Full Waveform Inversion of Crosswell Seismic Data Using Automatic Differentiation

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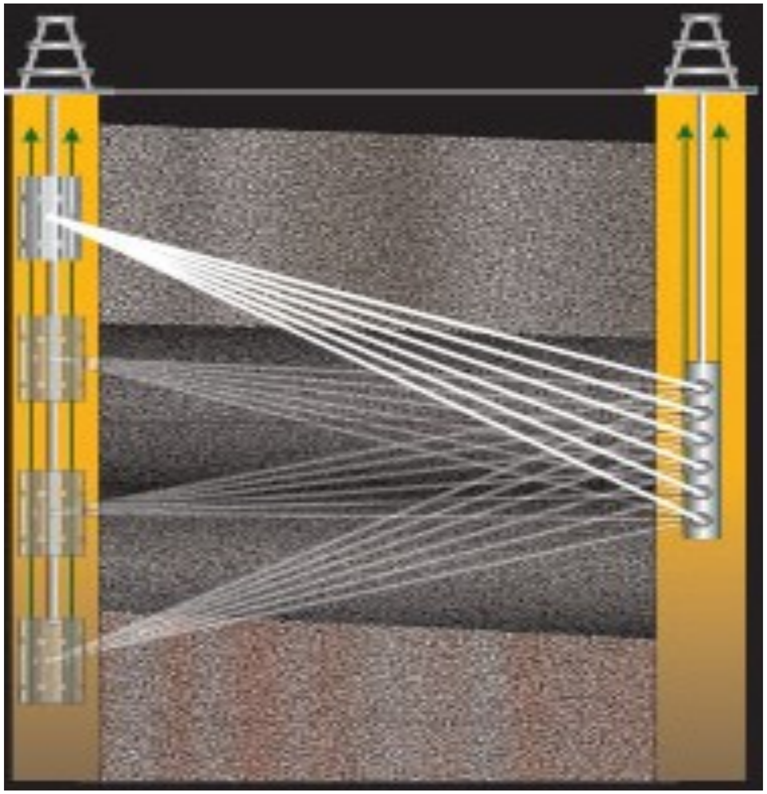
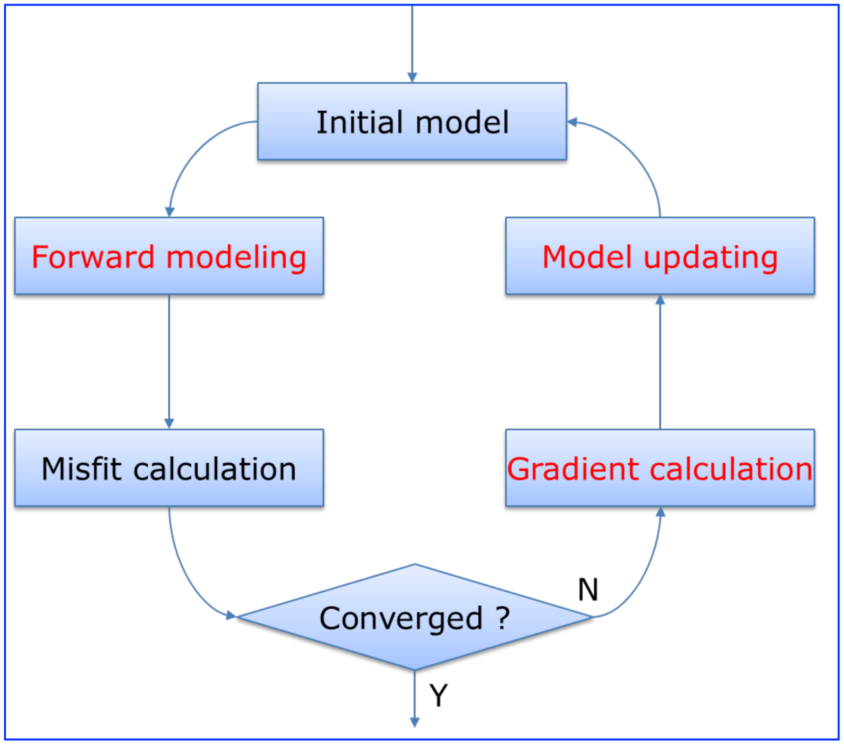
Outline

1. Introduction
2. Adjoint State Method
3. Automatic Differentiation(AD)
4. FWI using AD
5. Model Test
6. Conclusions

Introduction





Fixed Receivers – varying sources

Workflow of FWI

Introduction

- Mathematical Formulation: PDE-constrained Optimization

$$\mathcal{J}(m) = \frac{1}{2}\int_0^{t_f}\sum_{i=1}^{N_r}(d^i_{obs} - d^i_{cal}(m))^2\,dt + \kappa\|m\|$$

where

m : Model parameter(wave velocity)

d^i_{obs} : Observational data

$d^i_{cal}(m)$: Synthetic seismogram based on m through the wave eq.

$\kappa\|m\|$: Regularity term (Optional, depending on prior knowledge)

The inverse problem is solved through

$$\min_{m\in\mathbb{H}(\Omega)} \mathcal{J}(m)$$

Introduction

- PDE-Constrained Optimization: Gradient Calculation

$$\frac{\partial \mathcal{J}}{\partial m} = -\int_0^{t_f} \sum_{i=1}^{N_r} \left((d^i_{obs} - d^i_{cal}) \cdot \frac{\partial d^i_{cal}}{\partial u} \cdot \frac{\partial u}{\partial m} \right) dt + \kappa \frac{\partial \|m\|}{\partial m}$$

Direct computation of $\frac{\partial u}{\partial m}$ is difficult and expensive!

Adjoint-state method is an effective way to resolve this issue

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Adjoint State Method

- Define the cost functional as

$$\mathcal{J}(m) = \mathcal{J}(u(m), m)$$

which may depend on the model parameter implicitly if no regularity term.

The governing PDE(acoustic wave equation in this case) is stated as

$$\mathbb{L}(u(m), m) = 0$$

Here $\mathbb{L}$ is an operator defining the initial-boundary value problem of the wave equation.

Adjoint State Method: Perturbation Theory

Introduce a perturbation to the parameter:

$$\delta m \Rightarrow \delta u \Rightarrow \delta \mathcal{J}$$

$$\mathbb{L}(u, m) = 0 \qquad + \qquad \mathbb{L}(u + \delta u, m + \delta m) = 0$$

$$\delta \mathcal{J} = \left(\frac{\partial \mathcal{J}(u, m)}{\partial m} - \left\langle \xi, \frac{\partial \mathbb{L}(u, m)}{\partial m} \right\rangle \right) \delta m$$

where the adjoint-state variable is defined as

$$\left[\left(\frac{\partial \mathbb{L}(u, m)}{\partial u} \right)^* \right] \xi = \left[\frac{\partial \mathcal{J}(u, m)}{\partial u} \right] \Rightarrow \xi = \left[\left(\frac{\partial \mathbb{L}(u, m)}{\partial u} \right)^* \right]^{-1} \left[\frac{\partial \mathcal{J}(u, m)}{\partial u} \right]$$

Adjoint State Method: Lagrange Multipliers

Redefine a new cost functional as

$$\tilde{\mathcal{J}}(u, m, \xi) = \mathcal{J}(u, m) - \langle \xi, \mathbb{L}(u, m) \rangle$$

Solving the unconstrained optimization problem we obtain the gradient as

$$\frac{\partial \tilde{\mathcal{J}}(u, m, \xi)}{\partial m} = \frac{\partial \mathcal{J}(u, m)}{\partial m} - \left\langle \xi, \frac{\partial \mathbb{L}(u, m)}{\partial m} \right\rangle$$

where

$$\frac{\partial \tilde{\mathcal{J}}(u, m, \xi)}{\partial u} = 0 \Rightarrow \frac{\partial \mathcal{J}(u, m)}{\partial u} - \left(\frac{\partial \mathbb{L}(u, m)}{\partial u} \right)^* \xi = 0 \Rightarrow \xi = \left[\left(\frac{\partial \mathbb{L}(u, m)}{\partial u} \right)^* \right]^{-1} \frac{\partial \mathcal{J}(u, m)}{\partial u}$$

Adjoint –> Discretization or Discretization -> Adjoint

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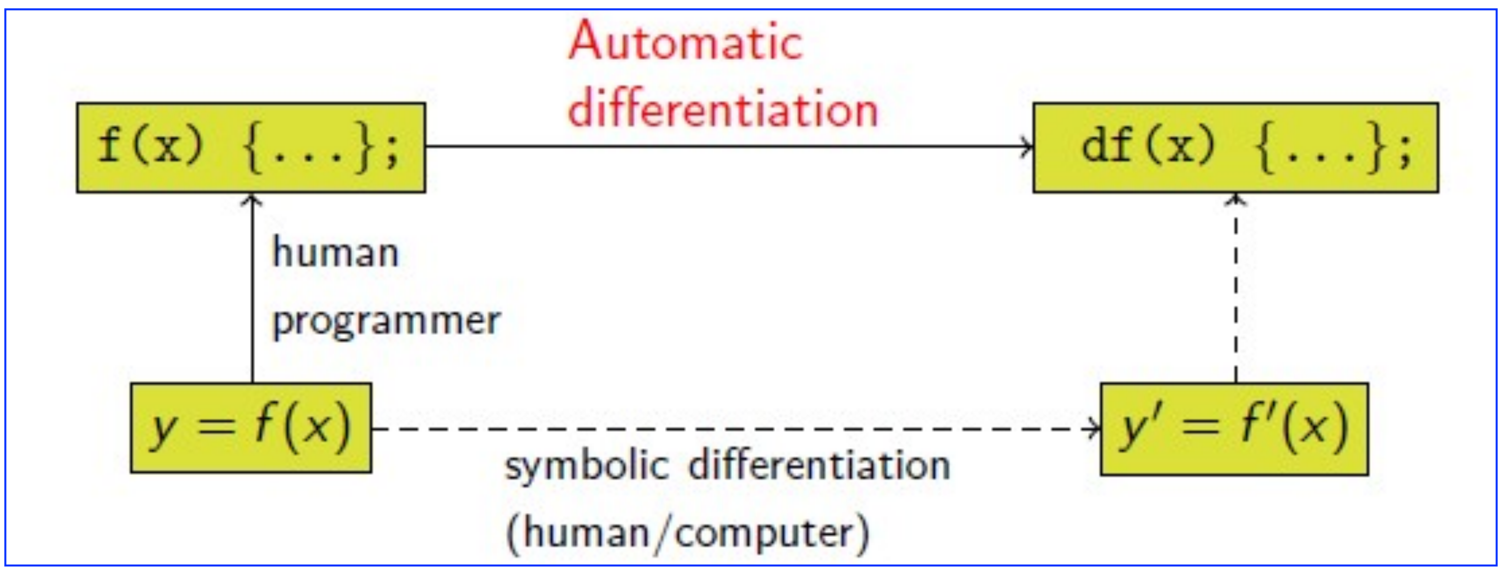
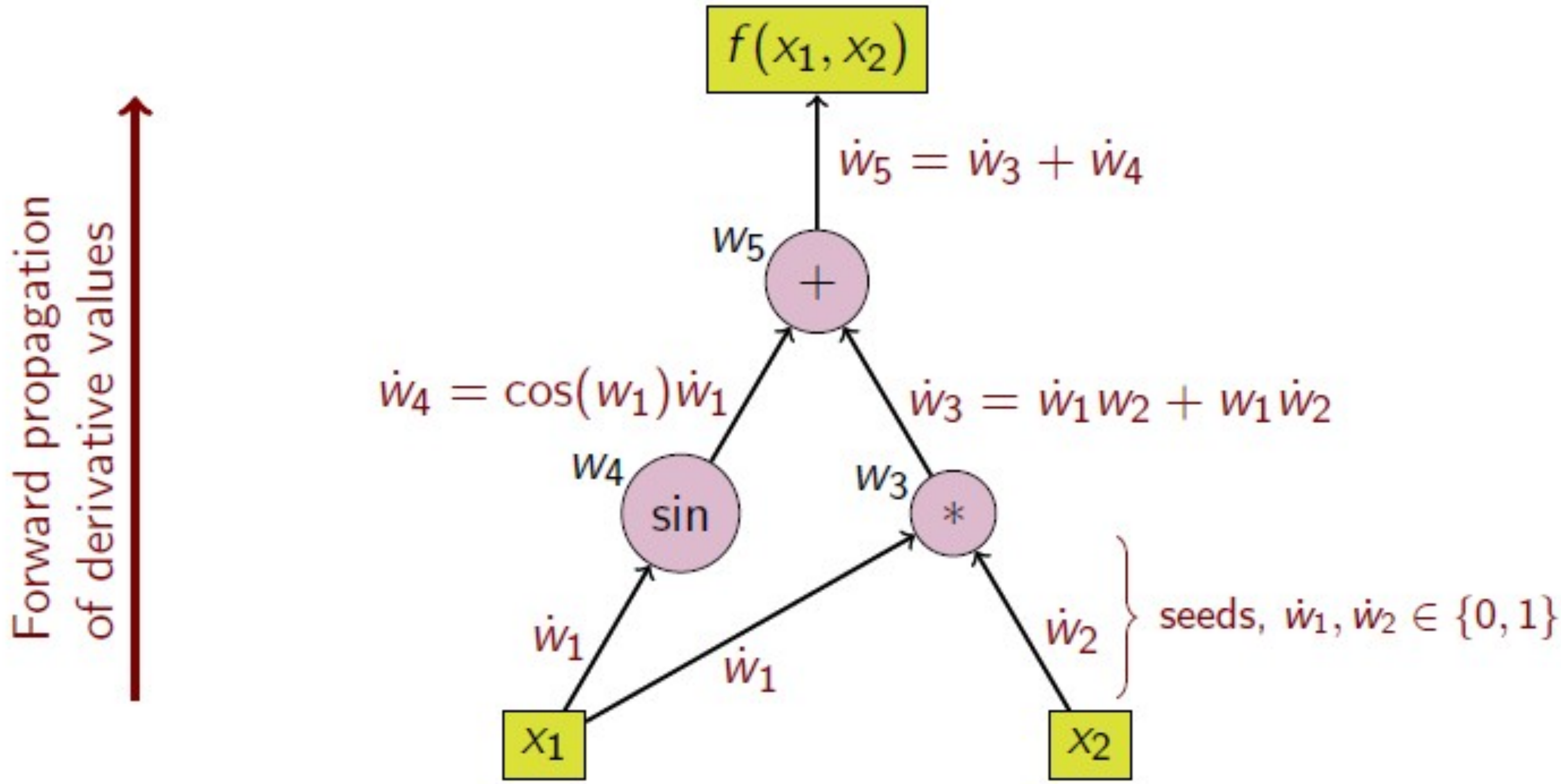
Automatic differentiation

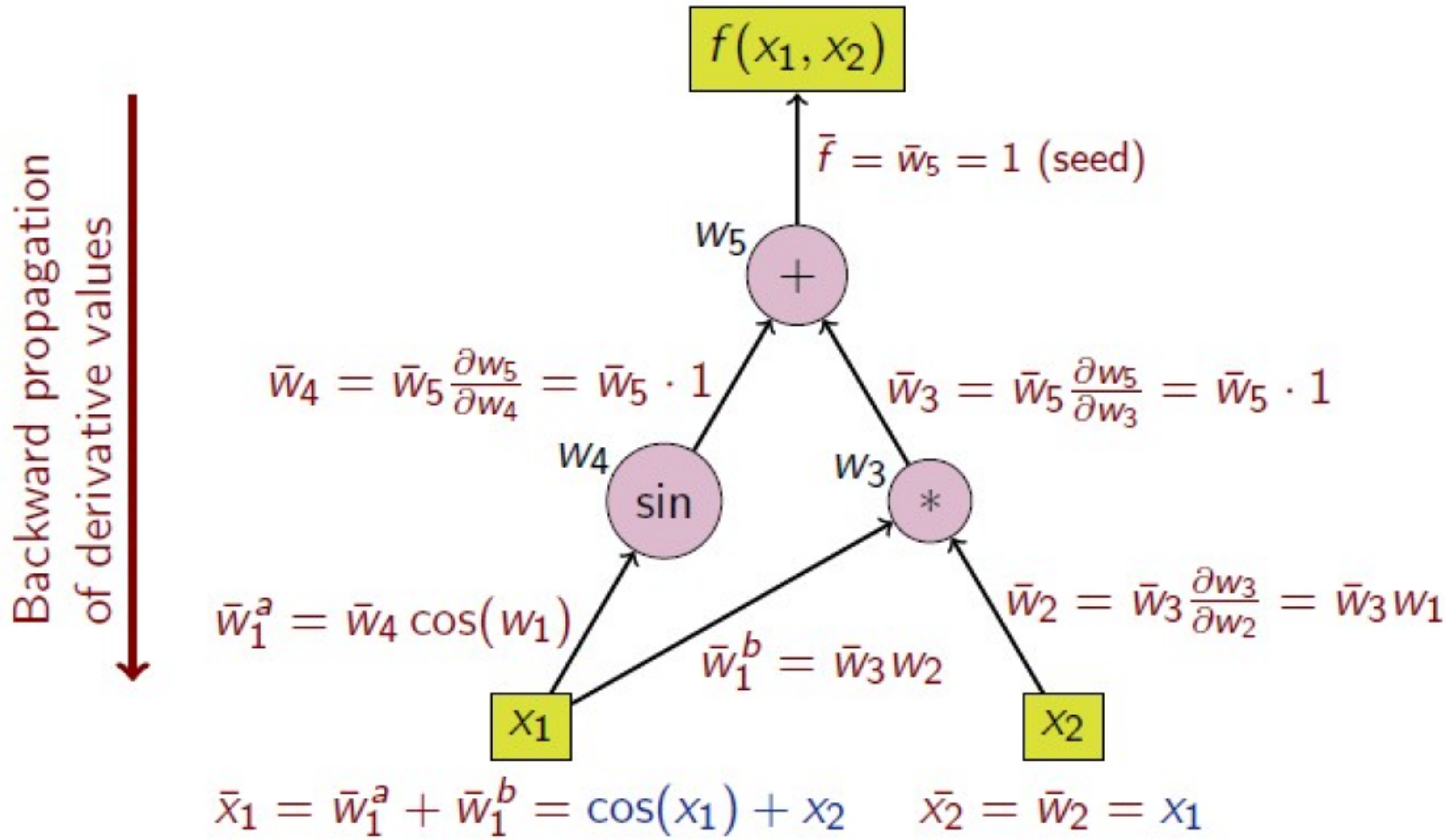
- Automatic Differentiation (AD), sometimes alternatively called algorithmic differentiation, is a set of techniques to numerically evaluate the derivative of a function specified by a computer program.



Forward mode AD



Reverse mode AD



AD tools

- AD Model Builder (C/C++)
- ADC (C/C++)
- ADF (Fortran77,Fortran95)
- ADIC (C/C++)
- ADIFOR (Fortran77)
- ADiMat (MATLAB)
- ADMAT / ADMIT (MATLAB)
- ADOL-C (C/C++)
- ADOL-F (Fortran95)
- APMonitor (Interpreted)
- AUTODIF (C/C++)
- AutoDiff .NET (.NET)
- AUTO_DERIV (Fortran77/95)
- ColPack (C/C++)
- COSY INFINITY (Fortran77/95,C/C++)
- CppAD (C/C++)
- CTaylor (C/C++)
- FAD (C/C++)
- FADBAD/TADIFF (C/C++)
- FFADLib (C/C++)
- GRESS (Fortran77)
- HSL_AD02 (Fortran95)
- INTLAB (MATLAB)
- NAGWare Fortran 95 (Fortran77,Fortran95)
- OpenAD (C/C++,Fortran77/95)
- PCOMP (Fortran77)
- pyadolc (python)
- pycppad (Interpreted,python)
- Rapsodia (C/C++,Fortran95)
- Sacado (C/C++)
- TAF (Fortran77,Fortran95)
- TAMC (Fortran77)
- TAPENADE (C/C++,Fortran77/95)
- TaylUR (Fortran95)
- The Taylor Center (independent)
- TOMLAB /MAD (MATLAB)
- TOMLAB /TomSym (MATLAB)
- Treeverse / Revolve (C/C++,Fortran77/95)
- YAO (C/C++)

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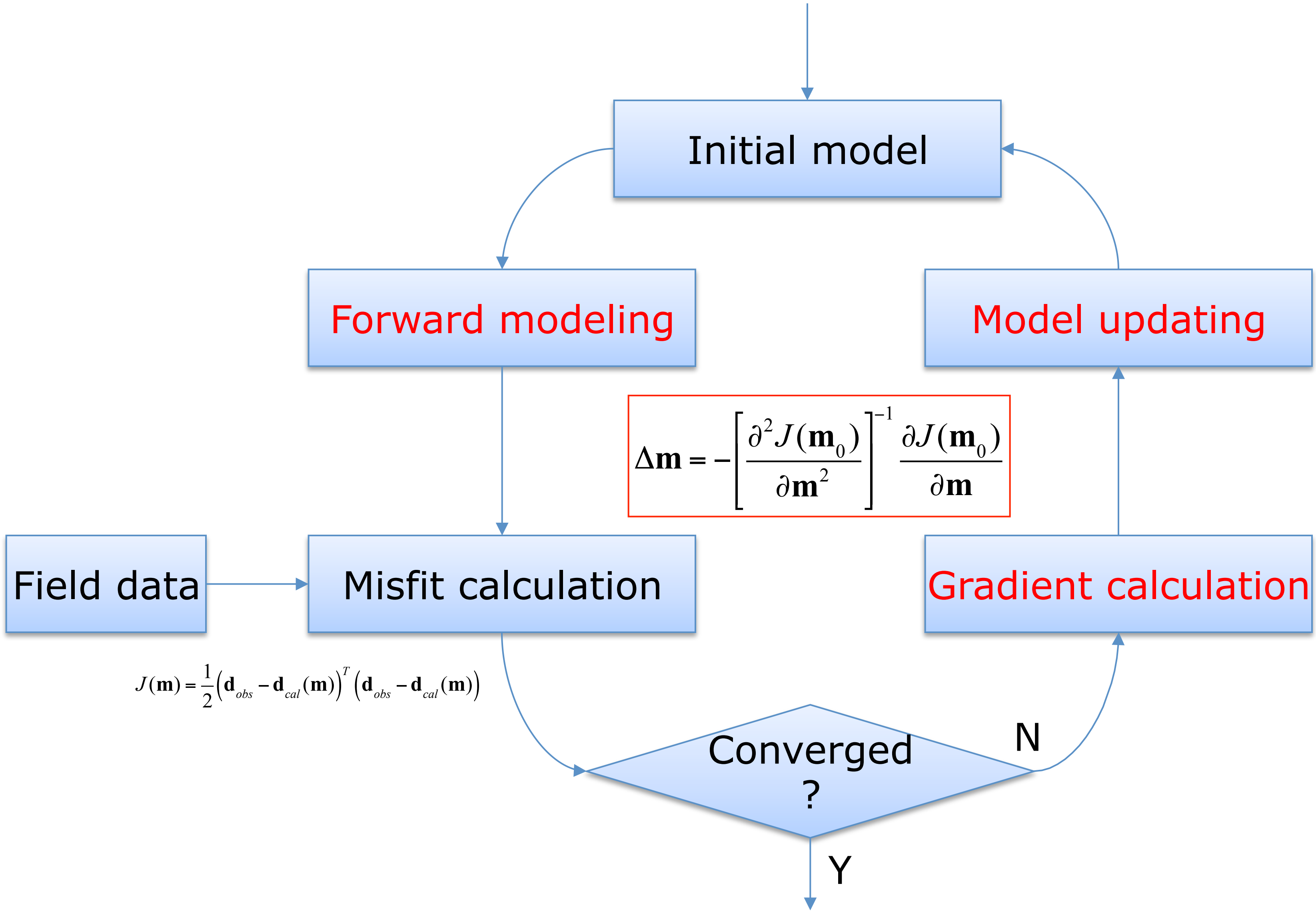
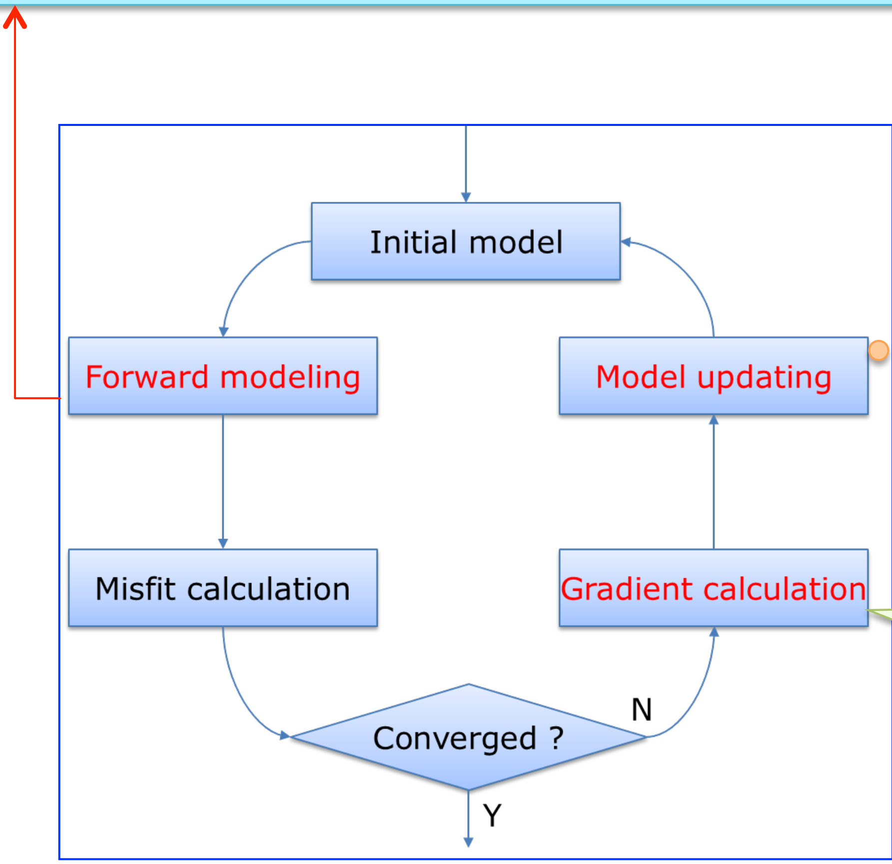
Workflow of FWI



FWI solution one by one

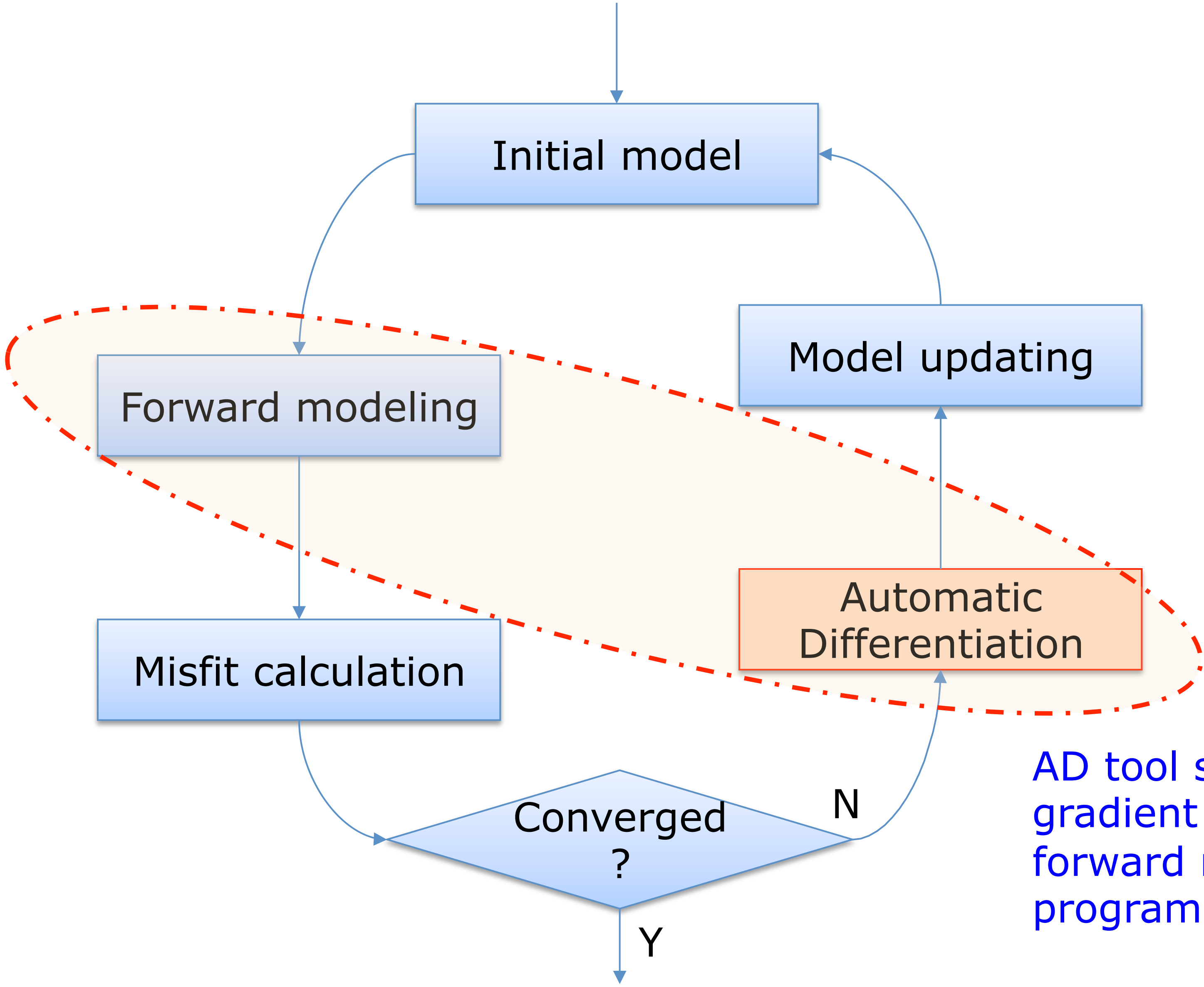
Forward Modeling:
- 2^{nd}-order in time and 4^{th}-order in space
- Stagger-grid finite difference
- PML absorbing boundary



Model Updating:
- **L-BFGS selected**
- **Limited memory required**
- **Quasi-Newton method**

Gradient Calculation:
- **Adjoint state method**
- **AD tools used**
- **TAPENADE test**

FWI workflow with AD

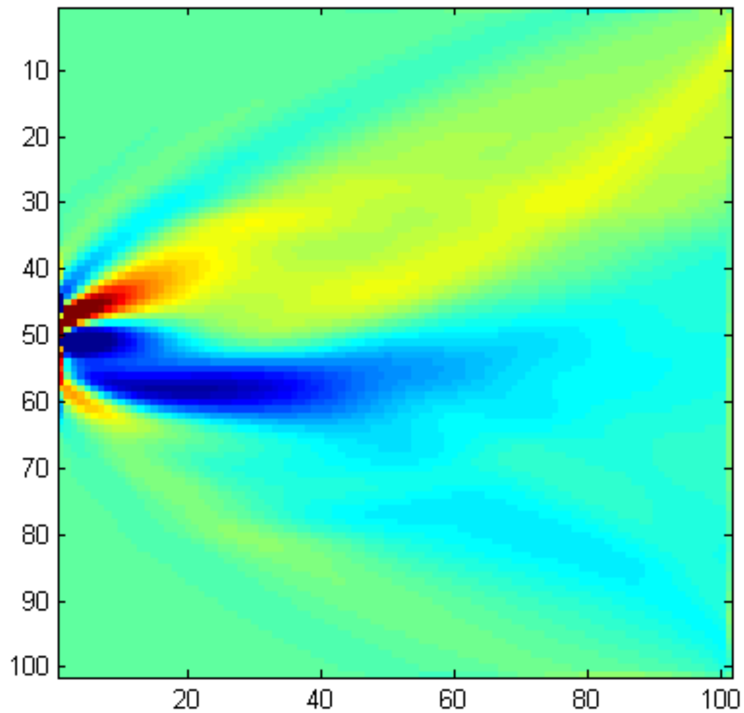


AD tool solve the gradient according to the forward modeling program

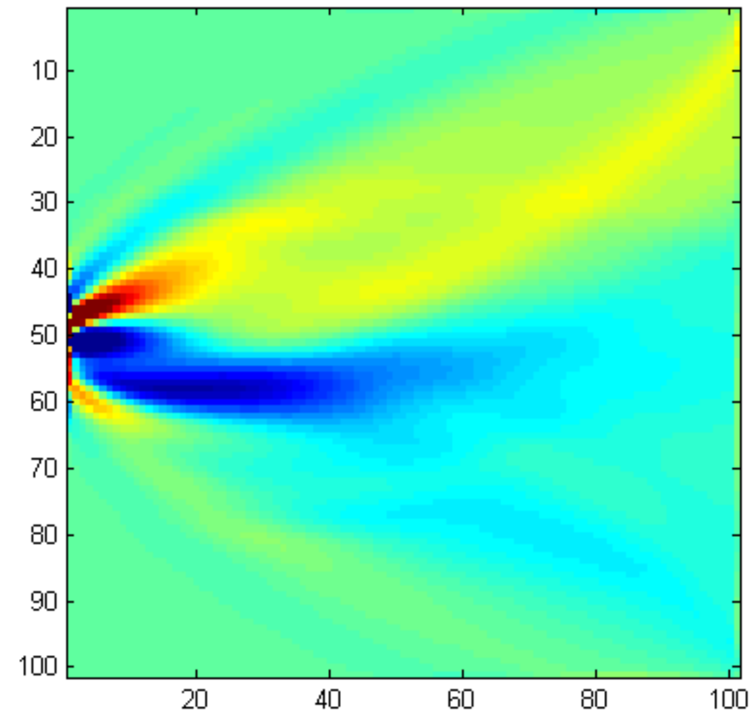
Benefit of FWI with AD

- Simplify the gradient calculation
- Focus on forward modeling and optimization method
- High efficiency forward modeling program will lead to high efficiency gradient calculation code
- FWI workflow is simplified

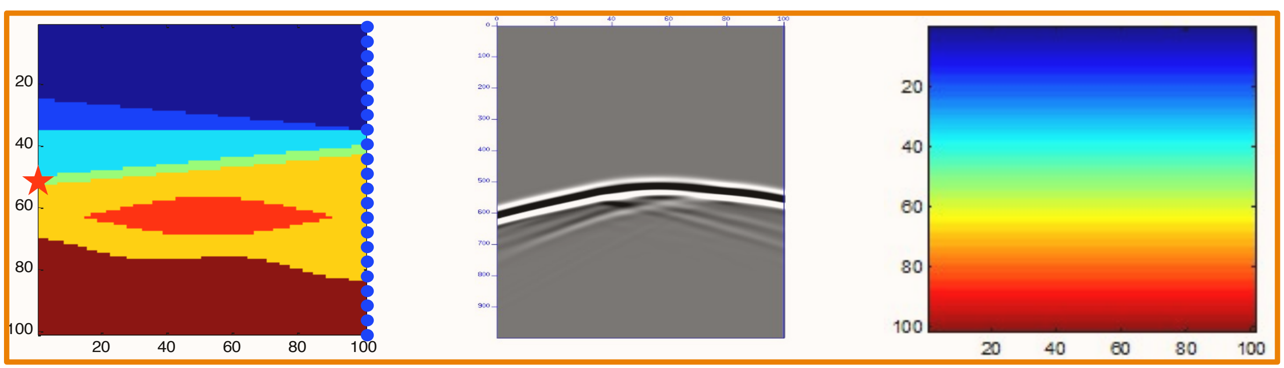
Accuracy of Gradient calculation



Gradient by TAPENADE



Gradient by central difference quotient



Gradient calculation:
-True model
-Synthetic record
-Initial model

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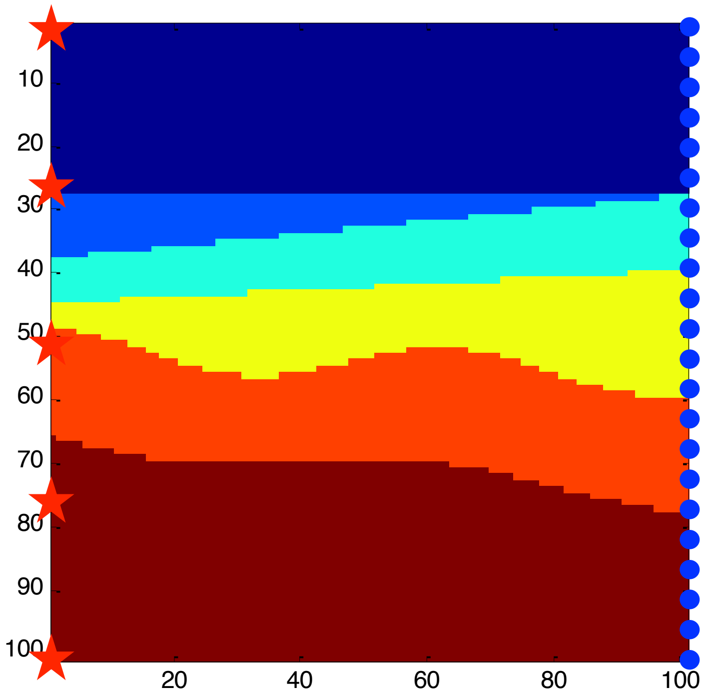
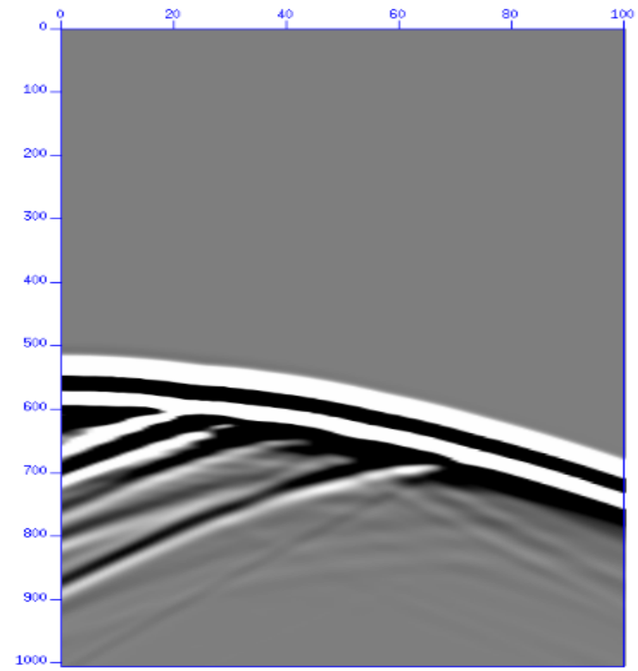
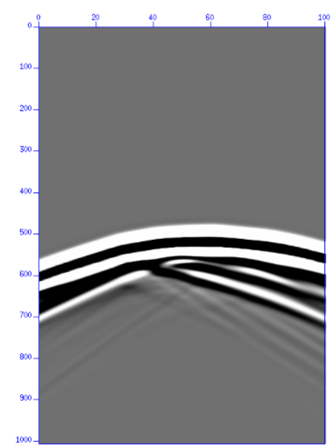
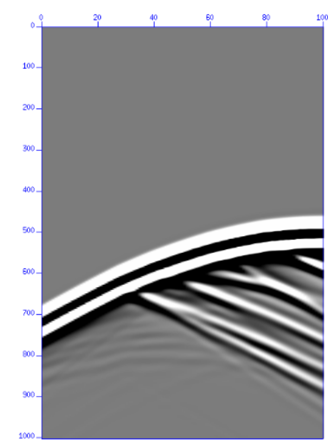
Model test 1



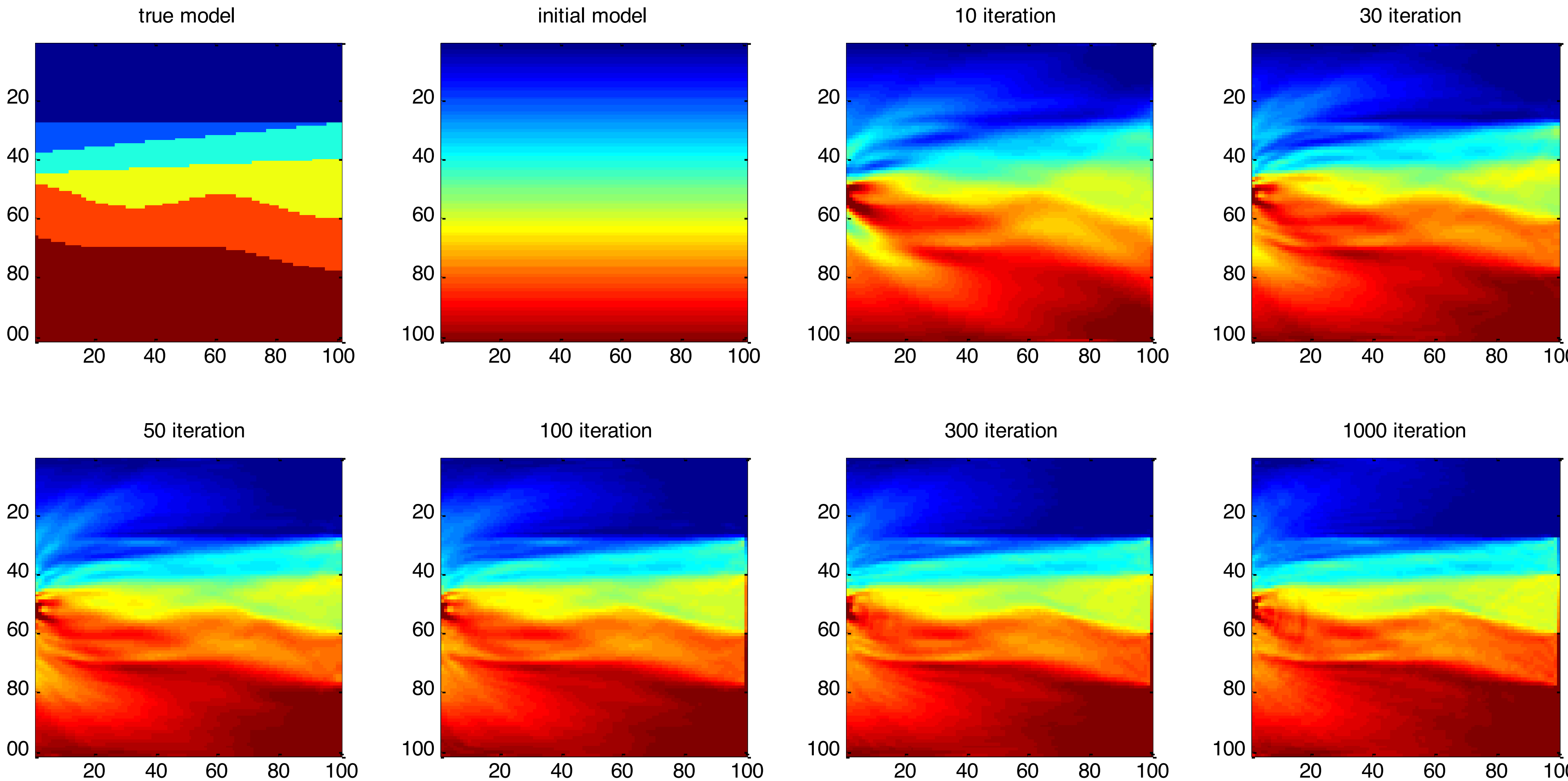


Model:101 X 101
Spatial sample:1m
Time sample:0.1ms
Source： ricker wavelet
Main frequency: 180 Hz
Boundary: PML

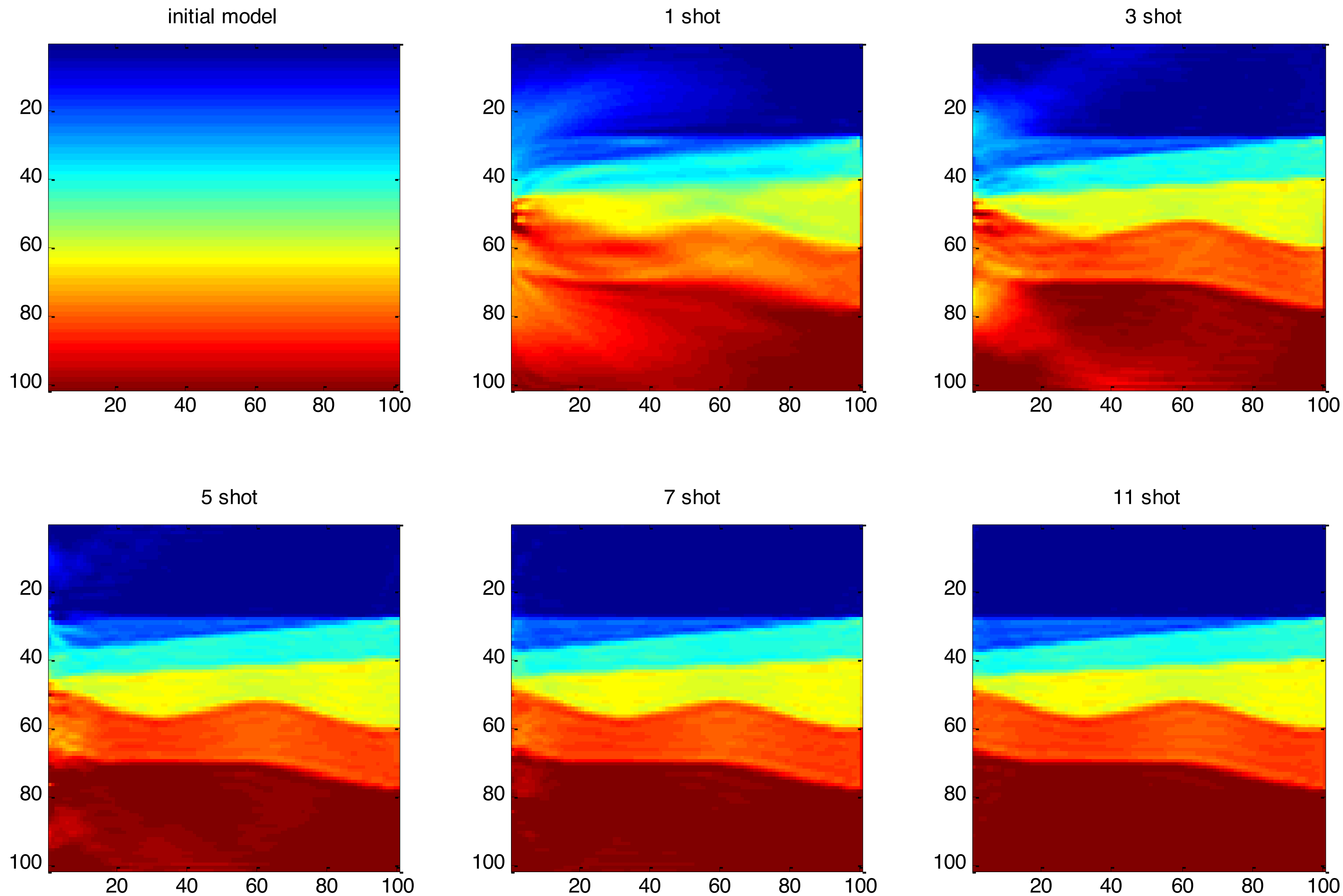




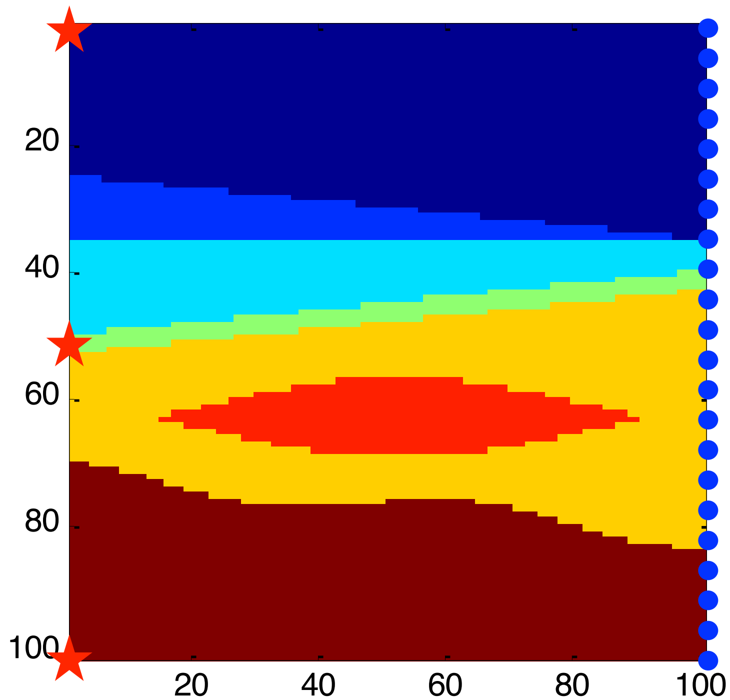
Inversion result - 1 shot

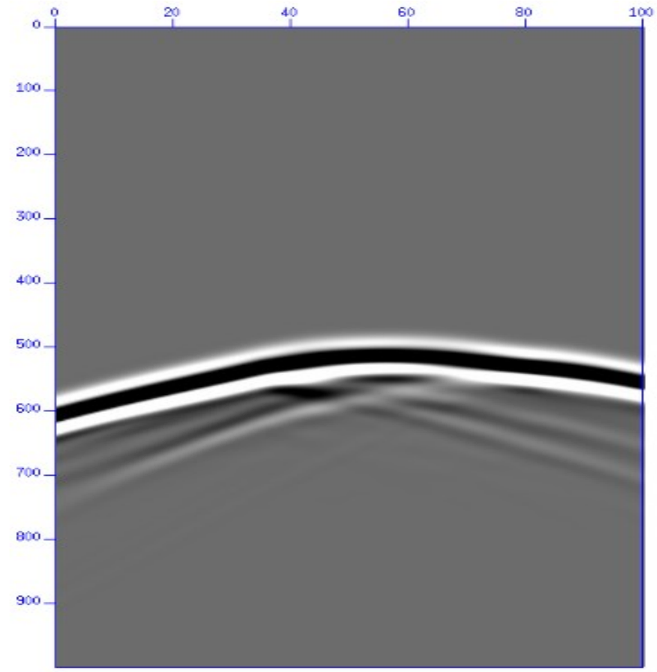


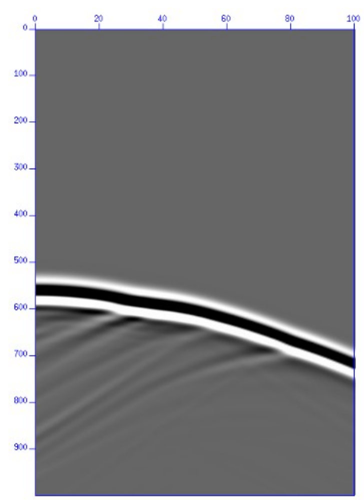
Inversion result - different shot

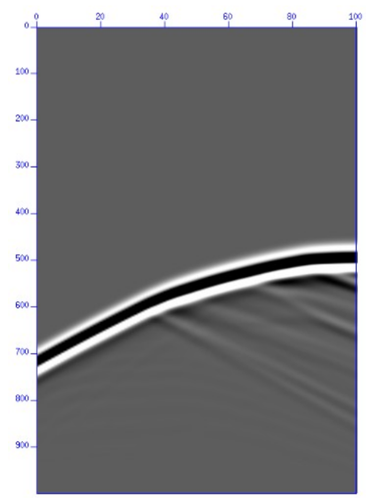


Model test 2



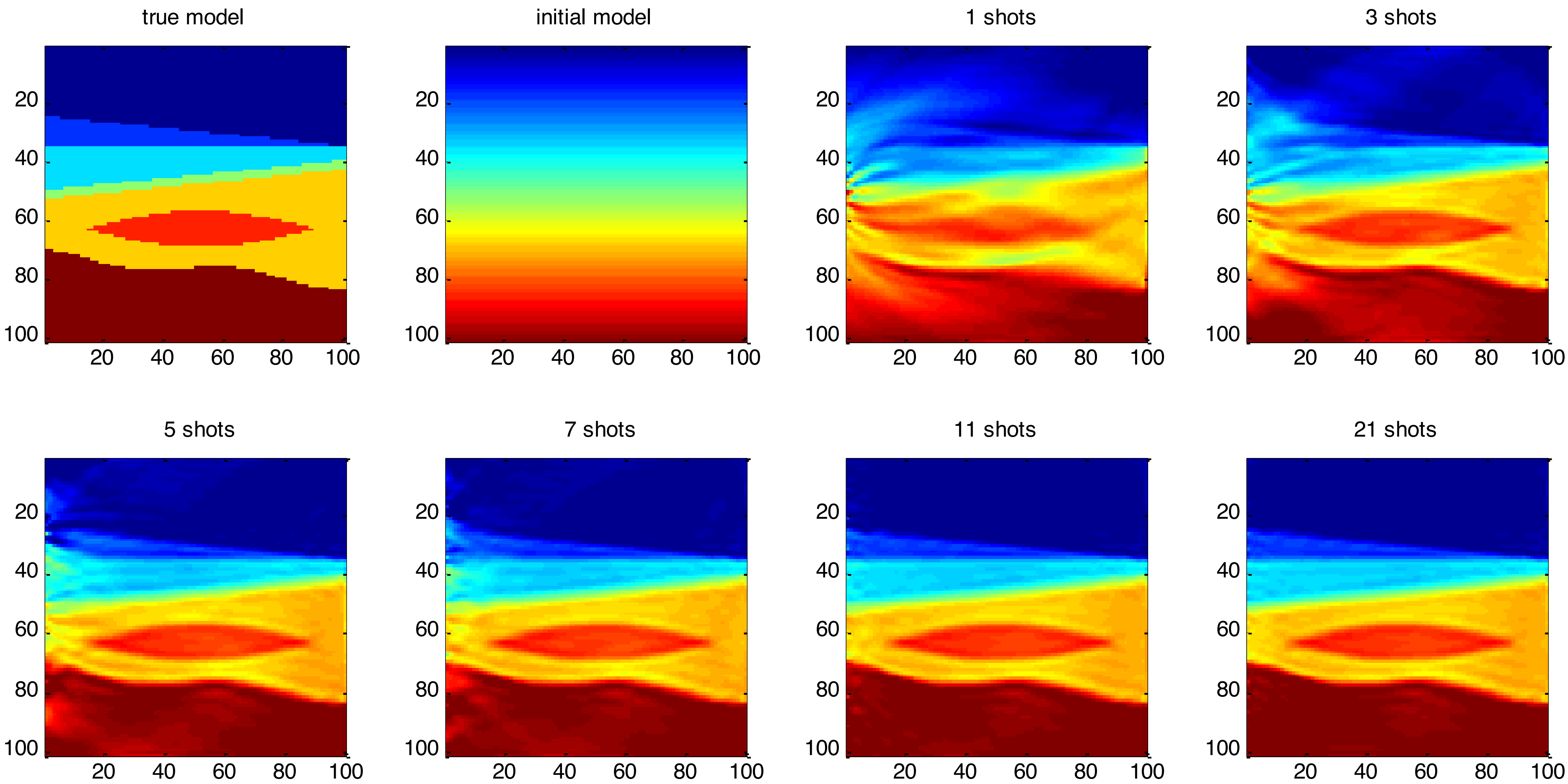




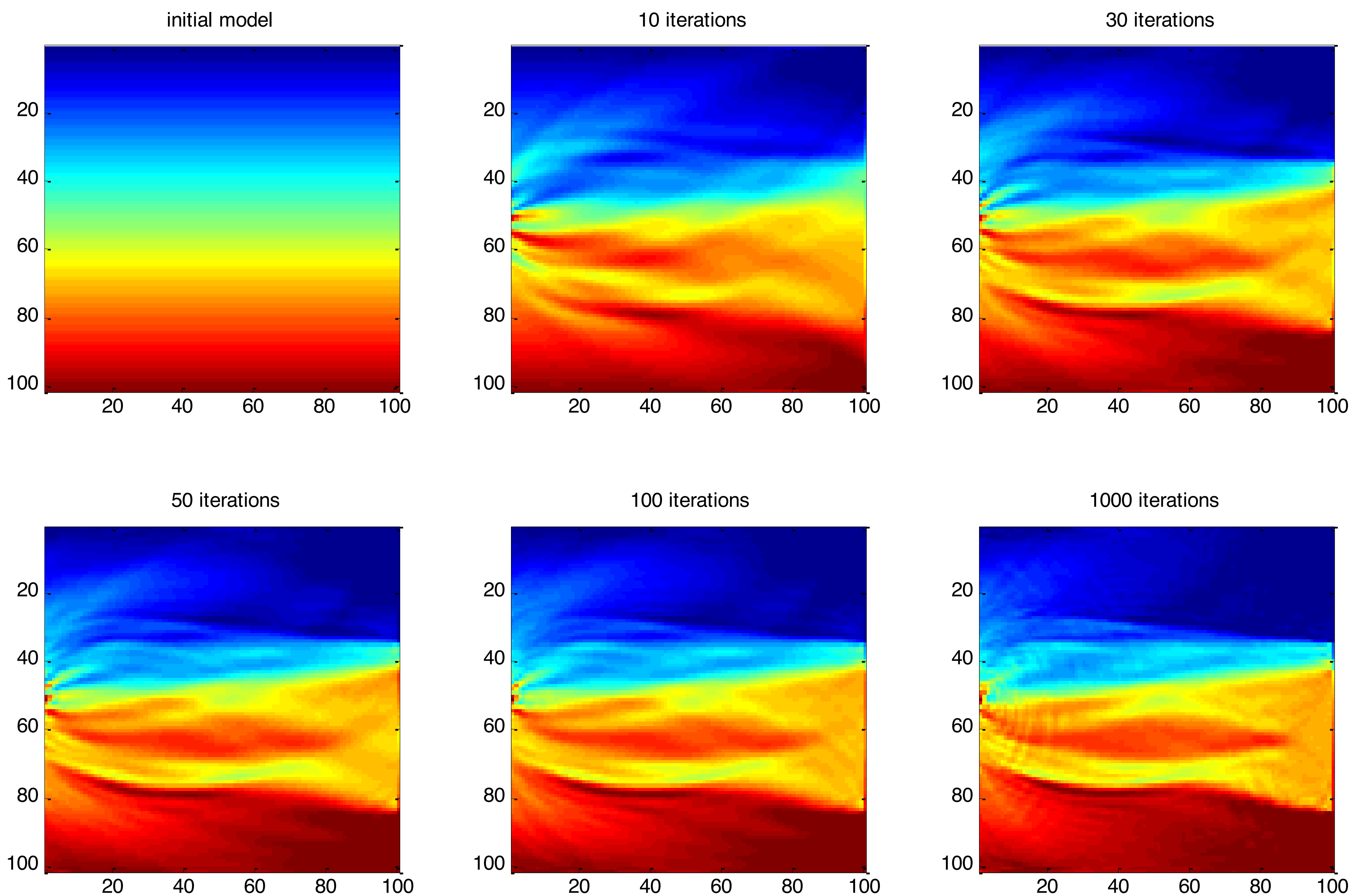


Model:101 X 101
Spatial sample:1m
Time sample:0.1ms
Source： ricker wavelet
Main frequency: 180 Hz
Boundary: PML

Inversion result - different shot



Inversion result - 1 shot



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Conclusion

- Automatic differentiation (AD) is a promising yet not popular approach in Geoscience.
- The gradient calculated through AD is accurate.
- The full waveform inversion workflow is simplified with the usage of the AD tool.
- Model tests show that the full waveform inversion method with AD is effective and efficient in the inversion of the crosswell seismic data.

Future work

- Improve the forward modeling: finite difference 4th-order in time
- Test the large-scale data inversion using checkpoint technology
- Test with other AD tools, and Optimization algorithms
- Test the surface seismic inversion
- Address inverse modeling issues under the current framework
- Test on other types of wave equations

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