Characterization of poroelastic targets for P- and S-waves using linear and non-linear AVO methods

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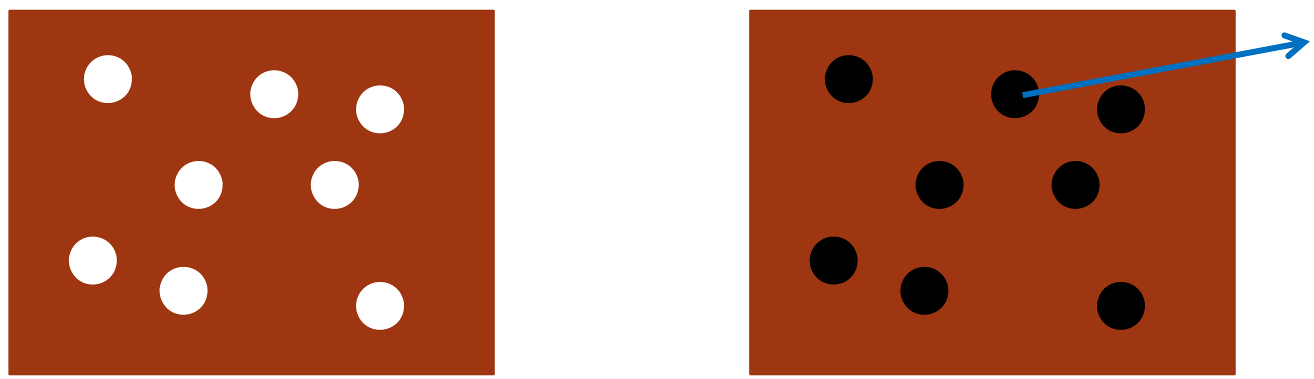
Outline

- Review of poroelasticity
  - Biot (1941), Gassmann (1951)
  - Russell et al. (2011)
- Review of AVO (full and linearized)
  - Zoeppritz
  - Aki and Richards
  - Russell
- Research objectives
  - Implementation of poroelasticity into full elastic equations
  - Extension – nonlinear AVO
- A different model of poroelastic reflections

Poroelasticity

- Biot (1941) and Gassmann (1951) devise relationships of elastic moduli to poroelastic moduli
  - Dry modulus (skeleton framework) to a saturated modulus (fluid filled)



<u>Fluids</u>
- Air
- Brine
- Gas

H_{dry} $\quad$ H_{sat}

$$H = \text{modulus}$$

Poroelasticity

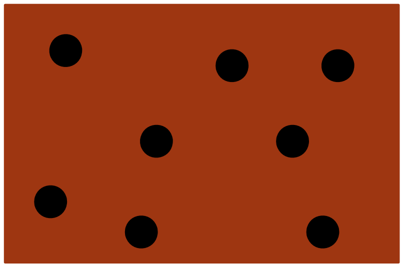
- Biot (1941) and Gassmann (1951) devise relationships of elastic moduli to poroelastic moduli

$$\boxed{\mu_{sat} = \mu_{dry} \qquad \lambda_{sat} = \lambda_{dry} + f \qquad K_{sat} = K_{dry} + f}$$

$$f = \alpha^2 M$$

$$\alpha = 1 - \frac{K_{dry}}{K_m} \qquad M^{-1} = \frac{\alpha - \phi}{K_m} + \frac{\phi}{K_{fl}}$$

$f = 0$



$f = f_{brine}$

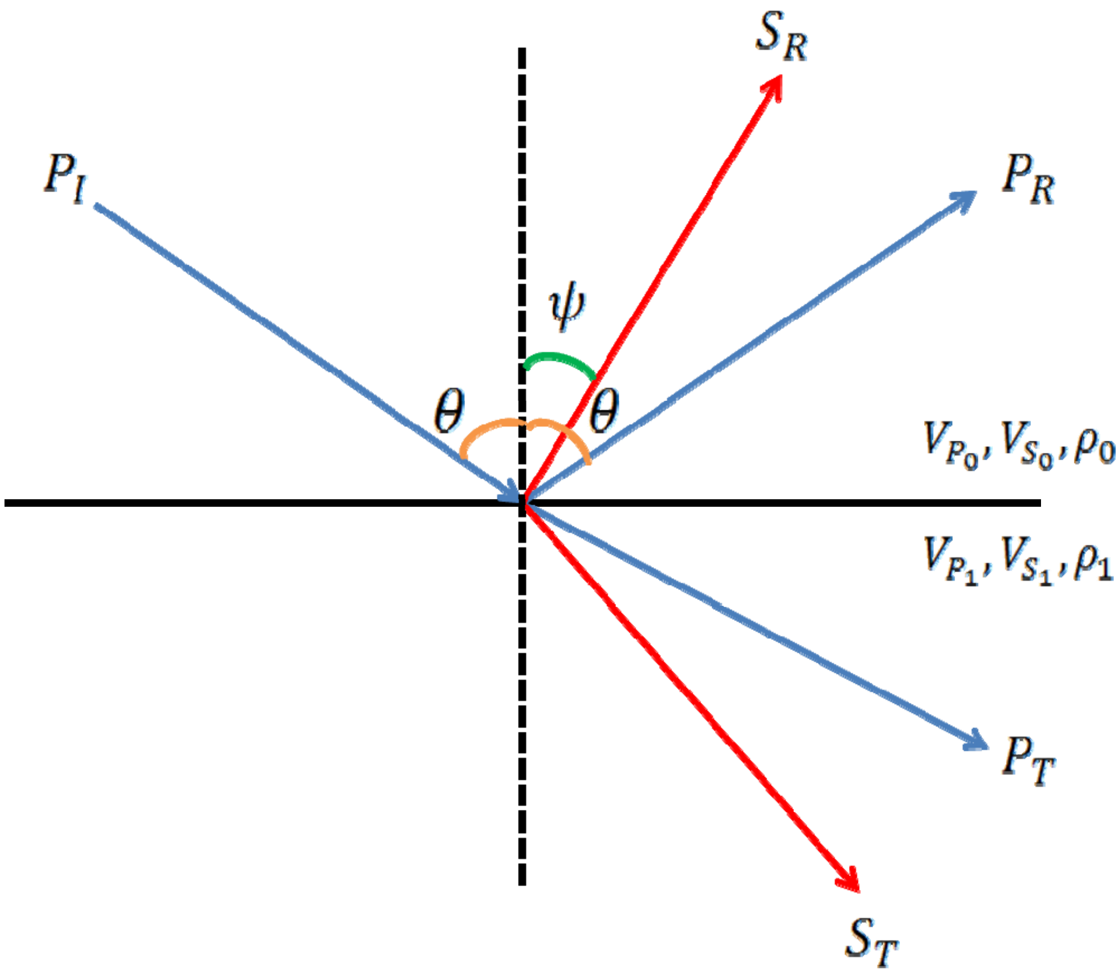
Poroelasticity

$$V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + (4/3)\mu}{\rho}} \qquad V_S = \sqrt{\frac{\mu}{\rho}}$$

$$\downarrow \qquad\qquad \downarrow$$

$$(V_P)_{sat} = \sqrt{\frac{\lambda_{dry} + f + 2\mu}{\rho_{sat}}} = \sqrt{\frac{K_{dry} + f + (4/3)\mu}{\rho_{sat}}} \qquad (V_S)_{sat} = \sqrt{\frac{\mu_{dry}}{\rho_{sat}}}$$

Incident P-wave model



<u>Exact</u>

- Zoeppritz

<u>Approx.</u>

- Aki and Richards
- Shuey
- Russell

Zoeppritz equations

- Keys (1989)

$$\begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & -\sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & DX \\ 2B^2X\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & -AD(1-2D^2X^2) \\ -(1-2B^2X^2) & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & 2AD^2X\sqrt{1-D^2X^2} \end{bmatrix} \begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2B^2X^2 \end{bmatrix}$$

$A = \frac{\rho_1}{\rho_0}$	$B = \frac{V_{S0}}{V_{P0}}$	$C = \frac{V_{P1}}{V_{P0}}$	$D = \frac{V_{S1}}{V_{P0}}$	$X = sin\theta$
constant	constant	constant	constant	varies

<u>Recall:</u>
Assumes homogeneity and isotropy

Linearized AVO

- Aki and Richards (2002)

$$R_{PP}(\theta) \approx (1+\tan^2\theta)\frac{\Delta V_P}{2V_P} + \left(-\frac{8\sin^2\theta}{\gamma_{sat}^2}\right)\frac{\Delta V_S}{2V_S} + \left(1-\frac{4\sin^2\theta}{\gamma_{sat}^2}\right)\frac{\Delta\rho}{2\rho}$$

$$\gamma_{sat}^2 = \left(\frac{V_{P\,avg}}{V_{S\,avg}}\right)_{sat}^2 \qquad \gamma_{dry}^2 = \left(\frac{V_{P\,avg}}{V_{S\,avg}}\right)_{dry}^2$$

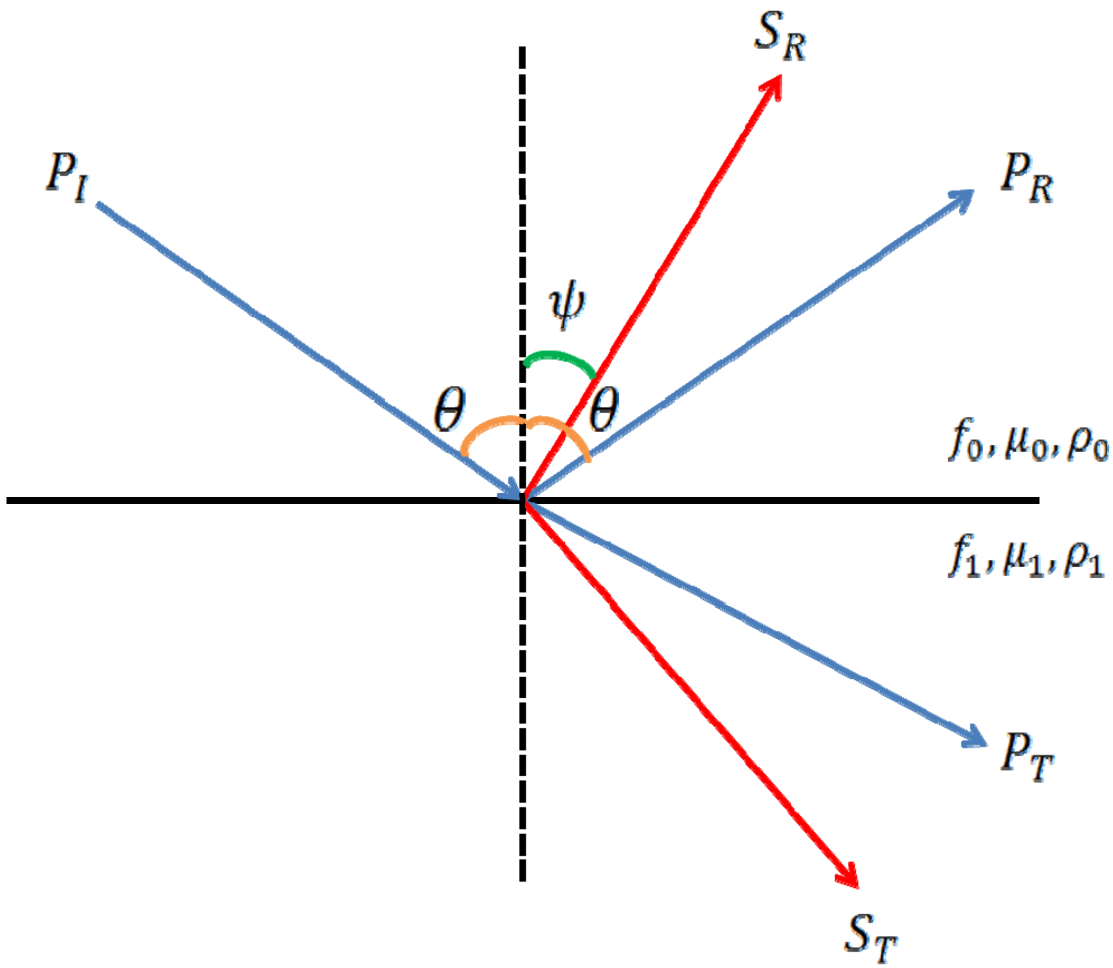
Linearized AVO and poroelasticity

- Russell's AVO approximation (fluid-mu-rho equation)

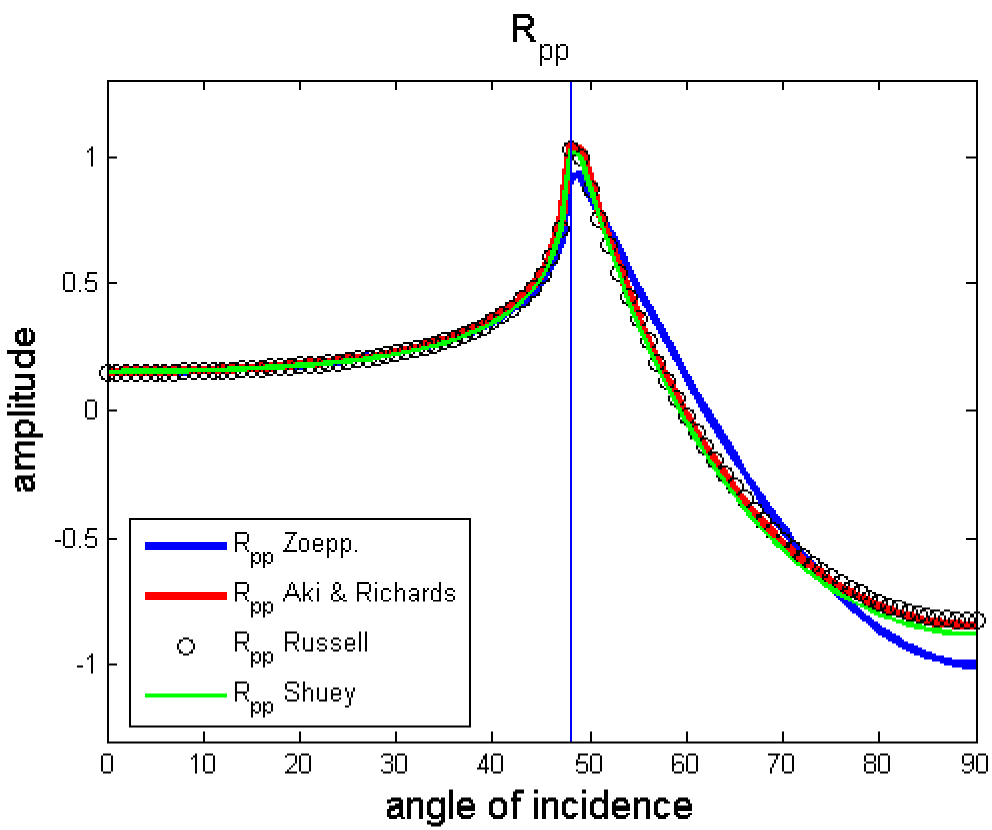
$$R_{PP}(\theta) \approx \left[\left(1-\frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)\frac{\sec^2\theta}{4}\right]\frac{\Delta f}{f} + \left[\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}\sec^2\theta - \frac{2}{\gamma_{sat}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu} + \left[\frac{1}{2} - \frac{\sec^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

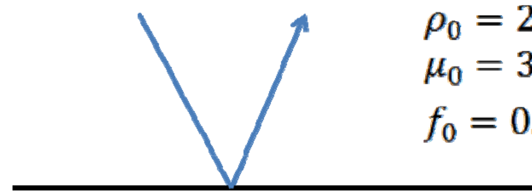
$$\begin{bmatrix} R_{PP}(\theta_1) \\ R_{PP}(\theta_2) \\ \vdots \\ R_{PP}(\theta_N) \end{bmatrix} = \begin{bmatrix} c_1(\theta_1) & c_2(\theta_1) & c_3(\theta_1) \\ c_1(\theta_2) & c_2(\theta_2) & c_3(\theta_2) \\ \vdots & \vdots & \vdots \\ c_1(\theta_1) & c_2(\theta_N) & c_3(\theta_N) \end{bmatrix} \begin{bmatrix} \Delta f/f \\ \Delta\mu/\mu \\ \Delta\rho/\rho \end{bmatrix}$$

Incident P-wave model



Example poroelastic AVO curve





$\rho_0 = 2235\ kg/m^3$
$\mu_0 = 3\ GPa$
$f_0 = 0.659\ GPa$

$\rho_1 = 2263\ kg/m^3$
$\mu_1 = 3\ GPa$
$f_1 = 7.02\ GPa$

Research objectives

- Adapt poroelastic reflection modelling into the exact elastic equations
- Produce series expansions of Rpp in orders of $\Delta f / f$, $\Delta\mu/\mu$, and $\Delta\rho/\rho$ (reflectivity series) and a_f, a_μ, a_ρ (perturbation series)
- Compare with the Russell Approximation
- Observe the role of nonlinear terms
  - Numerical accuracy
  - Geophysical interpretability

Zoeppritz equations

- What is the poroelastic AVO problem?

$$a_f = 1 - \frac{f_0}{f_1} \qquad a_\mu = 1 - \frac{\mu_0}{\mu_1} \qquad a_\rho = 1 - \frac{\rho_0}{\rho_1}$$

$$A = \frac{\rho_1}{\rho_0} = (1 - a_\rho)^{-1} \qquad B = \frac{V_{S_0}}{V_{P_0}} = \gamma_{sat}^{-1}$$

$$C = \frac{V_{P_1}}{V_{P_0}} = \left\{(1 - a_\rho)\left[\left(\frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)(1 - a_\mu)^{-1} + \left(1 - \frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)(1 - a_f)^{-1}\right]\right\}^{\frac{1}{2}}$$

$$D = \frac{V_{S_1}}{V_{P_0}} = \gamma_{sat}^{-1}\left[(1 - a_\mu)^{-1}(1 - a_\rho)\right]^{\frac{1}{2}}$$

Zoeppritz equations

- These elastic constants which now contain poroelastic elements can be replaced into the Zoeppritz equations
- Using Cramer's rule, we can solve for linearized expressions of Rpp, Rps, Tpp, Tps

$$R_{pp} = \frac{\det\left(\begin{bmatrix} X & -\sqrt{1-B^2X^2} & CX & -\sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & DX \\ 2B^2X\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & -AD(1-2D^2X^2) \\ (1-2B^2X^2) & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & 2AD^2X\sqrt{1-D^2X^2} \end{bmatrix}\right)}{\det\left(\begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & -\sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & DX \\ 2B^2X\sqrt{1-X^2} & B(1-2B^2X^2) & 2AD^2X\sqrt{1-C^2X^2} & -AD(1-2D^2X^2) \\ -(1-2B^2X^2) & 2B^2X\sqrt{1-B^2X^2} & AC(1-2D^2X^2) & 2AD^2X\sqrt{1-D^2X^2} \end{bmatrix}\right)}$$

Linearized poroelastic AVO

$$R_{PP}^{(1)}(\theta) \approx \left[\left(1-\frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)\frac{1+\sin^2\theta}{4}\right]\frac{\Delta f}{f} + \left[\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}(1+\sin^2\theta) - \frac{2}{\gamma_{sat}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu}$$

$$+ \left[\frac{1}{4}-\frac{\sin^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

Linearized poroelastic AVO

Derived from Zoeppritz

$$W_{\Delta 1} = \left[\left(1 - \frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)\frac{1 + \sin^2\theta}{4}\right]\frac{\Delta f}{f}$$

$$W_{\Delta 2} = \left[\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}(1 + \sin^2\theta) - \frac{2}{\gamma_{sat}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu}$$

$$W_{\Delta 3} = \left[\frac{1}{4} - \frac{\sin^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

Derived from Aki-Richards (Russell)

$$W_{R1} = \left[\left(1 - \frac{\gamma_{dry}^2}{\gamma_{sat}^2}\right)\frac{\sec^2\theta}{4}\right]\frac{\Delta f}{f}$$

$$W_{R2} = \left[\frac{\gamma_{dry}^2}{4\gamma_{sat}^2}\sec^2\theta - \frac{2}{\gamma_{sat}^2}\sin^2\theta\right]\frac{\Delta\mu}{\mu}$$

$$W_{R3} = \left[\frac{1}{2} - \frac{\sec^2\theta}{4}\right]\frac{\Delta\rho}{\rho}$$

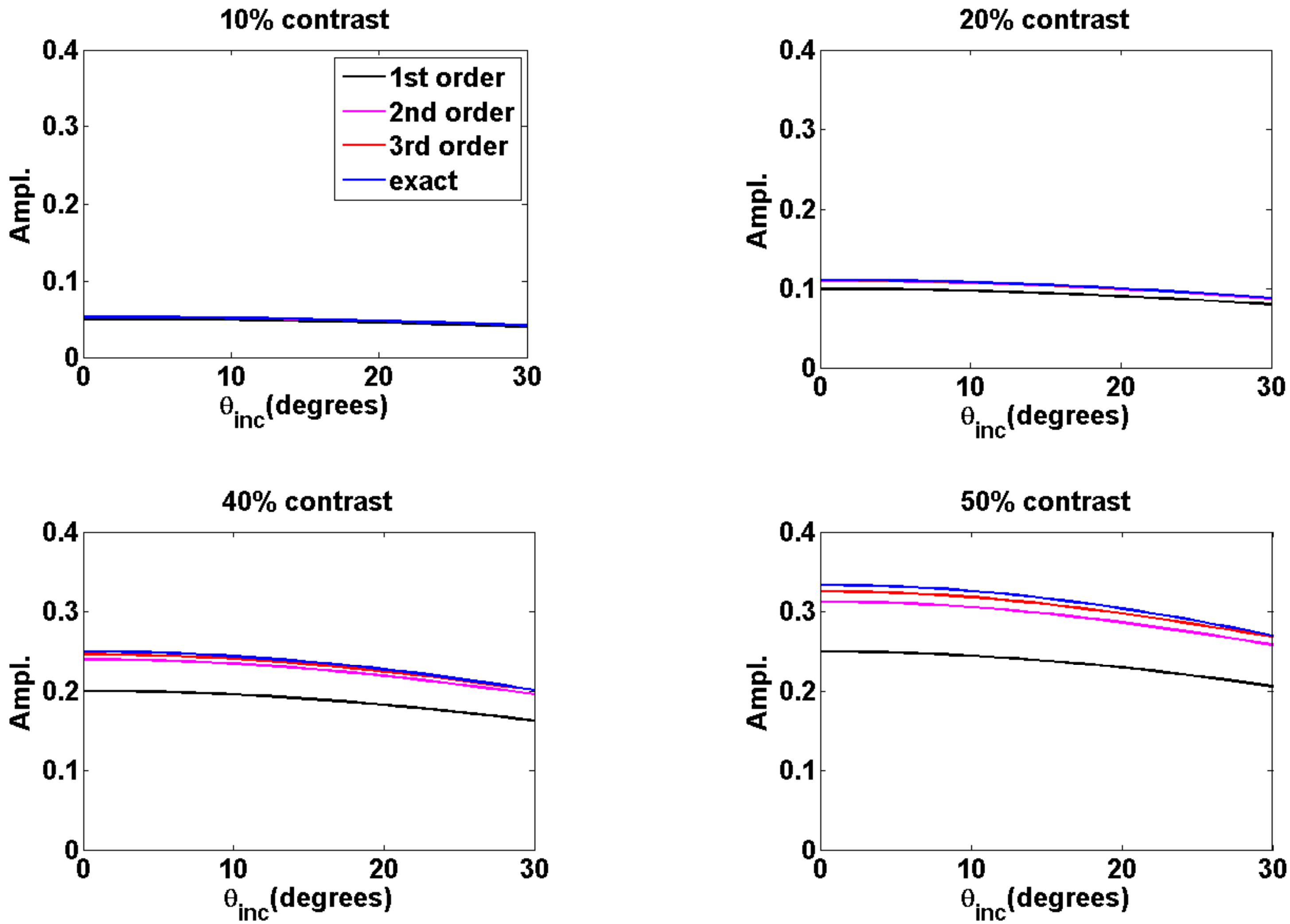
$$R_{pp}^{(1)}(\theta) = R_{pp}^{(Ru)}(\theta)$$

Non-Linear poroelastic AVO

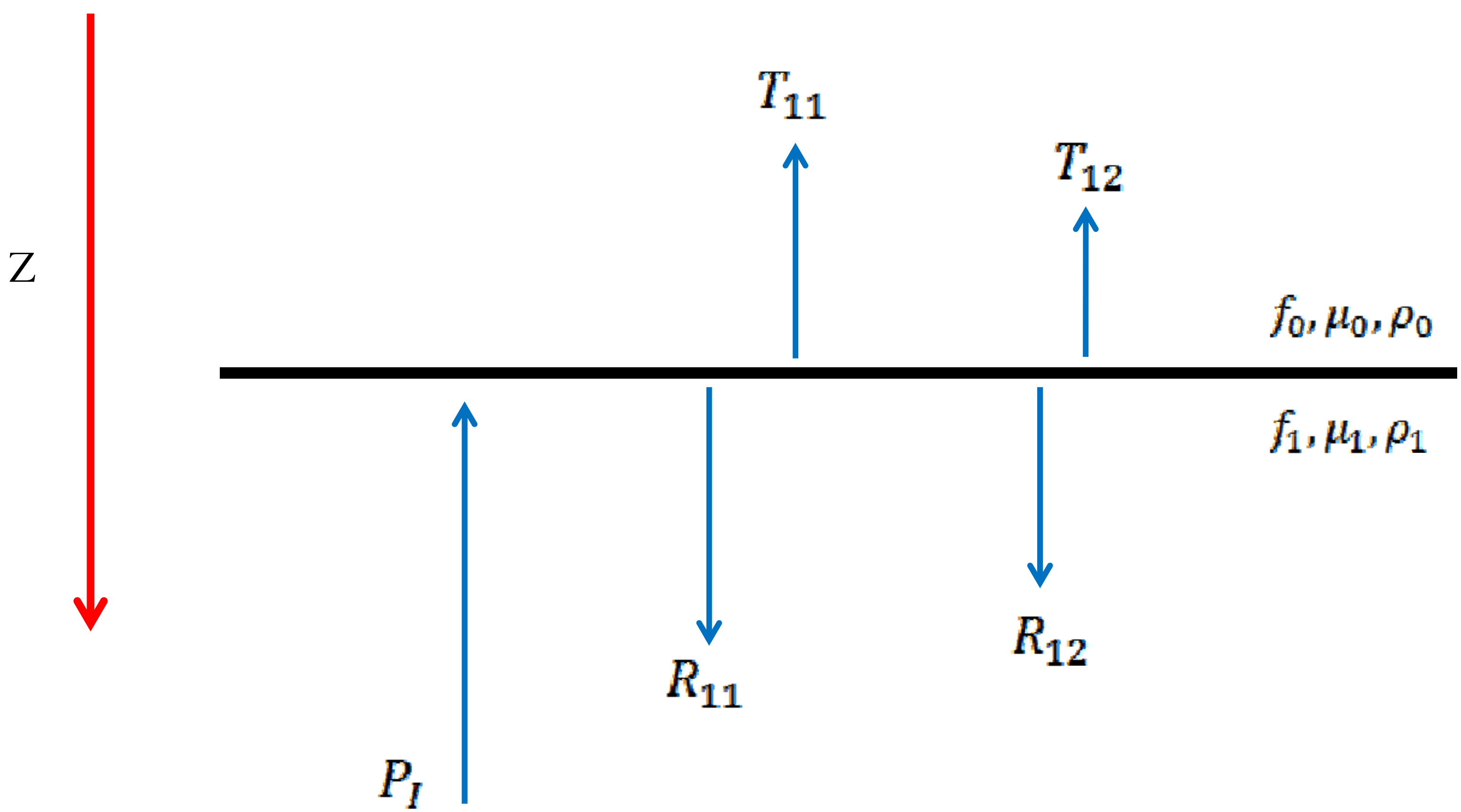
$$R_{PP}^{(2)} \approx W_{\Delta 1}\frac{\Delta f}{f} + W_{\Delta 2}\frac{\Delta \mu}{\mu} + W_{\Delta 3}\frac{\Delta \rho}{\rho} + W_{\Delta 4}\left(\frac{\Delta f}{f}\right)^2 + W_{\Delta 5}\left(\frac{\Delta \mu}{\mu}\right)^2 + W_{\Delta 6}\left(\frac{\Delta \rho}{\rho}\right)^2 + W_{\Delta 7}\frac{\Delta f}{f}\frac{\Delta \mu}{\mu} + W_{\Delta 8}\frac{\Delta f}{f}\frac{\Delta \rho}{\rho} + W_{\Delta 9}\frac{\Delta \mu}{\mu}\frac{\Delta \rho}{\rho}$$

$$R_{PP}^{(3)} \approx W_{\Delta 1}\frac{\Delta f}{f} + W_{\Delta 2}\frac{\Delta \mu}{\mu} + W_{\Delta 3}\frac{\Delta \rho}{\rho} + W_{\Delta 4}\left(\frac{\Delta f}{f}\right)^2 + W_{\Delta 5}\left(\frac{\Delta \mu}{\mu}\right)^2 + W_{\Delta 6}\left(\frac{\Delta \rho}{\rho}\right)^2 + W_{\Delta 7}\frac{\Delta f}{f}\frac{\Delta \mu}{\mu} + W_{\Delta 8}\frac{\Delta f}{f}\frac{\Delta \rho}{\rho} + W_{\Delta 9}\frac{\Delta \mu}{\mu}\frac{\Delta \rho}{\rho} + W_{\Delta 10}\left(\frac{\Delta f}{f}\right)^3 + W_{\Delta 11}\left(\frac{\Delta \mu}{\mu}\right)^3 + W_{\Delta 12}\left(\frac{\Delta \rho}{\rho}\right)^3$$
$$+ W_{\Delta 13}\left(\frac{\Delta f}{f}\right)^2\frac{\Delta \mu}{\mu} + W_{\Delta 14}\left(\frac{\Delta f}{f}\right)^2\frac{\Delta \rho}{\rho} + W_{\Delta 15}\left(\frac{\Delta \mu}{\mu}\right)^2\frac{\Delta f}{f} + W_{\Delta 16}\left(\frac{\Delta \mu}{\mu}\right)^2\frac{\Delta \rho}{\rho} + W_{\Delta 17}\left(\frac{\Delta \rho}{\rho}\right)^2\frac{\Delta f}{f} + W_{\Delta 18}\left(\frac{\Delta \rho}{\rho}\right)^2\frac{\Delta \mu}{\mu} + W_{\Delta 19}\frac{\Delta f}{f}\frac{\Delta \mu}{\mu}\frac{\Delta \rho}{\rho}$$

Numerical analysis



A different perspective



A different perspective

- Gurevich et al. (2002) studied fluid-saturated porous media for normal incidence reflection and transmission coefficients
  - Frequency dependent
  - Show relative amplitude displacement for fast and slow P-waves
  - Bound to Biot's critical frequency (experiments below 0.1Mhz)
  - For effective elastic conditions
- Objectives
  - Express in terms similar to Russell
  - Differences/Similarities (Russell and Gurevich)
  - Expand and analyze

A different perspective

$$R_{11}(\omega) = \frac{\rho_1 v_1 - (1 - X)\rho_0 v_0}{\rho_1 v_1 + (1 + X)\rho_0 v_0}$$

A different perspective

$$R_{11}(\omega)=\frac{\rho_1 v_1-\left(1-\dfrac{\left(K_{1dry}+\frac{4}{3}\mu+f_1\right)(k_1)_{fast}\left(\dfrac{\alpha_0 M_0}{K_{0dry}+\frac{4}{3}\mu_0+f_0}-\dfrac{\alpha_1 M_1}{K_{1dry}+\frac{4}{3}\mu_1+f_1}\right)^2}{\dfrac{N_0}{\sqrt{N_0}}\sqrt{\dfrac{i\omega\eta_0}{\kappa_0}}+\dfrac{N_1}{\sqrt{N_1}}\sqrt{\dfrac{i\omega\eta_1}{\kappa_1}}}\right)\rho_0 v_0}{\rho_1 v_1+\left(1+\dfrac{\left(K_{1dry}+\frac{4}{3}\mu+f_1\right)(k_1)_{fast}\left(\dfrac{\alpha_0 M_0}{K_{0dry}+\frac{4}{3}\mu_0+f_0}-\dfrac{\alpha_1 M_1}{K_{1dry}+\frac{4}{3}\mu_1+f_1}\right)^2}{\dfrac{N_0}{\sqrt{N_0}}\sqrt{\dfrac{i\omega\eta_0}{\kappa_0}}+\dfrac{N_1}{\sqrt{N_1}}\sqrt{\dfrac{i\omega\eta_1}{\kappa_1}}}\right)\rho_0 v_0}$$

$$\frac{N}{\sqrt{N}}=\frac{M\left(1-\dfrac{f}{K_{dry}+\frac{4}{3}\mu+f}\right)}{\sqrt{M}\sqrt{1-\dfrac{f}{K_{dry}+\frac{4}{3}\mu+f}}}$$

Ongoing research

- Linear and non-linear inversion
- Field cases
- Incorporated into SYNGRAM as synthetic tool

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