A framework for AVO analysis of time-lapse difference data when contrasts are large

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Outline

- Introduction and review
- Time lapse amplitudes and the work of Landrø (2001)
- A framework for time-lapse AVO
- Results
- Summary
- Future work

Time-lapse

- Repeated seismic surveys over calendar time
- The baseline and monitor survey
- Monitoring the pressure, fluid saturation, and temperature changes
- CO_2 monitoring, EOR, production monitoring
- Leads to changes in seismic parameters from baseline to monitor survey

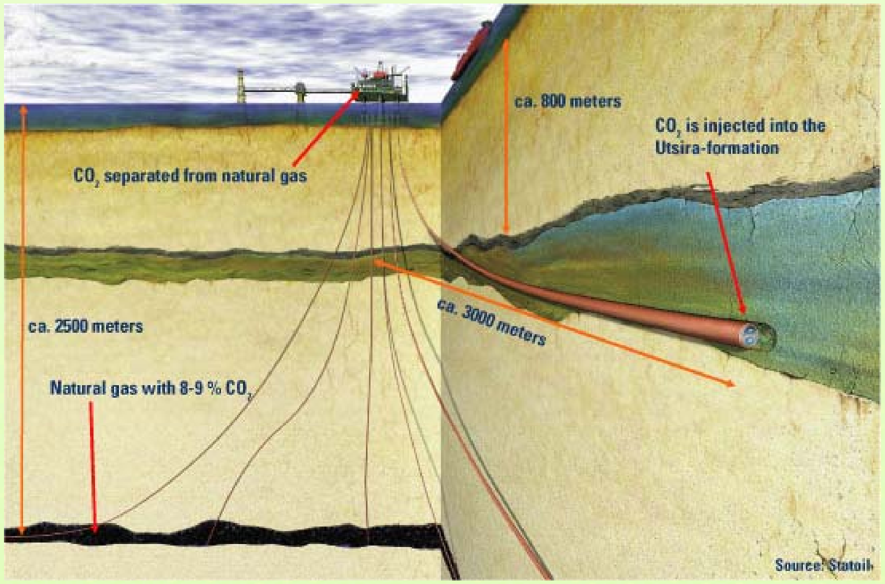


Figure 1: CO2 storage in the Sleipner gas field ((*image courtesy of StatoilHydro).*

AVO : Amplitude Versus Offset

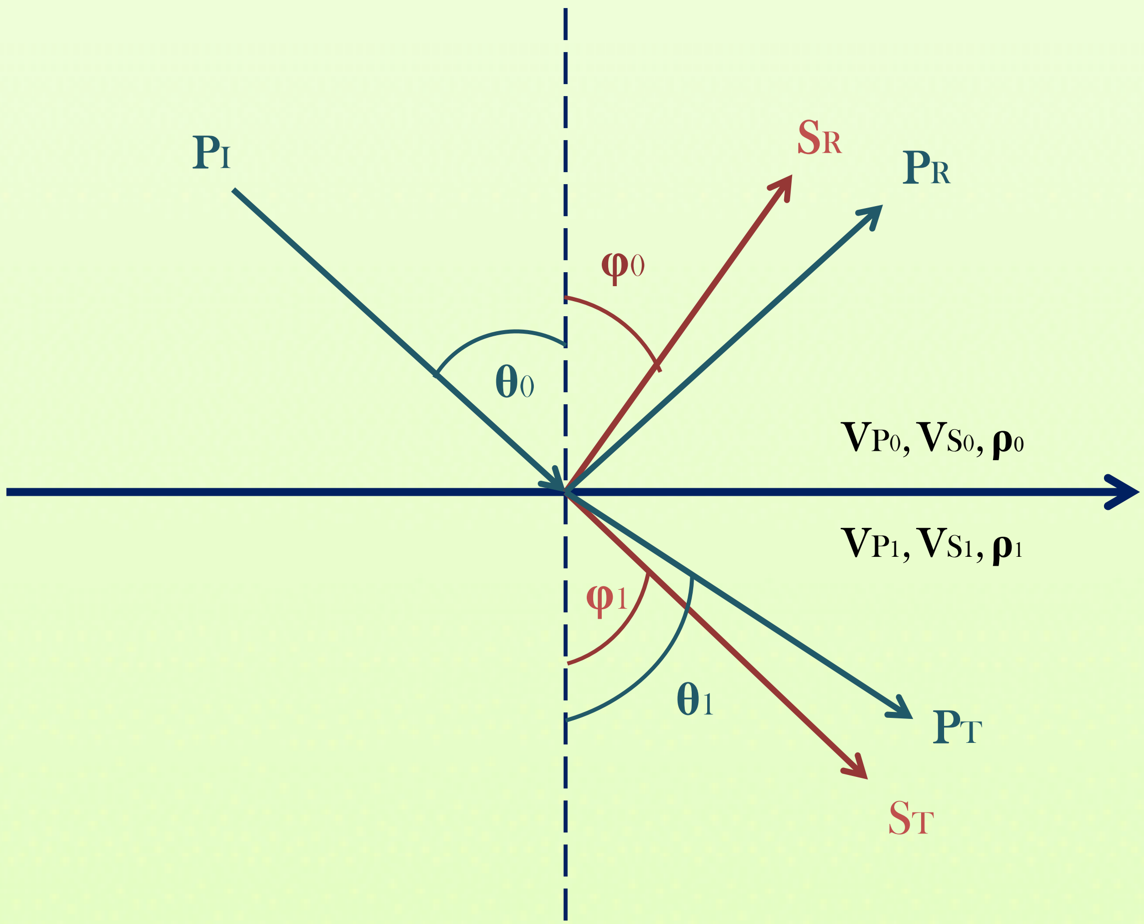


Figure 2. Reflected and transmitted P and S wave for an incident P Wave

Landrø et al., 2001

- Gullfaks field: water injection
- A time-lapse problem
  - Fluid saturation change: from 10% to 70-80%
  - Net pressure change: -5 Mpa to +5 MPa
- Recovery factor: 27%

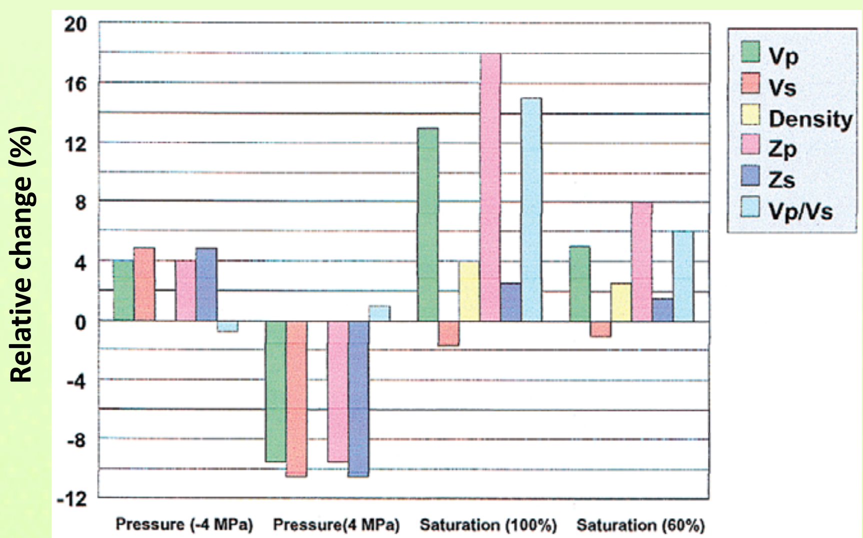
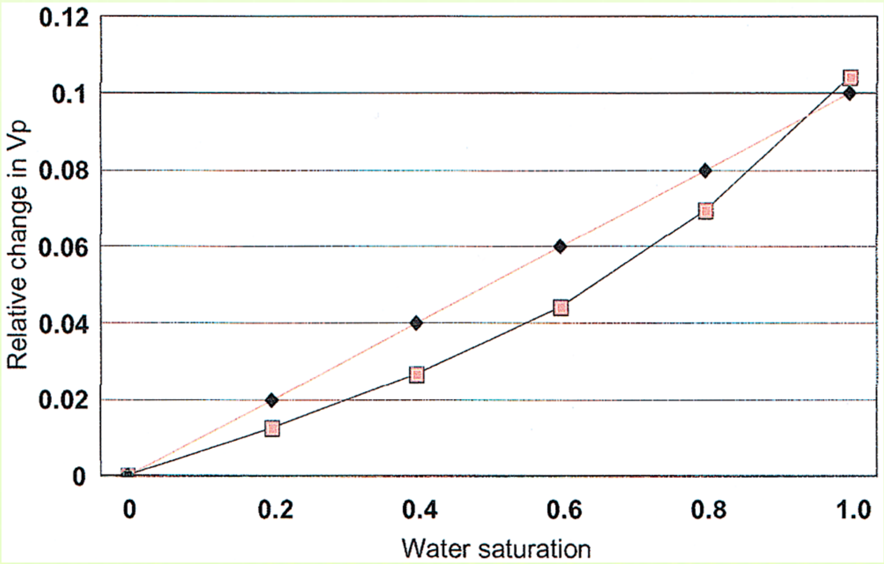


Figure 3: Expected changes in various seismic parameters In Gullfaks field. (Landrø et al., 2001)

Saturation and pressure changes versus seismic parameters changes



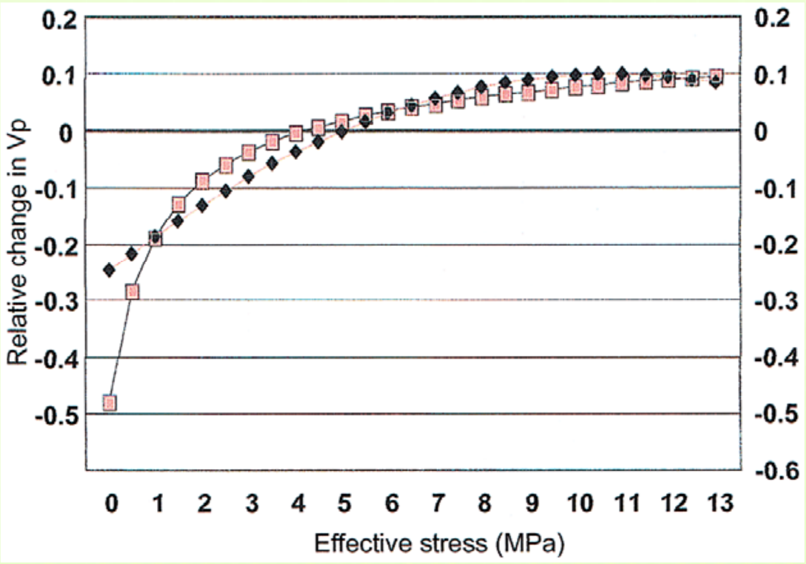
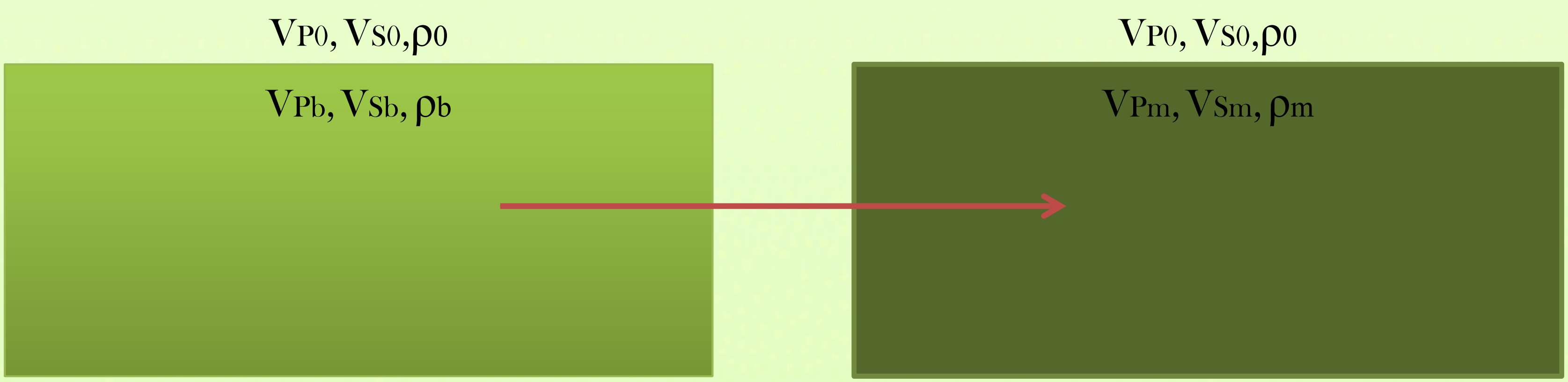


Figure 4: Relationship between (A) relative change in P-wave velocity and water saturation (B) relative change in P-wave velocity versus changes in net pressure based upon a calibrated Gassmann Model (Landrø 2001)

Landrø's model

- Two-layer model
  - A cap-rock layer
  - A reservoir layer
- Seismic parameters changes only in the reservoir



Perturbations in Landrø's work

Baseline Perturbation

$$\Delta V_{Pb} = V_{Pb} - V_{P_0}$$

$$\Delta V_{Sb} = V_{Sb} - V_{S_0}$$

$$\Delta \rho_b = \rho_b - \rho_0$$

Monitoring Perturbation

$$\Delta V_{Pm} = V_{Pm} - V_{P_0}$$

$$\Delta V_{Sm} = V_{Sm} - V_{S_0}$$

$$\Delta \rho_m = \rho_m - \rho_0$$

Time lapse Perturbation

$$\delta V_P = V_{Pm} - V_{P_b}$$

$$\delta V_S = V_{Sm} - V_{S_b}$$

$$\delta \rho = \rho_m - \rho_b$$

Time lapse change in reflectivity: first order (Landrø et al. 2001)

- ΔR_{PP}: Change from baseline to monitoring

- Fluid saturation changes: shear modulus is constant

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)+\frac{\delta V_P}{2V_P}\tan^2\theta$$

- Pressure changes: density is constant

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{2}\frac{\delta V_P}{V_P}-4\frac{V_S^2}{V_P^2}\frac{\delta V_S}{V_S}\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$

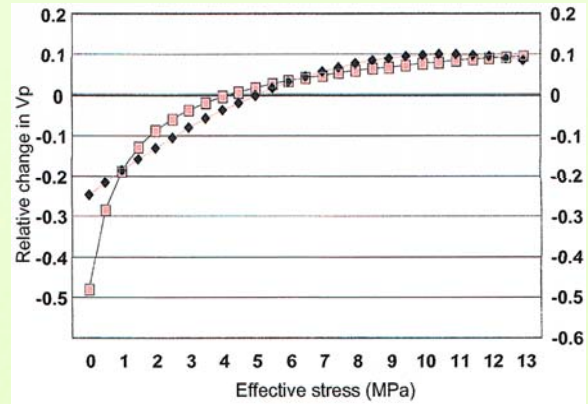
- Fluid saturation and pressure changes

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$

- Notice: ΔR_{PP} independent of $\frac{\Delta V_P}{V_P}, \frac{\Delta V_S}{V_S}, \frac{\Delta\rho}{\rho}$

A general framework for time-lapse AVO

- Large time lapse changes in V_P are possible (Landrø, 2001).



- Linearized equation is inaccurate for large contrast

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\tan^2\theta$$

Goals

- Establishing a framework for approximating $\Delta R_{PP}(\theta)$ that holds for large time-lapse contrasts
- This should reduce to Landrø's form as contrasts shrink

A general framework for time-lapse AVO

- Review a procedure for driving Aki-Richards approximation and nonlinear correction from Zoeppritz equations.
- Adapt the procedure to coincide with the time-lapse problem treated by Landrø.
- Examine linear and nonlinear terms for:
  - Agreement with Landrø at small contrast
  - Behavior of large contrast

Procedure for deriving A-R from Zoeppritz equations

- Zoeppritz Equation for incident P-wave

$$P\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_P \qquad\qquad R_{PP}(\theta) = \frac{\det(P_P)}{\det(P)}$$

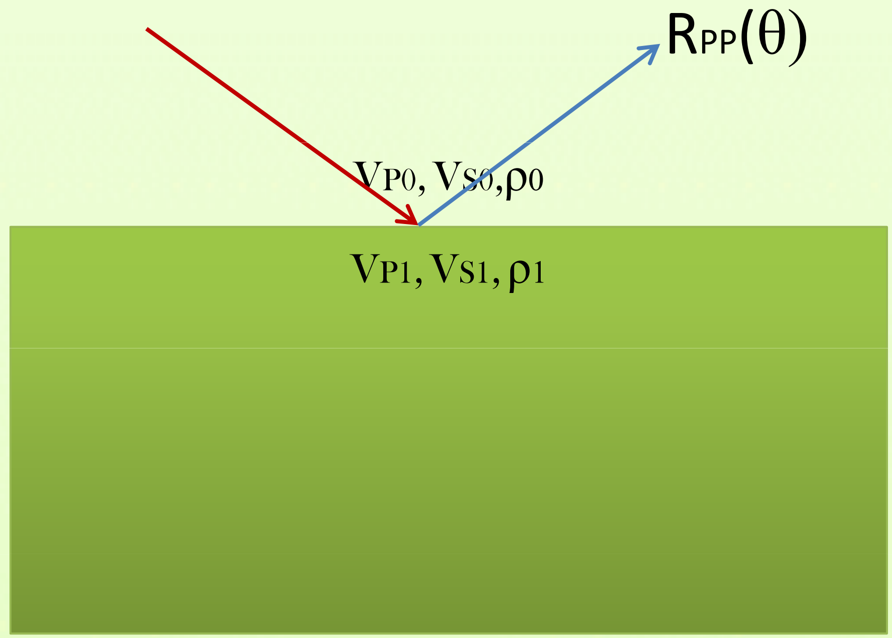
- Matrix P and vector b_P contains incidence and target elastic properties V_P, V_S, ρ, and $\sin\theta$
- Matrix P_P is matrix P with column 1 replaced by b_P

Perturbations in V_P, V_S, ρ

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_1}^2}$$

$$a_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_1}^2}$$

$$a_{\rho} = 1 - \frac{\rho_0}{\rho_1}$$



- The perturbations, which are in a convenient form, are related to the more familiar relative changes:

$$\frac{\Delta V_P}{V_P}, \frac{\Delta V_S}{V_S}, \frac{\Delta \rho}{\rho}$$

Expansion of R_{PP} in orders of perturbations in V_P, V_S, ρ

$$R_{PP}(\theta) = R_{PP}^{(1)}(\theta) + R_{PP}^{(2)}(\theta) + \ldots$$

$$R_{PP}^{(1)}(\theta) = \frac{1}{4}\left(1 + \sin^2\theta\right) a_{VP} - 2\left(\frac{V_{S_0}}{V_{P_0}}\sin\theta\right)^2 a_{VS} + \left(\frac{1}{2} - 2\left(\frac{V_{S_0}}{V_{P_0}}\sin\theta\right)^2\right) a_\rho$$

First order: Equivalent to the Aki-Richards approximation

$$R_{PP}^{(2)}(\theta) = \frac{1}{4}\left(\frac{1}{2} + \sin^2\theta\right) a_{VP}^2 + \left(\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2\theta - 2\left(\frac{V_{S_0}}{V_{P_0}}\sin\theta\right)^2\right) a_{VS}^2$$

$$+ \left(\frac{1}{4} - \frac{1}{4}\left(\frac{V_{S_0}}{V_{P_0}}\right)\sin^2\theta - \left(\frac{V_{S_0}}{V_{P_0}}\sin\theta\right)^2 + \left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2\theta\right) a_\rho^2 + \left(2\left(\frac{V_{S_0}}{V_{P_0}}\right)^3 \sin^2\theta - \left(\frac{V_{S_0}}{V_{P_0}}\sin\theta\right)^2\right) a_\rho a_{VS}$$

R_{PP} with linear and second order approximation

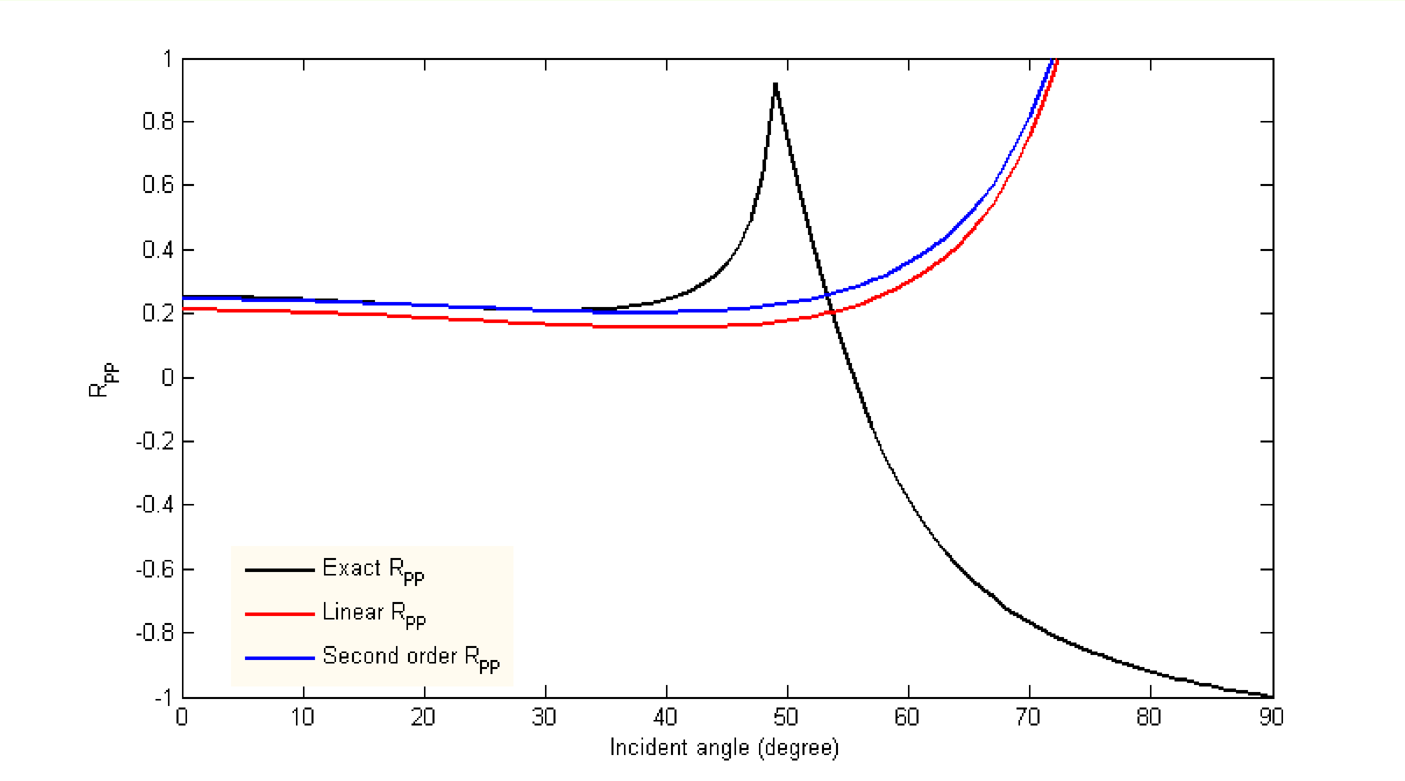
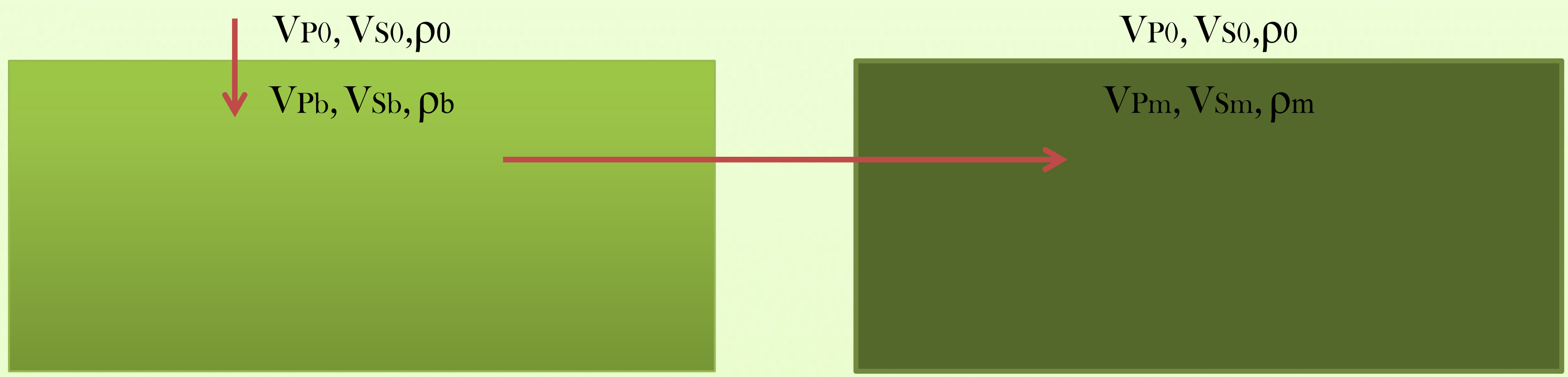


Figure 5. R_{PP} with linear and second order approximation, Elastic incidence parameters: V_{P0} = 3000m/s, V_{S0} = 1500m/s and ρ_0 = 2.0gm/cc ; V_{P1} = 4000m/s, V_{S1} = 2000m/s and ρ_1 = 2.5gm/cc .

Adapting the procedure to the time-lapse problem



Baseline Survey

Monitor Survey

Baseline perturbation

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{Pb}^2}, \quad a_{VS} = 1 - \frac{V_{S_0}^2}{V_{Sb}^2}, \quad a_{\rho} = 1 - \frac{\rho_0}{\rho_b},$$

Time-lapse perturbation

$$b_{VP} = 1 - \frac{V_{Pb}^2}{V_{Pm}^2}, \quad b_{VS} = 1 - \frac{V_{Sb}^2}{V_{Sm}^2}, \quad b_{\rho} = 1 - \frac{\rho_b}{\rho_m},$$

Adapting the procedure to the time-lapse problem

- Zoeppritz equations for baseline and monitoring targets:

$$P_{BL}\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_{BL} \quad R_{PP}^{BL}(\theta) = \frac{\det(P_P)}{\det(P)} \qquad P_{M}\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_{M} \quad R_{PP}^{M}(\theta) = \frac{\det(P_P)}{\det(P)}$$

- Expand $\Delta R_{PP}(\theta)$ in orders of a_{VP}, … and b_{VP},…

$$\Delta R_{PP}(\theta) = R_{PP}^{M}(\theta) - R_{PP}^{BL}(\theta)$$

R_{PP} for the Baseline and Monitor survey and ΔR_{PP}

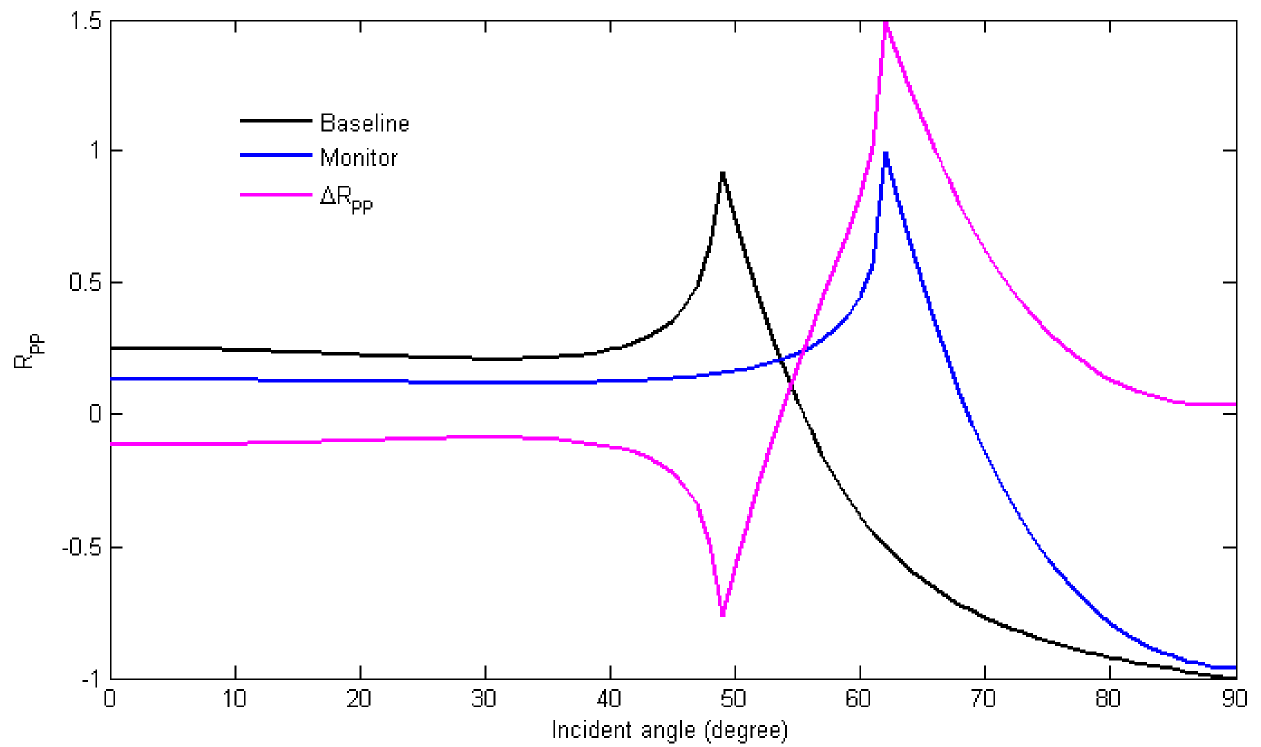


Figure 6. R_{PP} for the Baseline and Monitor survey and ΔR_{PP} ,
Elastic incidence parameters: V_{P0} = 3000m/s, V_{S0} = 1500m/s and ρ_0 = 2.0gm/cc ;
Baseline parameters: V_{Pb} = 4000m/s, V_{Sb} =2000m/s and ρ_b = 2.5 gm/cc ;
Monitor parameters: V_{Pm} = 3400m/s, V_{Sm} = 1700m/s and ρ_m = 2.4 gm/cc.

Examine linear and nonlinear terms

$$\Delta R_{PP}^{(1)}(\theta)=\frac{1}{4}\left(1+sin^2\theta\right)b_{VP}-2\left(\frac{V_{S_0}}{V_{P_0}}sin\theta\right)^2 b_{VS}+\left(\frac{1}{2}-2\left(\frac{V_{S_0}}{V_{P_0}}sin\theta\right)^2\right)b_\rho$$

$$\Delta R_{PP}^{(2)}(\theta)=\mathrm{K}_{VP}\left(b_{VP}^2\right)+\mathrm{K}_{VS}\left(b_{VS}^2\right)+\mathrm{K}_{\rho}\left(b_{\rho}^2\right)+\mathrm{K}_{\rho VS}\left(b_{\rho}b_{VS}\right)+\mathrm{K}_{VSS}\left(a_{VS}b_{VS}\right)$$
$$+\mathrm{K}_{VPP}\left(a_{VP}b_{VP}\right)+\mathrm{K}_{\rho VS}\left(a_{\rho}b_{VS}\right)+\mathrm{K}_{VS\rho}\left(b_{\rho}a_{VS}\right)+\mathrm{K}_{\rho\rho}\left(a_{\rho}b_{\rho}\right)$$

Agreement of linear term in ΔR_{PP} with Landrø's work

$$a_{VP} = 2\left(\frac{\Delta V_P}{V_P}\right) - 2\left(\frac{\Delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_P}{V_P}\right)^3 - \ldots \qquad b_{VP} = 2\left(\frac{\delta V_P}{V_P}\right) - 2\left(\frac{\delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\delta V_P}{V_P}\right)^3 - \ldots$$

... ...

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{4}\left(1+\sin^2\theta\right) b_{VP} - 2\left(\left(\frac{V_S}{V_P}\sin\theta\right)^2\right) b_{VS} + \left(\frac{1}{2} - 2\sin^2\theta\right) b_\rho$$

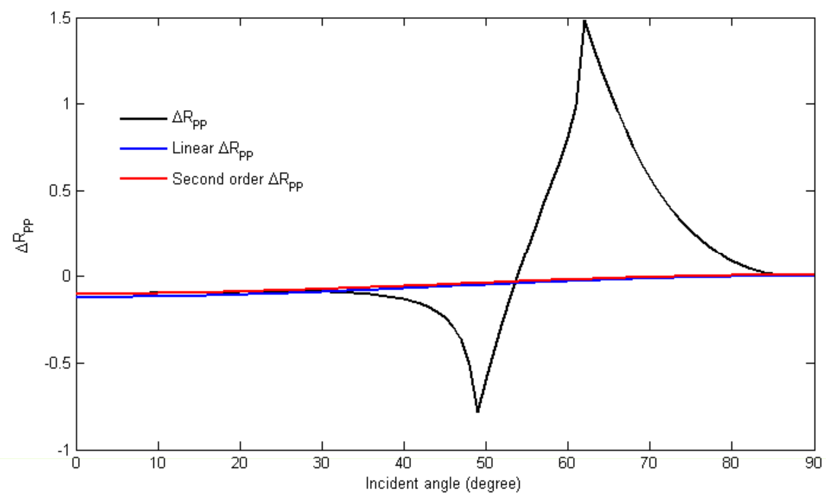
$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}\left(\frac{\delta\rho}{\rho} + \frac{\delta V_P}{V_P}\right) - 2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho} + 2\frac{\delta V_S}{V_S}\right)\sin^2\theta + \frac{\delta V_P}{2V_P}\tan^2\theta$$

Second order approximation in orders of relative changes

$$\Delta R_{PP}^{(2)}(\theta) = \mathrm{K}_{VP}\left(b_{VP}^{2}\right) + \mathrm{K}_{VS}\left(b_{VS}^{2}\right) + \mathrm{K}_{\rho}\left(b_{\rho}^{2}\right) + \mathrm{K}_{\rho VS}\left(b_{\rho} b_{VS}\right) + \mathrm{K}_{VSS}\left(a_{VS} b_{VS}\right)$$
$$+ \mathrm{K}_{VPP}\left(a_{VP} b_{VP}\right) + \mathrm{K}_{\rho VS}\left(a_{\rho} b_{VS}\right) + \mathrm{K}_{VS\rho}\left(b_{\rho} a_{VS}\right) + \mathrm{K}_{\rho\rho}\left(a_{\rho} b_{\rho}\right)$$

$$\Delta R_{PP}^{(2)}(\theta) = \Gamma_{\delta VP}\left(\frac{\delta V_P}{V_P}\right)^2 + \Gamma_{\delta VS}\left(\frac{\delta V_S}{V_S}\right)^2 + \Gamma_{\delta\rho}\left(\frac{\delta\rho}{\rho}\right)^2 + \Gamma_{\delta\rho VS}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)$$
$$+ \Gamma_{\delta\rho\Delta VP}\left(\frac{\delta V_P}{V_P}\right)\left(\frac{\Delta V_P}{V_P}\right) + \Gamma_{\delta\rho\Delta VS}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta V_S}{V_S}\right) + \Gamma_{\delta\rho\Delta\rho}\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\Delta\rho}{\rho}\right)$$
$$+ \Gamma_{\delta VS\Delta\rho}\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta\rho}{\rho}\right) + \Gamma_{\delta VS\Delta VS}\left(\frac{\delta V_S}{V_S}\right)\left(\frac{\Delta V_S}{V_S}\right)$$

ΔR_{PP} for the exact, linear, and second order approximation



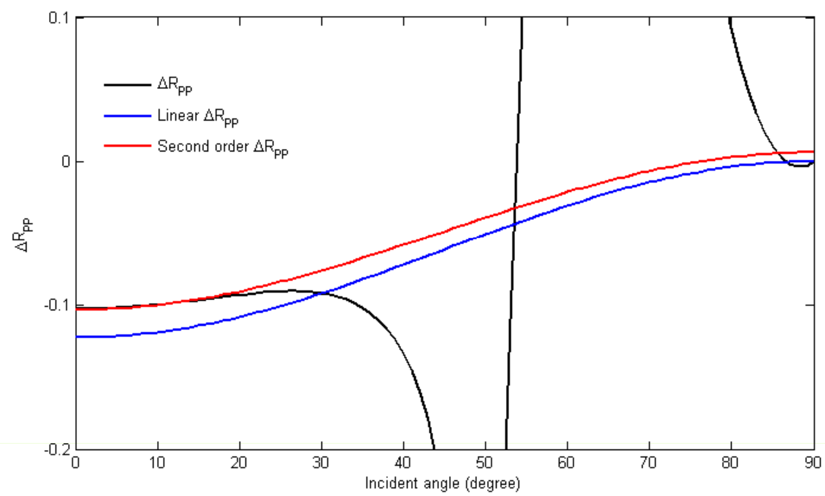


Figure 7. ΔR_{PP} for the exact, linear, and second order approximation,
Elastic incidence parameters: V_{P0} = 3000m/s, V_{S0} = 1500m/s and ρ_0 = 2.0gm/cc ;
Baseline parameters: V_{Pb} = 4000m/s, V_{Sb} =2000m/s and ρ_b = 2.5 gm/cc ;
Monitor parameters: V_{Pm} = 3400m/s, V_{Sm} = 1700m/s and ρ_m = 2.4 gm/cc.

Summary

- A framework for linear and non linear time-lapse AVO analysis is formulated.
- Linear and higher order approximations are available for ΔR_{PP}, ΔR_{PS}, (ΔR_{SS} in 2013).
- Agreement of linear term in ΔR_{PP} with Landrø's work.
- We conclude that in many plausible time-lapse scenarios increase in accuracy associated with higher order corrections is non-negligible.

Future work

- Further numerical, analytical examination of ΔR_{PP}, ΔR_{PS}, ΔR_{SS}
- Validation of time-lapse AVO formula using physical modeling data
- Modeling of inversion of field data example

Acknowledgments

- Dr Kris Innanen
- CREWES

Questions

Zoeppritz matrix- Elastic parameters

$$P\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_P \qquad R_{PP}(\theta_0) = \frac{\det(P_P)}{\det(P)} \qquad b_P \equiv \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2(BX)^2 \end{bmatrix}$$

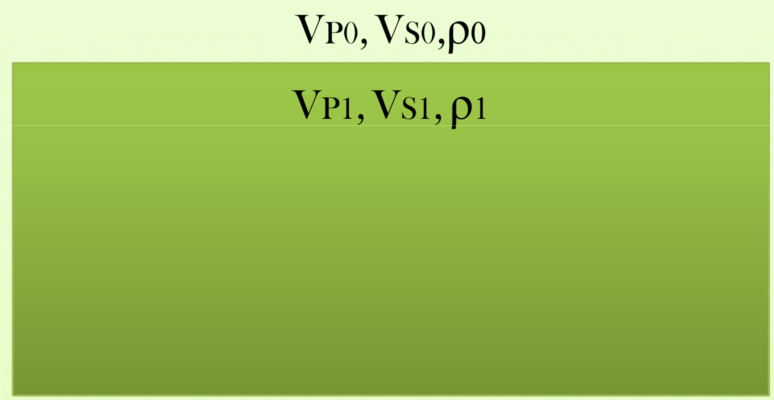
$$X = Sin(\theta_0),\ \ A \equiv \frac{\rho_1}{\rho_0},\ \ B \equiv \frac{V_{S_0}}{V_{P_0}},\ \ C \equiv \frac{V_{P_1}}{V_{P_0}},\ \ D \equiv \frac{V_{S_1}}{V_{P_0}},\ \ E \equiv \frac{V_{P_1}}{V_{S_0}},\ \ F \equiv \frac{V_{S_1}}{V_{S_0}}.$$

$$P \equiv \begin{bmatrix} -X & -\sqrt{1-(BX)^2} & CX & \sqrt{1-(DX)^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-(CX)^2} & -DX \\ 2B^2X\sqrt{1-X^2} & B\left(1-2(BX)^2\right) & 2AD^2X\sqrt{1-(CX)^2} & AD\left(1-2(DX)^2\right) \\ -\left(1-2(BX)^2\right) & 2B^2X\sqrt{1-(BX)^2} & AC\left(1-2(DX)^2\right) & -2AD^2X\sqrt{1-(DX)^2} \end{bmatrix}$$

Zoeppritz matrix- Perturbation parameters

$$a_{VP} = 1 - \frac{V_{P_0}^2}{V_{P_1}^2}, \quad a_{VS} = 1 - \frac{V_{S_0}^2}{V_{S_1}^2}, \quad a_\rho = 1 - \frac{\rho_0}{\rho_1},$$

$$A \equiv \frac{\rho_1}{\rho_0}, \; C \equiv \frac{V_{P_1}}{V_{P_0}}, \; D \equiv \frac{V_{S_1}}{V_{P_0}}.$$



$$A = \left(1 - a_\rho\right)^{-1}, \; C = \left(1 - a_{VP}\right)^{-\frac{1}{2}}, \; D = B \times \left(1 - a_{VS}\right)^{-\frac{1}{2}}$$

$$(1 - a_\rho)^{-1} = 1 + a_\rho + a_\rho^2 + \ldots$$

$$(1 - a_{VP})^{-\frac{1}{2}} = 1 + \frac{1}{2} a_{VP} + \frac{3}{8} a_{VP}^2 + \ldots$$

$$(1 - a_{VS})^{-\frac{1}{2}} = 1 + \frac{1}{2} a_{VS} + \frac{3}{8} a_{VS}^2 + \ldots$$

$$R_{PP}(\theta_0) = \frac{\det(P_P)}{\det(P)}$$

Elastic parameters in Monitor survey

$$A \equiv \frac{\rho_m}{\rho_0}\,,\;\; B \equiv \frac{V_{Sm}}{V_{P_0}}\,,\;\; C \equiv \frac{V_{Pm}}{V_{P_0}}\,,\;\; D \equiv \frac{V_{Sm}}{V_{P_0}}\,,\;\; E \equiv \frac{V_{Pm}}{V_{S_0}}\,,\;\; F \equiv \frac{V_{Sm}}{V_{S_0}}\,.$$

$$A \equiv \frac{\rho_m}{\rho_0} = \frac{\rho_m}{\rho_b} \times \frac{\rho_b}{\rho_0} = \left(1-b_\rho\right)^{-1} \times \left(1-a_\rho\right)^{-1}$$

$$C \equiv \frac{V_{Pm}}{V_{P_0}} = \frac{V_{Pm}}{V_{Pb}} \times \frac{V_{Pb}}{V_{P_0}} = \left(1-a_{VP}\right)^{-\frac{1}{2}} \times \left(1-b_{VP}\right)^{-\frac{1}{2}}$$

$$D \equiv B \times \frac{V_{Sm}}{V_{S_0}} = B \times \frac{V_{Sm}}{V_{Sb}} \times \frac{V_{Sb}}{V_{S_0}} = B \times \left(1-a_{VS}\right)^{-\frac{1}{2}} \times \left(1-b_{VS}\right)^{-\frac{1}{2}}$$

Examine linear and nonlinear terms

$$R_{PP}^{(1)}(\theta)=\frac{1}{4}\left(1+sin^2\theta\right)b_{VP}-2\left(\frac{V_{S_0}}{V_{P_0}}sin\theta\right)^2 b_{VS}+\left(\frac{1}{2}-2\left(\frac{V_{S_0}}{V_{P_0}}sin\theta\right)^2\right)b_\rho$$

$$\Delta R_{PP}^{(2)}(\theta)=\frac{1}{4}(\frac{1}{2}+X^2)b_{VP}^2+(B^3X^2-2(BX)^2)b_{VS}^2+\left(\frac{1}{4}-\frac{1}{4}BX^2-(BX)^2+B^3X^2\right)b_\rho^2$$
$$+(2B^3X^2-(BX)^2)b_\rho b_{VS}+(2B^3X^2-2(BX)^2)a_{VS}b_{VS}+(\frac{1}{4}X^2)a_{VP}b_{VP}$$
$$+(2B^3X^2-(BX)^2)a_\rho b_{VS}+(2B^3X^2-(BX)^2)b_\rho a_{VS}+(2B^3X^2-\frac{1}{2}BX^2)a_\rho b_\rho$$

Second order approximation

$$\Delta R_{PP}^{(2)}(\theta_0)=\frac{1}{4}(\frac{1}{2}+X^2)b_{VP}^2+(B^3X^2-2(BX)^2)b_{VS}^2+\left(\frac{1}{4}-\frac{1}{4}BX^2-(BX)^2+B^3X^2\right)b_{\rho}^2$$
$$+(2B^3X^2-(BX)^2)b_{\rho}b_{VS}+(2B^3X^2-2(BX)^2)a_{VS}b_{VS}+(\frac{1}{4}X^2)a_{VP}b_{VP}$$
$$+(2B^3X^2-(BX)^2)a_{\rho}b_{VS}+(2B^3X^2-(BX)^2)b_{\rho}a_{VS}+(2B^3X^2-\frac{1}{2}BX^2)a_{\rho}b_{\rho}$$

$$\Delta R_{PP}^{(2)}(\theta_0)=\left(X^2+\frac{1}{2}\right)\left(\frac{\delta V_P}{V_P}\right)^2+4\left(B^3X^2-2B^2X^2\right)\left(\frac{\delta V_S}{V_S}\right)^2$$
$$+\left(\frac{1}{4}-\frac{1}{4}BX^2-B^2X^2+B^3X^2\right)\left(\frac{\delta\rho}{\rho}\right)^2+2\left(2B^3X^2-B^2X^2\right)\left(\frac{\delta\rho}{\rho}\right)\left(\frac{\delta V_S}{V_S}\right)$$

$$X=Sin(\theta_0),B\equiv\frac{V_{S_0}}{V_{P_0}}.$$

Agreement of linear term in ΔR_{PP} with Landrø's work

$$a_{VP} = 2\left(\frac{\Delta V_P}{V_P}\right) - 2\left(\frac{\Delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_P}{V_P}\right)^3 - \ldots \qquad b_{VP} = 2\left(\frac{\delta V_P}{V_P}\right) - 2\left(\frac{\delta V_P}{V_P}\right)^2 + \frac{3}{2}\left(\frac{\delta V_P}{V_P}\right)^3 - \ldots$$

$$a_{VS} = 2\left(\frac{\Delta V_S}{V_S}\right) - 2\left(\frac{\Delta V_S}{V_S}\right)^2 + \frac{3}{2}\left(\frac{\Delta V_S}{V_S}\right)^3 - \ldots \qquad b_{VS} = 2\left(\frac{\delta V_S}{V_S}\right) - 2\left(\frac{\delta V_S}{V_S}\right)^2 + \frac{3}{2}\left(\frac{\delta V_S}{V_S}\right)^3 - \ldots$$

$$a_{\rho} = \left(\frac{\Delta\rho}{\rho}\right) - \frac{1}{2}\left(\frac{\Delta\rho}{\rho}\right)^2 + \frac{1}{4}\left(\frac{\Delta\rho}{\rho}\right)^3 + \ldots \qquad b_{\rho} = \left(\frac{\delta\rho}{\rho}\right) - \frac{1}{2}\left(\frac{\delta\rho}{\rho}\right)^2 + \frac{1}{4}\left(\frac{\delta\rho}{\rho}\right)^3 + \ldots$$

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{4}\left(1+\sin^2\theta\right)b_{VP} - 2\left(\left(\frac{V_S}{V_P}\sin\theta\right)^2\right)b_{VS} + \left(\frac{1}{2} - 2\sin^2\theta\right)b_{\rho}$$

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}\left(\frac{\delta\rho}{\rho} + \frac{\delta V_P}{V_P}\right) - 2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho} + 2\frac{\delta V_S}{V_S}\right)\sin^2\theta + \frac{\delta V_P}{2V_P}\tan^2\theta$$