.

Fourier transform from \( x_g, x_s \) coordinates to source-receiver wavenumbers \( k_g, k_s \).

\[ D(k_g, k_s, \omega) = \frac{1}{2\pi} \int dx_g \int dx_s e^{-ik_g x_g} D(x_g, x_s, \omega)e^{ik_s x_s}, \]

\[ = \int dx' \int dz' G^+_r(k_g, 0|x', z'; \omega)V(x', z'; \omega)G^+_r(x', z'|k_s, 0; \omega)S(\omega), \]

\[ k_m = k_g - k_s, \quad k_h = k_g + k_s, \quad x_m = \frac{x_g + x_s}{2}, \quad x_h = \frac{x_g - x_s}{2}, \]

Inversion is driven by mapping the spectrum of the data to the spectrum of the model.

\[ D(k_m, k_z, k_h) = \frac{-\rho_r S(\omega)}{C(k_m, k_z, k_h)} \left[ \frac{\omega^2}{\nu^2} a_K(k_m, k_z) + F(k_m, k_z, k_h)a_\rho(k_m, k_z) \right] \]
Inversion

When transformation variables are substituted in and simplified we obtain the system of equations which need to be inverted.

\[
D(k_m, k_z, k_h) = -\rho_r \left[ \sum_{i=1}^{2} A_i(k_m, k_h, k_z) a_i(k_m, k_z) \right] S(\omega),
\]

\[
A_1 = (k_m, k_h, k_z) = \frac{1}{4} \frac{(k_z^2 + k_h^2)(k_z^2 + k_m^2)}{k_z^4 - k_m^2 k_h^2}, \quad A_2 = (k_m, k_h, k_z) = \frac{1}{4} \frac{(k_z^2 - k_h^2)(k_z^2 + k_m^2)}{k_z^4 - k_m^2 k_h^2},
\]

After the deconvolution stage we are left with

\[
D'(k_m, k_z, k_h) = \left[ \sum_{i=1}^{2} A_i(k_m, k_h, k_z) a_i(k_m, k_z) \right],
\]

of which we need to determine the \(a_i(k_m, k_z)\) through perhaps a least squares method.
Synthetic Examples

- **Forward modelling:**
  - x line length of 2500m, Max depth of 1000m (t=0.8 sec),
  - source/receiver spacing of 10m, sampling time of 4ms.
  - F-D calculations: time step is 0.5ms computation grid of 10mx10m.
  - A band limited Ricker wavelet at 30Hz and 0.1 seconds was also added.
  - Scalar (1 parameter acoustic) model.

- **Pre-processing:**
  - Deconvolution (already know the type of wavelet).
  - Subtraction of direct wave from input dataset.

- **Inversion:**
  - Direct Fourier transform of the dataset.
  - Fill in model spectra on a regular \( k_x, k_z \) grid
  - Inverse Fourier transform back into the image space \( x,z \).
Four layer horizontal model

Figure 1: Four Layer Horizontal Velocity Model

Figure 2: Four Layer Shot Profile w Ricker wavelet
Four layer horizontal model
Four layer horizontal model

Figure 3: Four Layer, Deconvolved w/o Direct Wave
Four layer horizontal model

Figure 5: Four layer Horizontal Model, Migrated Image

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Shallow low velocity model
Shallow low velocity model

Figure 8: Shallow Lens, Deconvolved w/o Direct Wave
Shallow low velocity model

Figure 10: Shallow, Low Velocity Model, Migrated Image

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Anticline velocity model

Figure 11: Anticline Velocity Model
Anticline velocity model

Figure 13: Anticline Deconvolved w/o Direct Wave
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Anticline velocity model

Figure 15: Anticline Velocity Model, Migrated Image

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Conclusion

• Even with the relatively straightforward algorithm for a constant background, we are able to image the models to a fairly high degree of accuracy.
• Our simulations have not taken into account real world effects such as multiples and anisotropic and anelastic media which are variable background effects.
• We need to verify that the amplitudes of the reflections found in these results are true amplitudes.
• Next step incorporate material parameters such as bulk modulus, variable density into the forward model(true two parameter mode).
• test the accuracy of the inversions in producing the correct and physically realistic answers.
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References


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Appendix A: Inversion

The complete derivation from the Clayton and Stolt paper.
Appendix A: Inversion

\[ D = (G - G_r) S(\omega) = (G_r V G) S(\omega). \]

\[ D'(k_m, k_h, k_z) = -\frac{1}{\rho_r} \frac{D(k_m, k_h, \omega)}{S(\omega) + \epsilon}, \]

Deconvolve the source wavelet from data.

The first step: Fourier transform from \( x_g, x_s \) coordinates to source-receiver wavenumbers \( k_g, k_s \).

\[ D(k_g, k_s, \omega) = \frac{1}{2\pi} \int dx_g \int dx_s e^{-i k_g x_g} D(x_g, x_s, \omega) e^{i k_s x_s}, \]

\[ = \int dx' \int dz' G_r^+(k_g, 0|x', z'; \omega) V(x', z'; \omega) G_r^+(x', z'|k_s, 0; \omega) S(\omega), \]

with the Green's operators for a 2D constant background are given by

\[ G_r^+(k_g, 0|x', z'; \omega) = \frac{i \rho_r}{\sqrt{2\pi}} \frac{e^{-i(k_g x' - q_g |z'|)}}{2q_g}, \quad G_r^+(x', z'|k_s, 0; \omega) = \frac{i \rho_r}{\sqrt{2\pi}} \frac{e^{i(k_s x' + q_s |z'|)}}{2q_s}, \]

where

\[ q_g = \frac{\omega}{\nu_r} \sqrt{1 - \frac{\nu_r^2 k_g^2}{\omega^2}}, \quad q_s = \frac{\omega}{\nu_r} \sqrt{1 - \frac{\nu_r^2 k_s^2}{\omega^2}}, \]
Appendix A: Inversion

\[ \mathbf{V} = \frac{\omega^2}{K_r} a_K + \nabla \cdot \frac{a_\rho}{\rho_r} \nabla, \quad a_K = \left( \frac{K_r}{K(\mathbf{r}') - 1} \right), \quad a_\rho = \left( \frac{\rho_r}{\rho(\mathbf{r}') - 1} \right), \]

using the definitions for \( a_K \) and \( a_\rho \) then integration by parts we get

\[ D(k_g, k_s, \omega) = \frac{-\rho_r^2}{2\pi} \int \, dx' \int \, dz' \frac{e^{-i[(k_g - k_s)x' - (q_g + q_s)z']}}{4q_gq_s} \times \]

\[ \left[ \frac{\omega^2}{\nu_r^2} a_K(x', z') + (q_gq_s - k_gk_s)a_\rho(x', z') \right] S(\omega), \]

The above equation is of the same form as a double Fourier transform in the \( x' \) and \( z' \) variables if we do some rearranging the result of the evaluation yields

\[ D(k_g, k_s, \omega) = \frac{-\rho_r^2 S(\omega)}{4q_gq_s} \left[ \frac{\omega^2}{\nu_r^2} a_K(k_g - k_s, -q_g - q_s) \right. \]

\[ + (q_gq_s - k_gk_s)a_\rho(k_g - k_s, -q_g - q_s) \left. \right], \]
Appendix A: Inversion

Change of Coordinates to $\omega$, $q_g$, $q_s$ space

To solve for $a_K$ and $a_\rho$ change to midpoint/offset coordinates from the source/receiver system. We have the midpoint wavenumber $k_m = k_g - k_s$, the half offset wavenumber $k_h = k_g + k_s$ which in x,z domain corresponds to

$$x_m = \frac{x_g + x_s}{2}, \quad x_h = \frac{x_g - x_s}{2},$$

and a new independent variable, Solving for $\omega$, $q_g$ and $q_s$ we have the expressions

$$k_z = -q_g - q_s = -\frac{\omega}{\nu_r} \sqrt{1 - \frac{\nu_r^2 k_g^2}{\omega^2}} - \frac{\omega}{\nu_r} \sqrt{1 - \frac{\nu_r^2 k_s^2}{\omega^2}},$$

$$\omega(k_m, k_h, k_z) = -\frac{\nu_r k_z}{2} \sqrt{(1 + \frac{k_m^2}{k_z^2})(1 + \frac{k_h^2}{k_z^2})},$$

$$q_g(k_m, k_h, k_z) = -\frac{k_z}{2} \left(1 - \frac{k_m k_h}{k_z^2}\right), \quad q_s(k_m, k_h, k_z) = -\frac{k_z}{2} \left(1 + \frac{k_m k_h}{k_z^2}\right),$$

$$k_m = k_g - k_s, \quad k_h = k_g + k_s, \quad x_m = \frac{x_g + x_s}{2}, \quad x_h = \frac{x_g - x_s}{2},$$
Appendix A: Inversion

Now that we have expressions for \( \omega, q_g, q_s \) the Direct Fourier transform computed in these coordinates and data can be transformed back after the inversion.

\[
D(k_m, k_z, k_h) = \frac{-\rho_r^2 S(\omega)}{4q_g q_s} \left[ \frac{\omega^2}{\nu_r^2} a_K(k_m, k_z) + (q_g q_s - k_g k_s) a_\rho(k_m, k_z) \right],
\]

\[
D(k_m, k_z, k_h) = \frac{-\rho_r S(\omega)}{C(k_m, k_z, k_h)} \left[ \frac{\omega^2}{\nu_r^2} a_K(k_m, k_z) + F(k_m, k_z, k_h) a_\rho(k_m, k_z) \right],
\]

When transformation variables are substituted in and simplified we obtain the system of equations which need to be inverted.

\[
D(k_m, k_z, k_h) = -\rho_r \left[ \sum_{i=1}^{2} A_i(k_m, k_h, k_z) a_i(k_m, k_z) \right] S(\omega),
\]

\[
A_1 = (k_m, k_h, k_z) = \frac{1}{4} \left( \frac{k_z^2 + k_h^2}{k_z^4 - k_m^2 k_h^2} \right), \quad A_2 = (k_m, k_h, k_z) = \frac{1}{4} \left( \frac{k_z^2 - k_h^2}{k_z^4 - k_m^2 k_h^2} \right).
\]

After the deconvolution stage we are left with

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