Recovering Low Frequencies For Impedance Inversion By Frequency Domain Deconvolution

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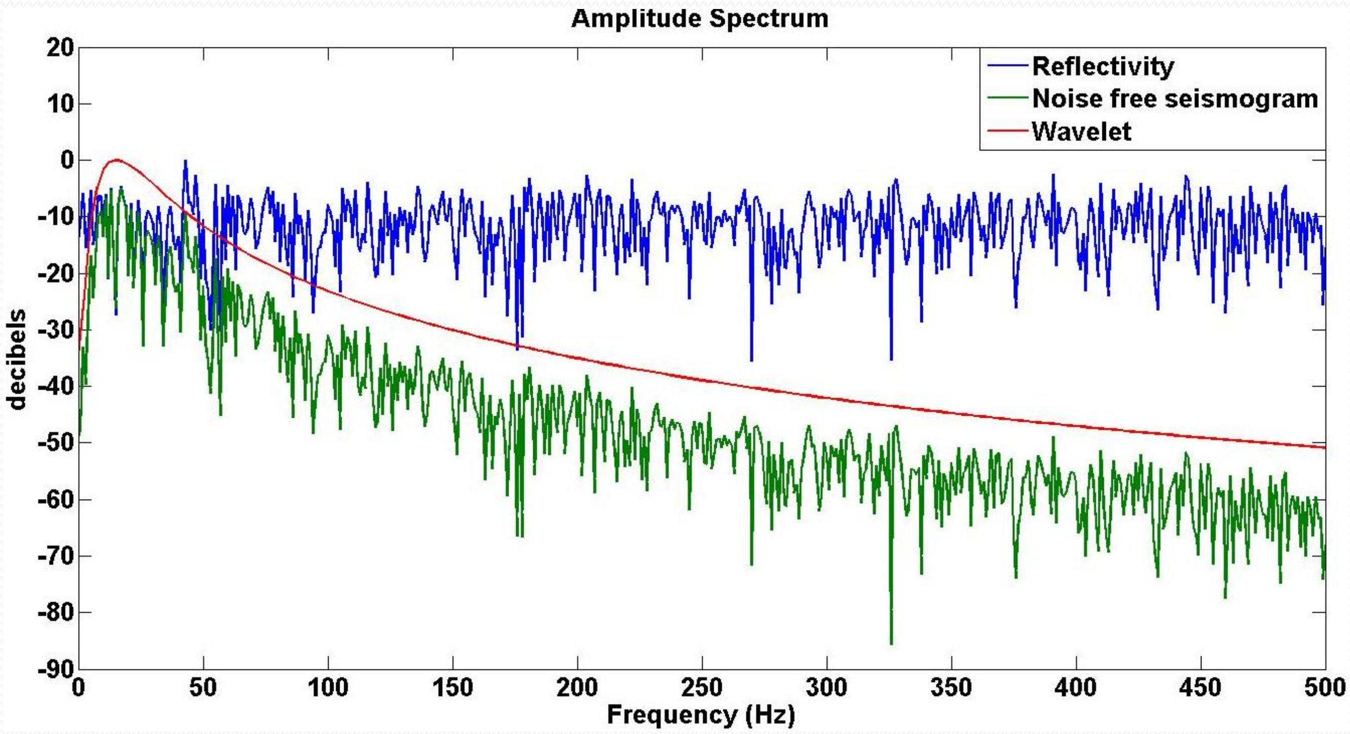
Outline

- Modeling of seismic trace
- Impedance inversion and estimation of reflectivity
- Frequency domain deconvolution
- Spectral white reflectivity estimation (random reflectivity)
- Spectral color reflectivity estimation (well log data)
- Impedance inversion results
- Conclusion

Modeling the Seismic trace

Convolutional Model: $s(t) = r(t) \bullet w(t)$

In frequency domain



Impedance Inversion

In homogenous medium for normal incident wavelet the reflection coefficient can be written as:

$$r_n = \frac{I_{n+1} - I_n}{I_{n+1} + I_n}$$

$I = v\rho$ Acoustic Impedance

I_n Acoustic Impedance of layer nth

I_{n+1} Acoustic Impedance of layer (n+1)th

$$I_{n+1} = I_n \left(\frac{1+r_n}{1-r_n} \right) = I_1 \prod_{i=1}^{n} \left(\frac{1+r_i}{1-r_i} \right) \approx I_1 \exp\left(2\sum_{i=1}^{n} r_i \right)$$

How to estimate the reflectivity?

$$S(t) = r(t) \bullet w(t)$$

We can define d(t) as:

"d" is the perfect deconvolution operator

$$w(t) \bullet d(t) = \delta(t) \Rightarrow d(t) = w^{-1}(t) \quad \text{(II)}$$

$$r(t) = s(t) \bullet d(t)$$

In practice, because of the **band-limited** nature of the wavelet, it is impossible to find d(t) which makes the right hand side of equation (II) equal to a delta function.

$$w(t) \bullet d(t) = w_d(t)$$

$$r(t) \neq s_d(t)$$

$$s_d(t) = r(t) \bullet w_d(t)$$

Deconvolution

- In mathematics, deconvolution is an algorithm-based process which is used to reverse the effects of convolution on the recorded data
- Deconvolution must estimate the wavelet from the data
- Amplitude spectrum of wavelet estimated by smoothing data amplitude spectrum.
- All spectral shape is attributed to the wavelet (white reflectivity assumption)

Frequency Domain Deconvolution

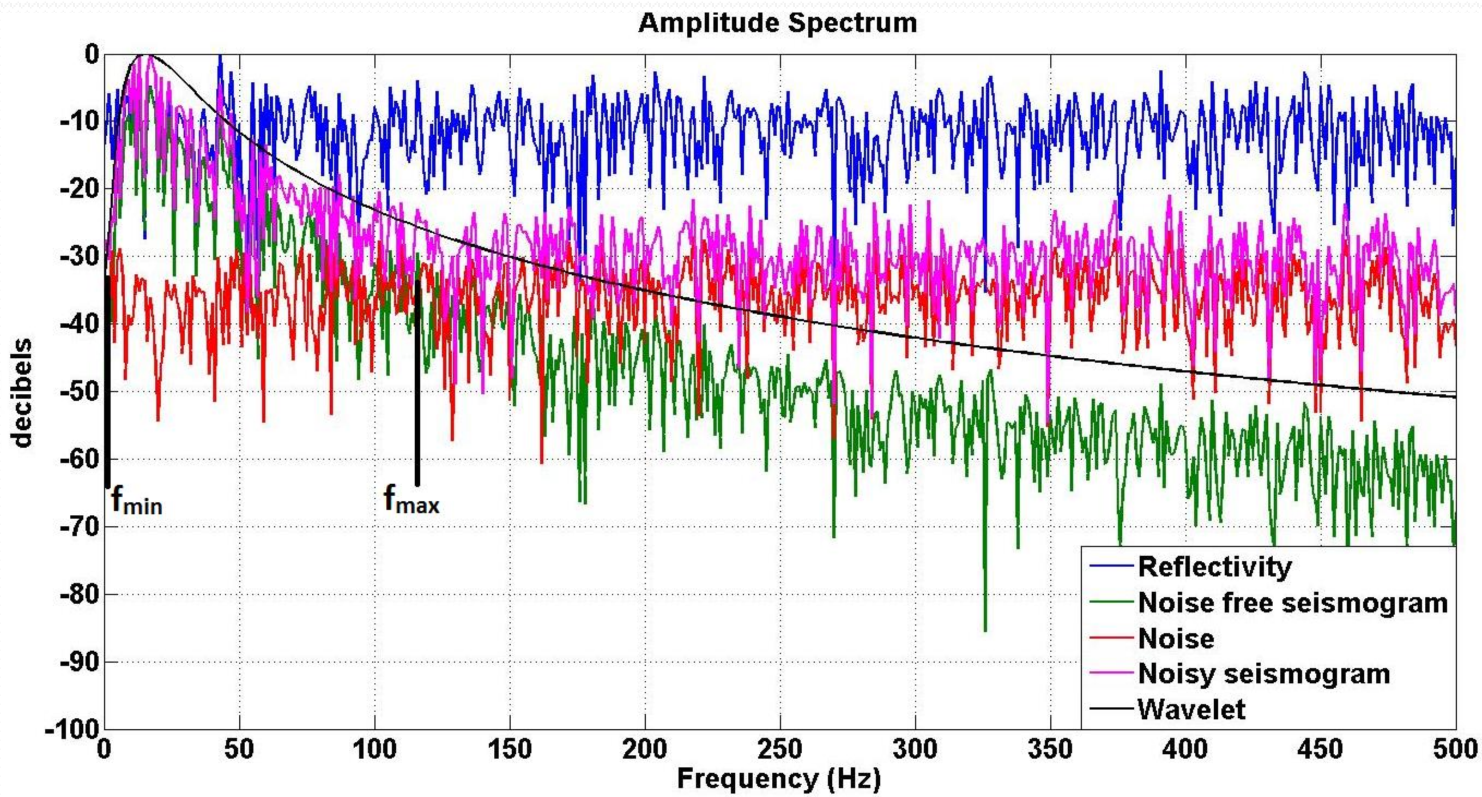
The assumptions:

- The wavelet should be minimum phase
- The wavelet spectrum should be smooth
- The wavelet should be stationary
- The reflectivity should be random so it has a white spectrum

Adding the noise to seismogram:

$$s(t) = r(t) \bullet w(t) + n(t)$$

$$S(f) = R(f)W(f) + N(f)$$



$$f_{\min} \le f \le f_{\max}$$

$$\underbrace{|S(f)|}_{\text{Noisy Seismogram}} \approx \underbrace{|R(f)||W(f)|}_{\text{Noise free Seismogram}}$$

The white reflectivity means: $\overline{|R(f)|} \approx 1$

$$\overline{|S(f)|} \approx |W(f)|$$

Frequency Domain Deconvolution

Then the estimated wavelet is defined as:

$$|w(f)|_{estimated} = \overline{|S(f)|}$$

By comparing with:

$$r(t) = s(t) \bullet w^{-1}(t)$$

$$|D(f)| = |W(f)|^{-1}_{estimated} = \overline{|S(f)|}^{-1}$$

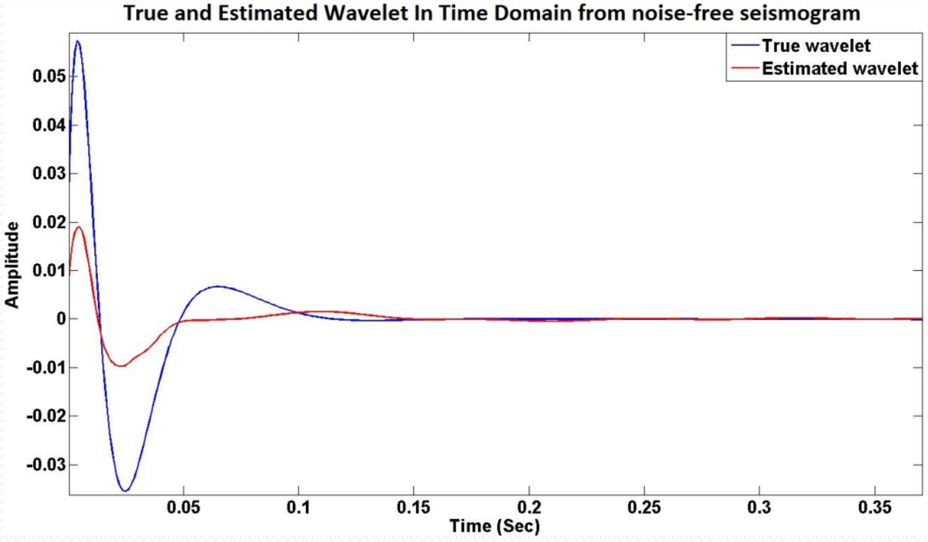
Deconvolution operator can be defined as (Margrave, 2002):

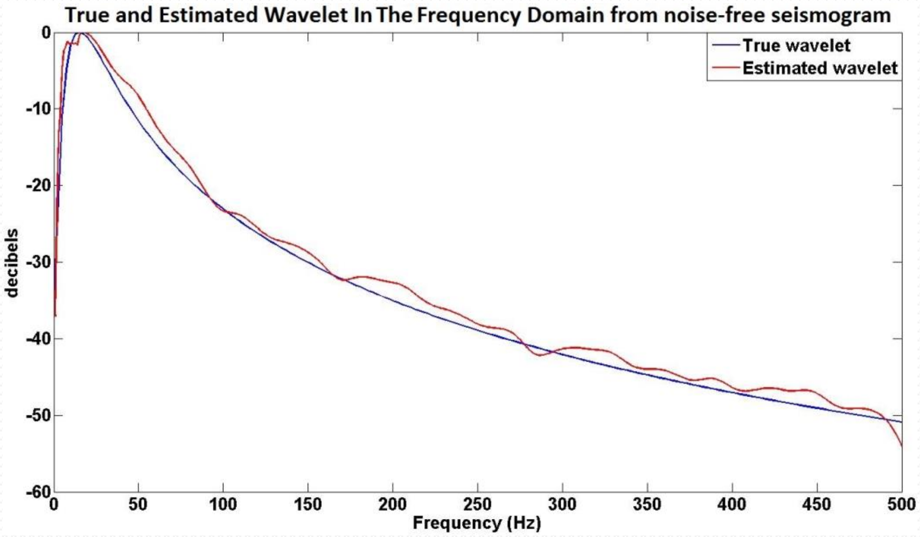
$$D(f) = \frac{1}{|W(f)|_{est} + \mu A_{\max}} e^{i\phi_D(f)} \quad \begin{cases} A_{\max} = \max\left(|W(f)|_{est}\right) \\ \phi_D(f) = H\left(\ln\left(|D(f)|\right)\right) \\ 0.01 < \mu < 0.000001 \end{cases}$$

Estimated wavelet

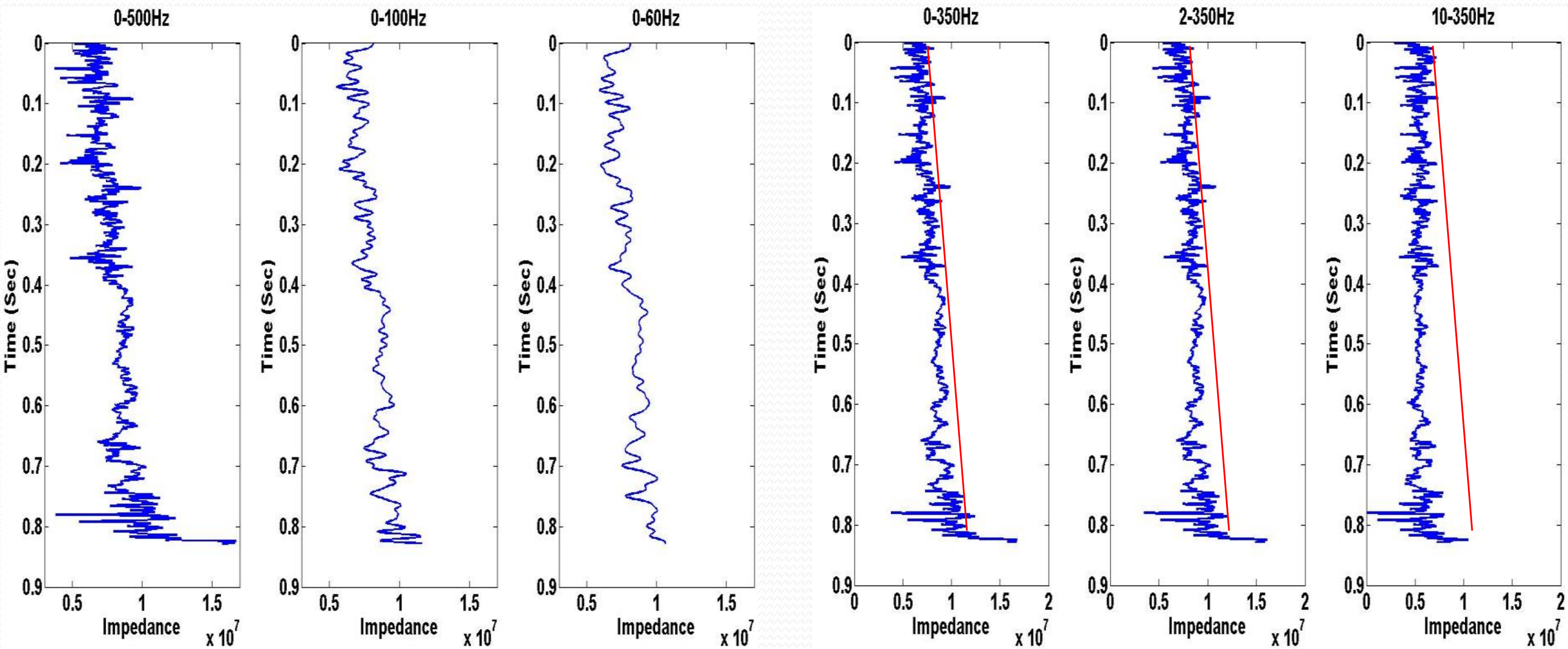
$$S(f)D(f) = R(f)W(f)D(f) = R(f)W_D(f)$$

$$W_D(f) = W(f)D(f) = \frac{W(f)}{\left|W(f)\right|_{est} + \mu A_{\max}} e^{i\phi_D(f)}$$





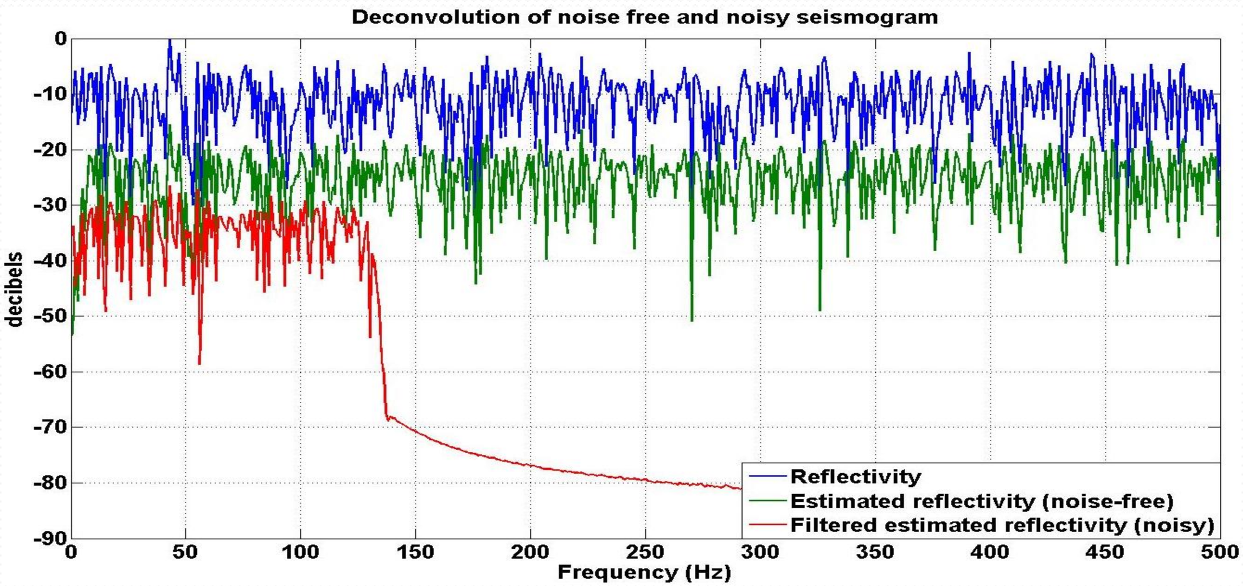
Missing High and Low Frequencies

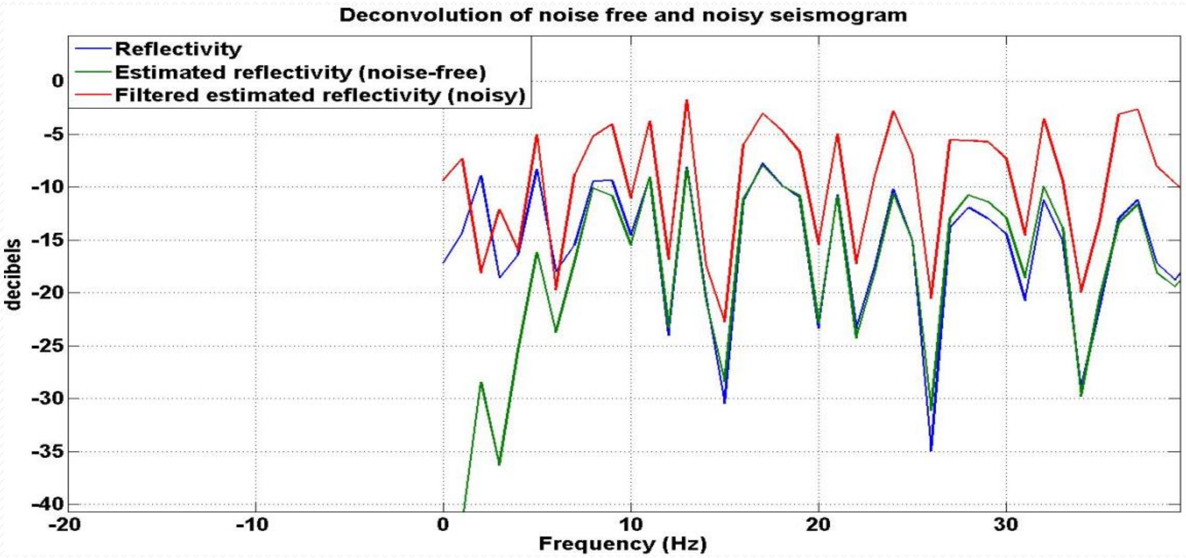


Missing high frequency causes reduced resolution

Missing low frequency causes reduced trend of data

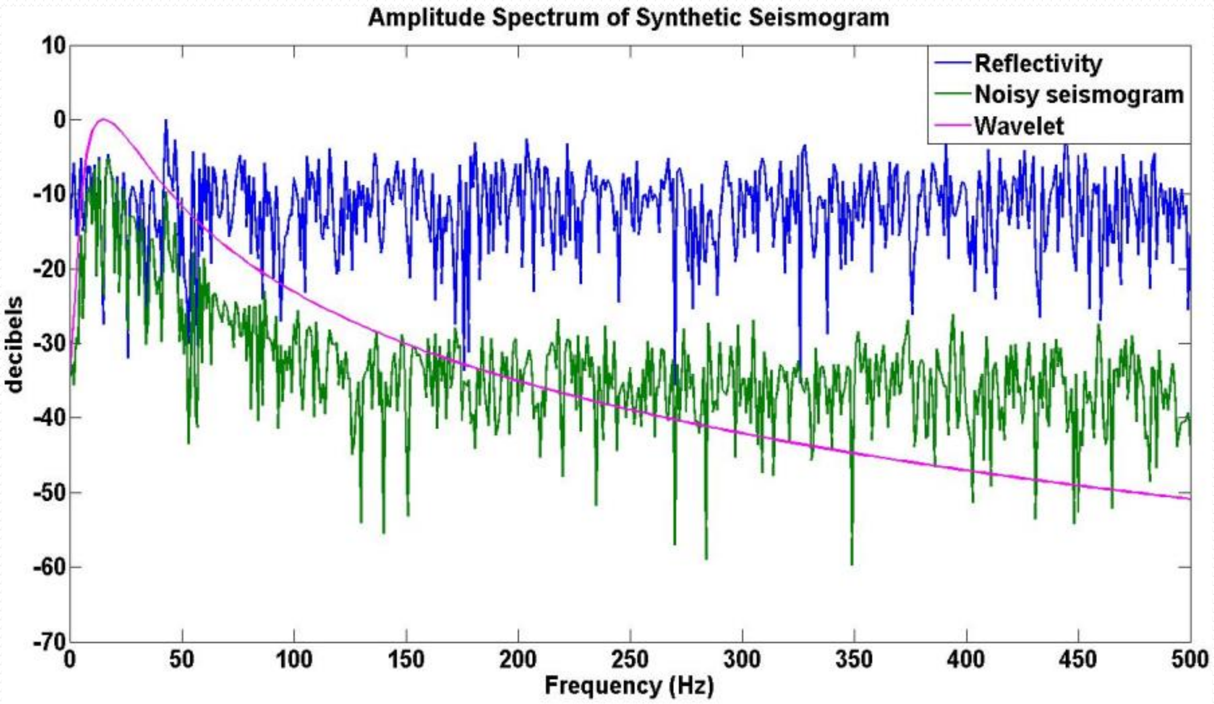
Frequency Domain Deconvolution (Old Version)





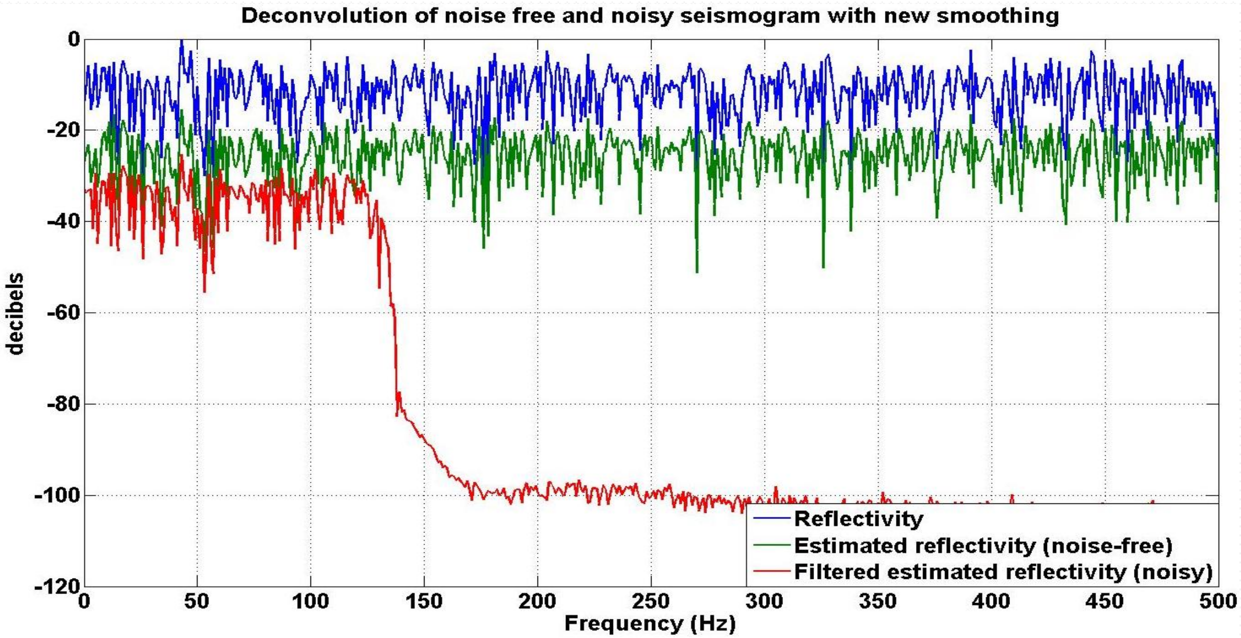
How possible to improve?

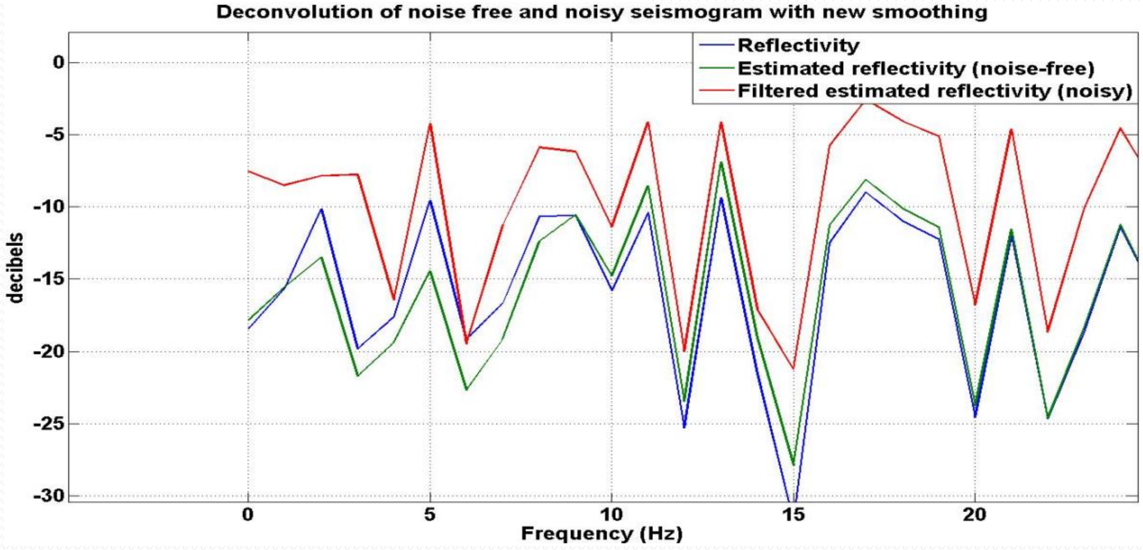
- Improving the process of smoothing amplitude spectrum of seismic trace for estimating wavelet

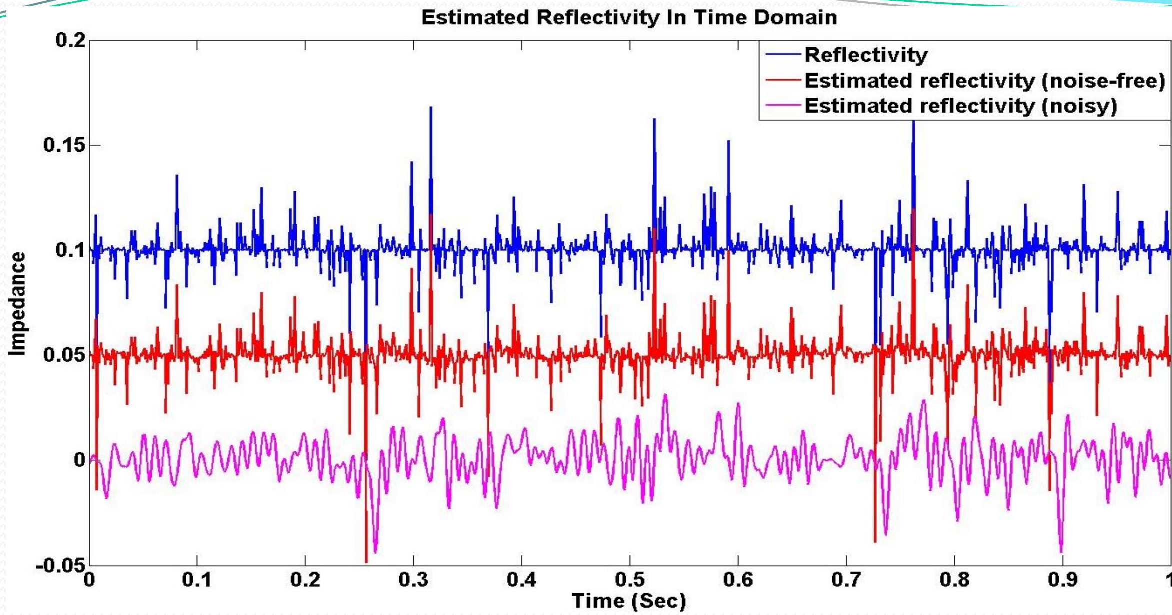


- Applying spectral color operator to Deconvolved data

Deconvolution with new smoothing

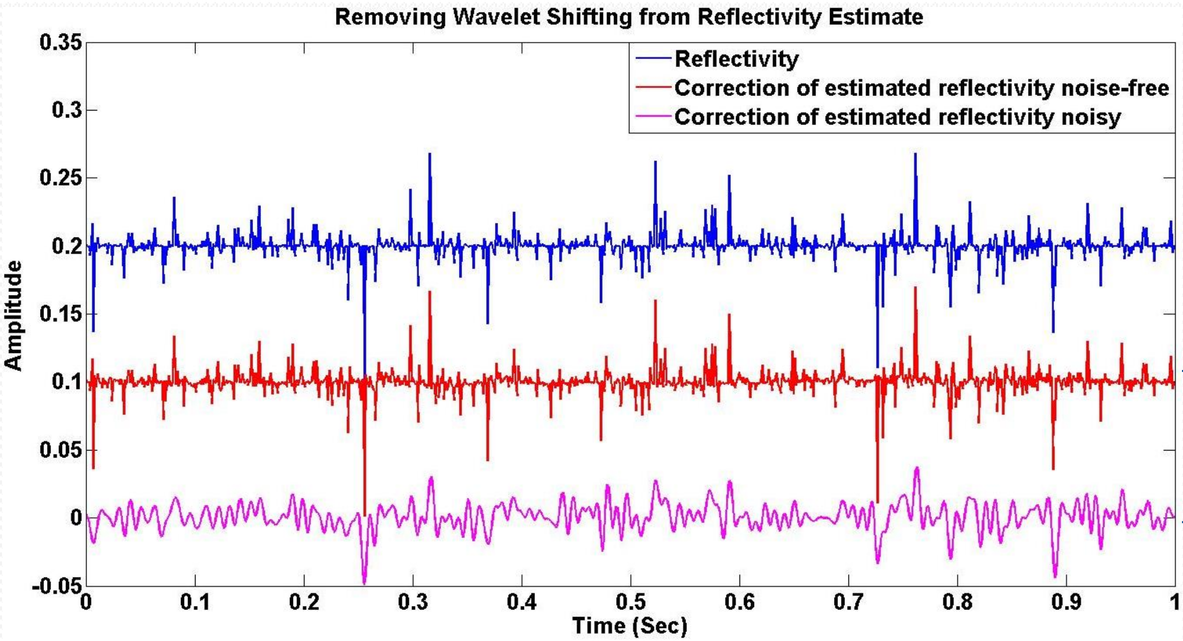






Maximum Correlation = 0.9865
Lag = 0.0000

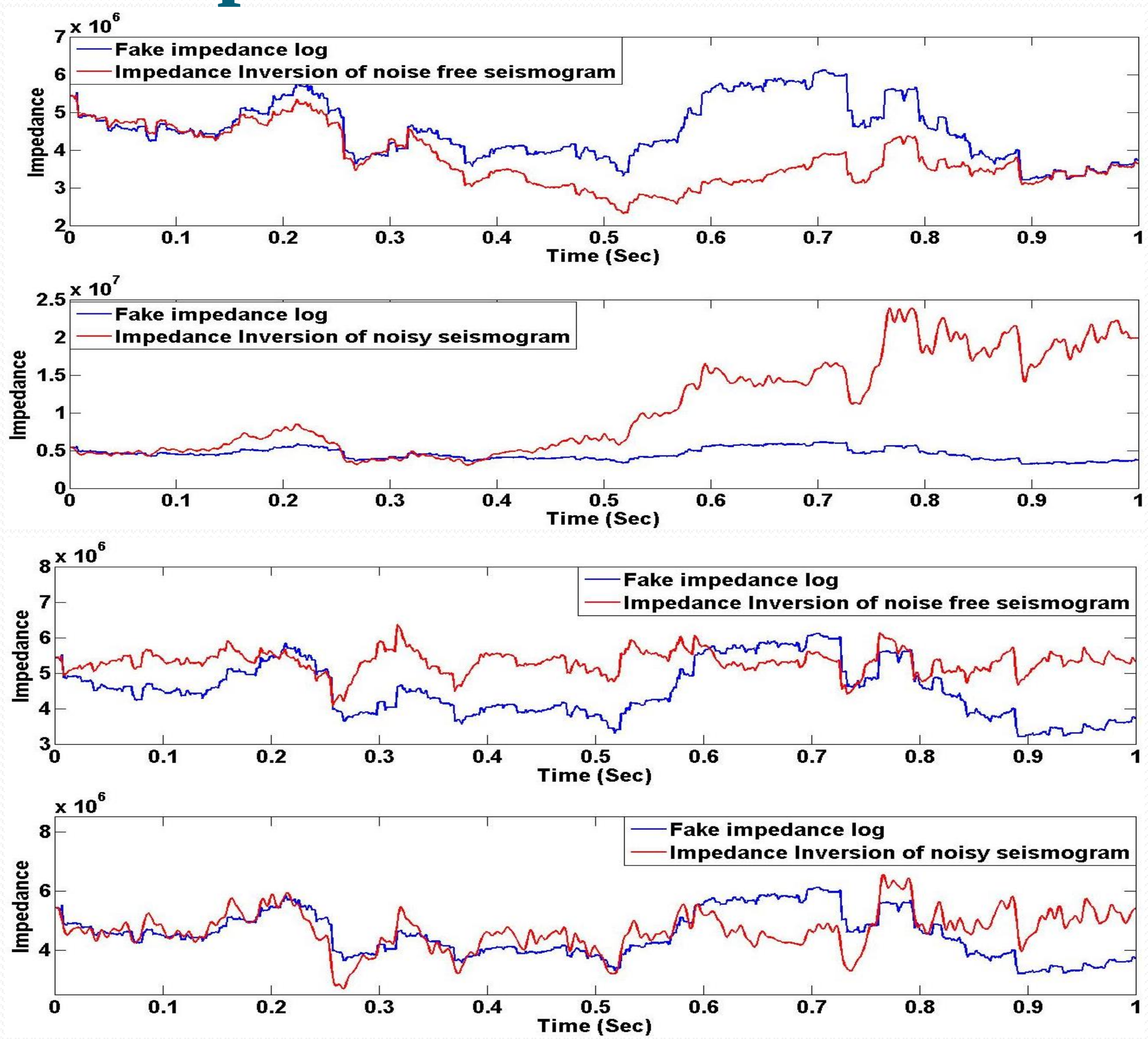
Maximum Correlation = 0.3950
Lag = 9.5000



Maximum Correlation = 0.9865
Lag = 0.0000

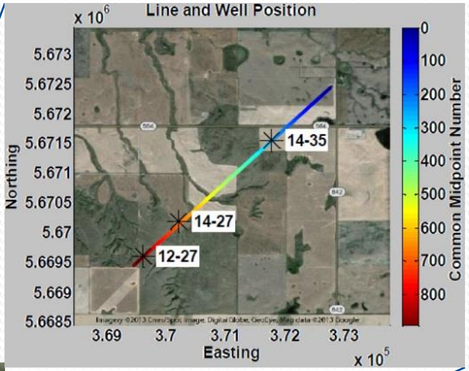
Maximum Correlation = 0.4900
Lag = 0.5000

Inverted Impedance for random reflectivity

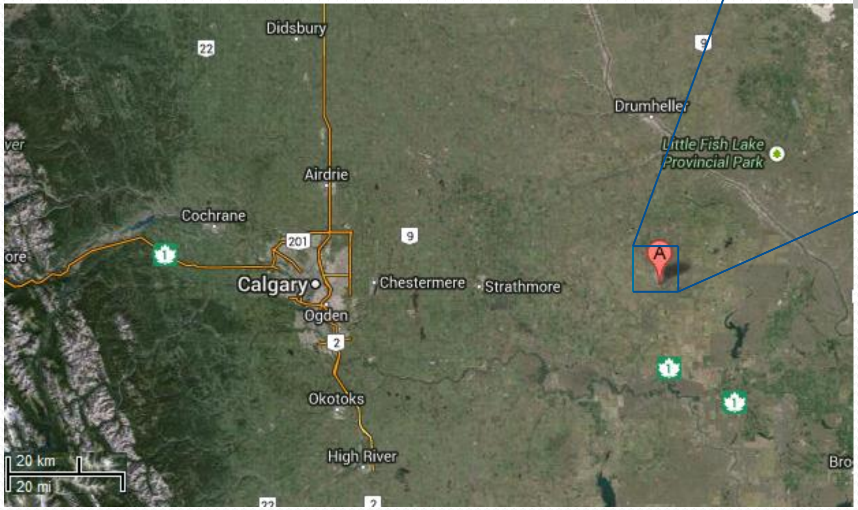


Real well log data

- The Real Data: Sonic and Density logs from the well 12-27 near Hussar

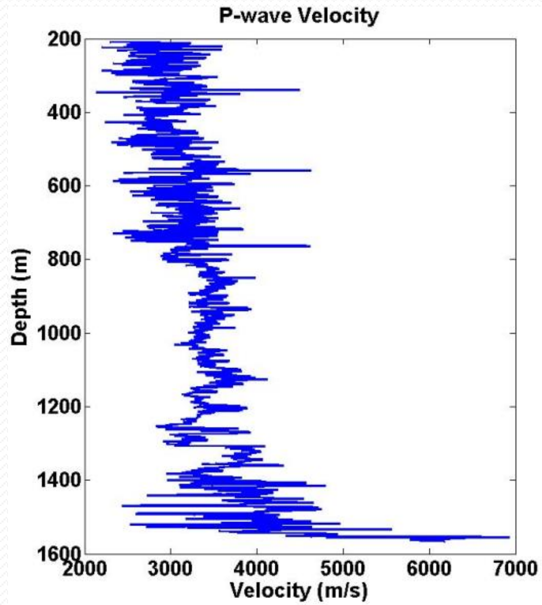


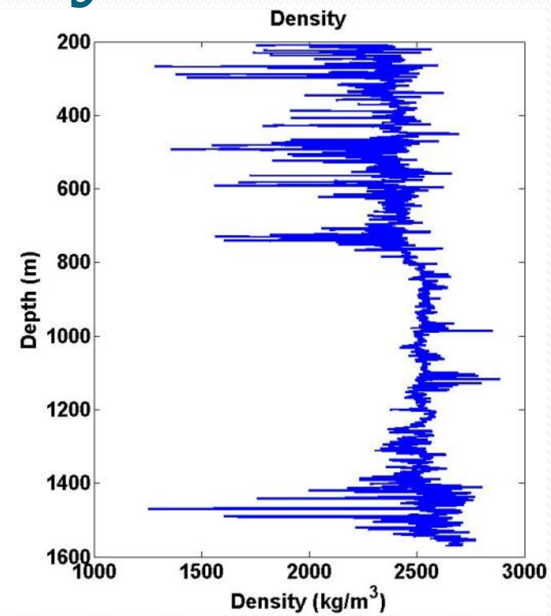
Lloyd, Heather, 2013



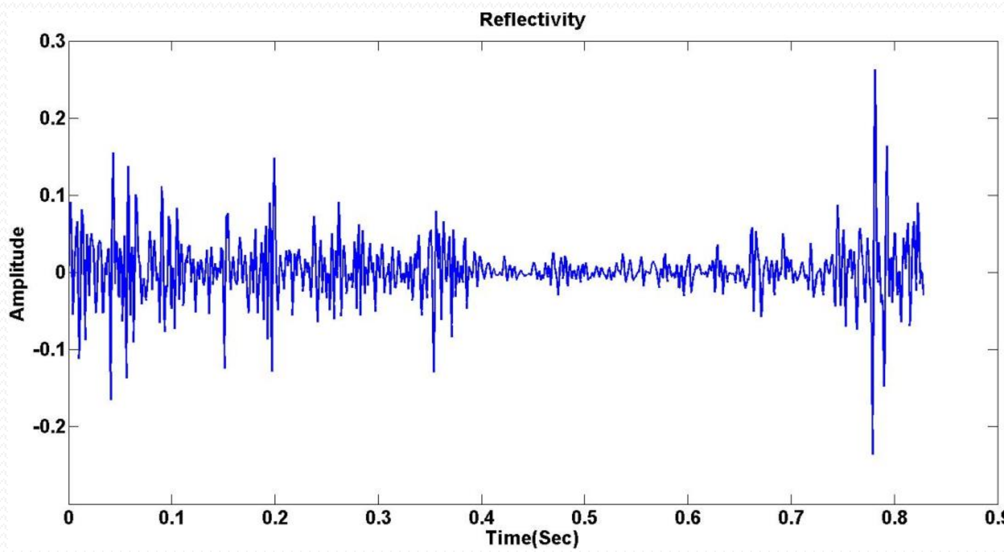
Location of the well 12-27 area near Hussar, Alberta, Canada indicated by the red marker

Spectral color reflectivity



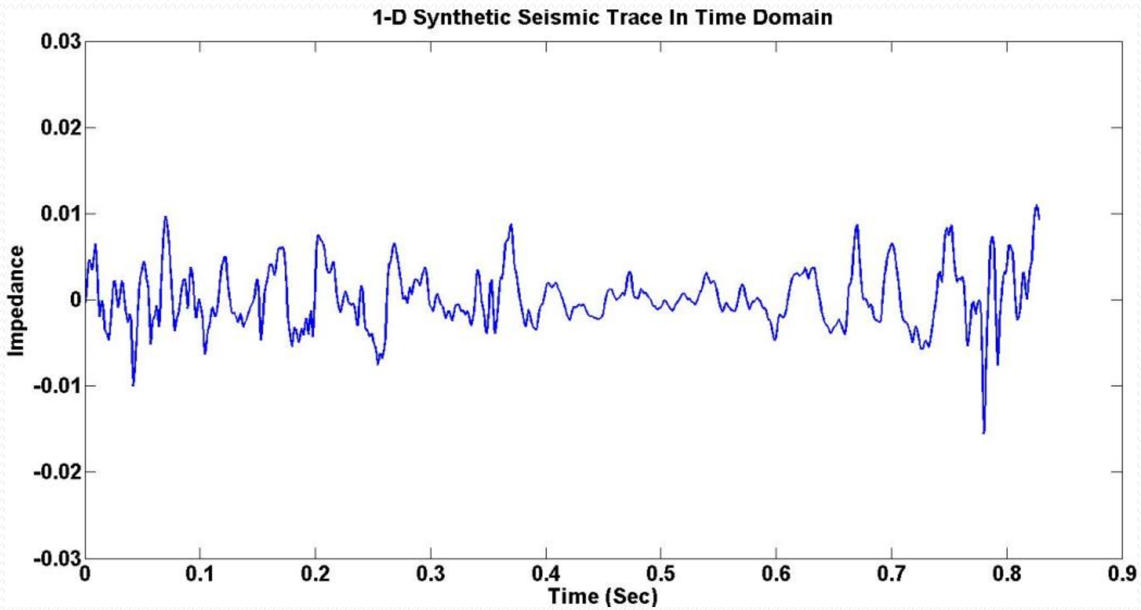


$$r_n = \frac{\rho_{n+1}v_{n+1} - \rho_n v_n}{\rho_{n+1}v_{n+1} + \rho_n v_n}$$

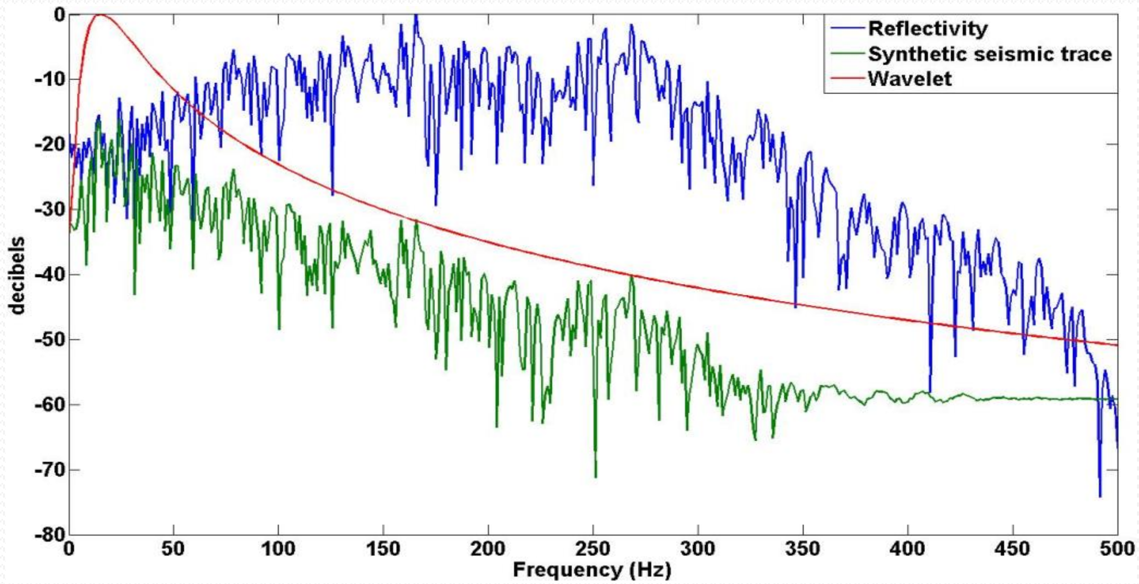


The Synthetic Seismic Trace

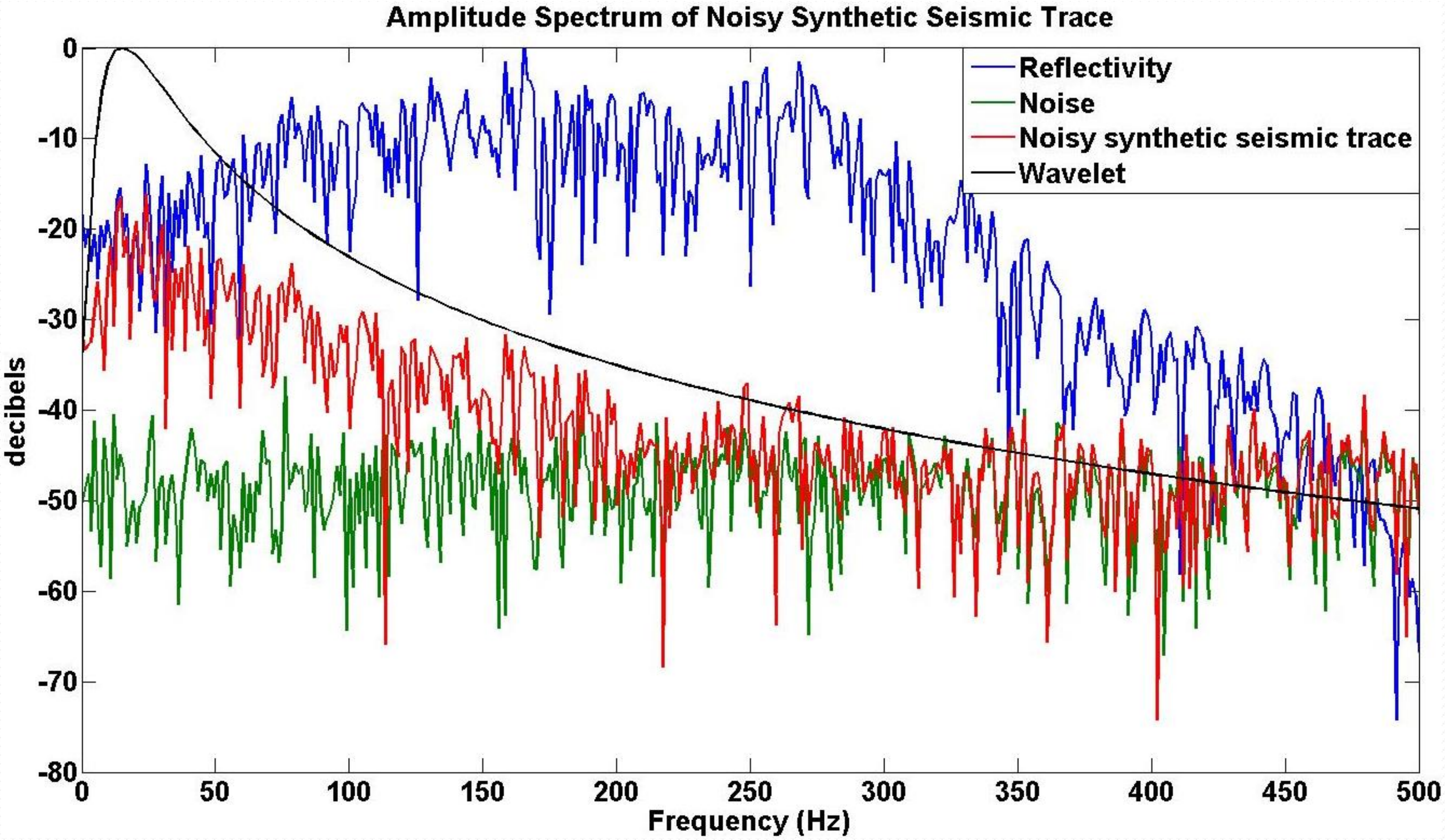
In time domain:



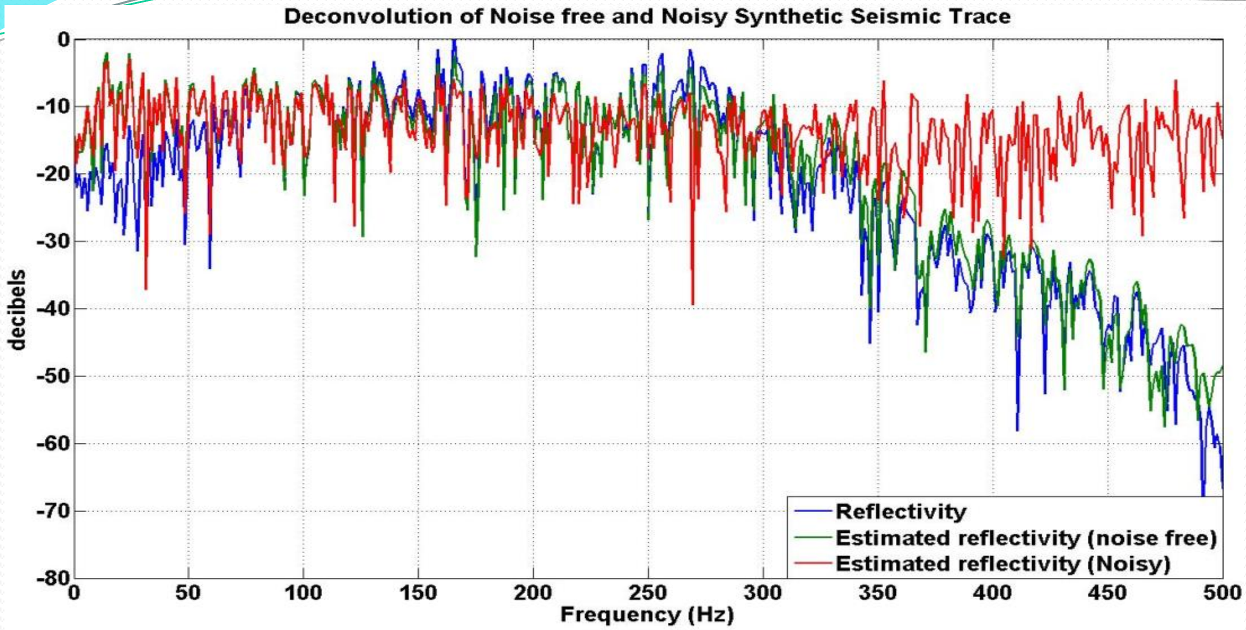
In frequency domain:



Adding Noise To Data

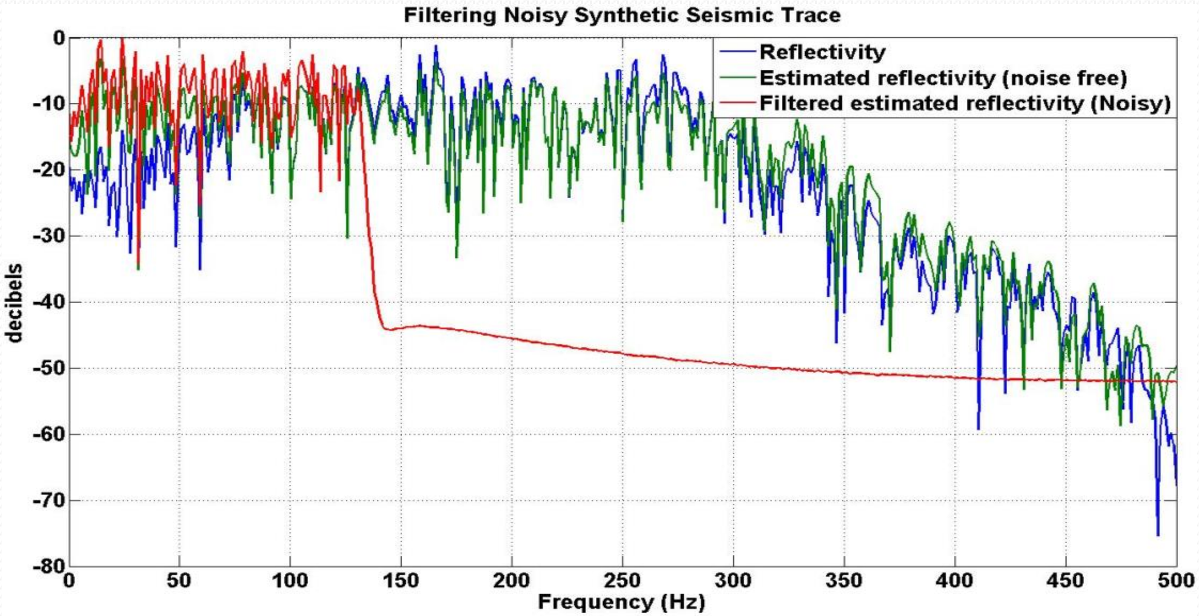


DECONVOLUTION

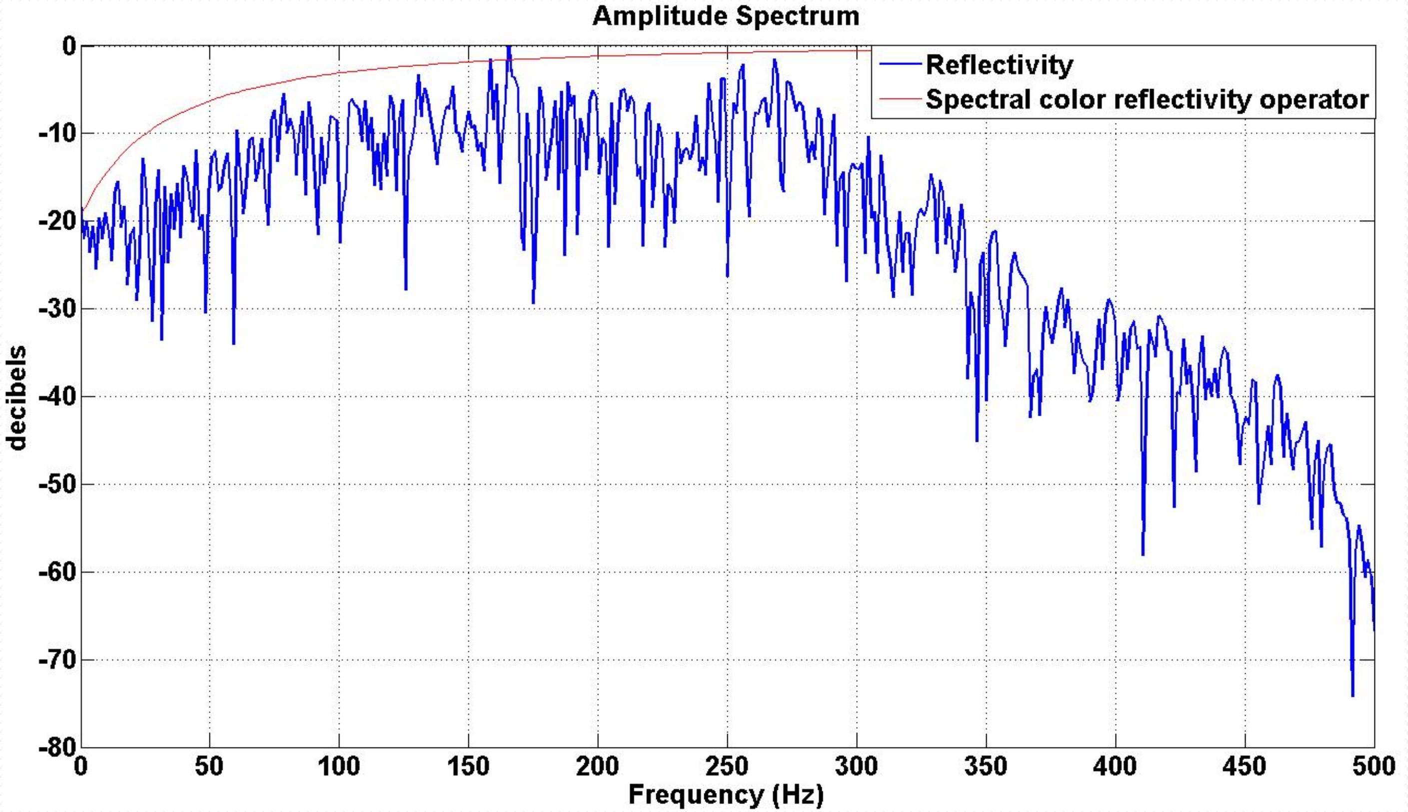


Deconvolution

Filtering

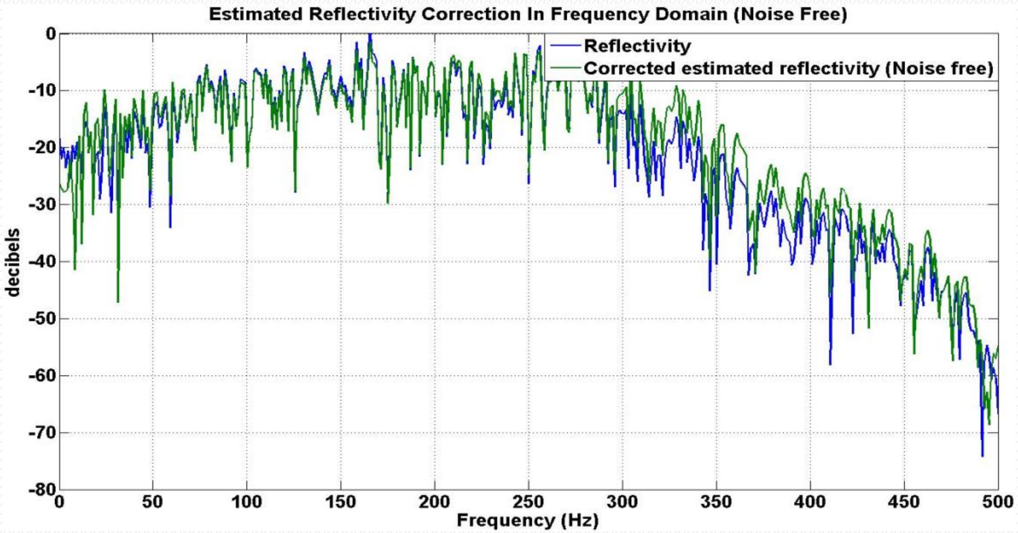


Spectral Color Operator

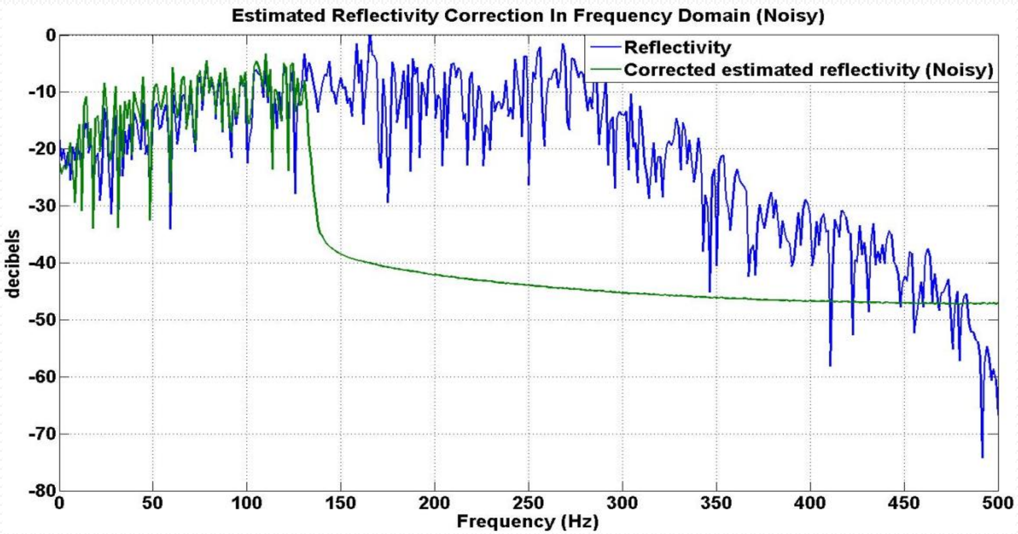


Applying Operator To Data

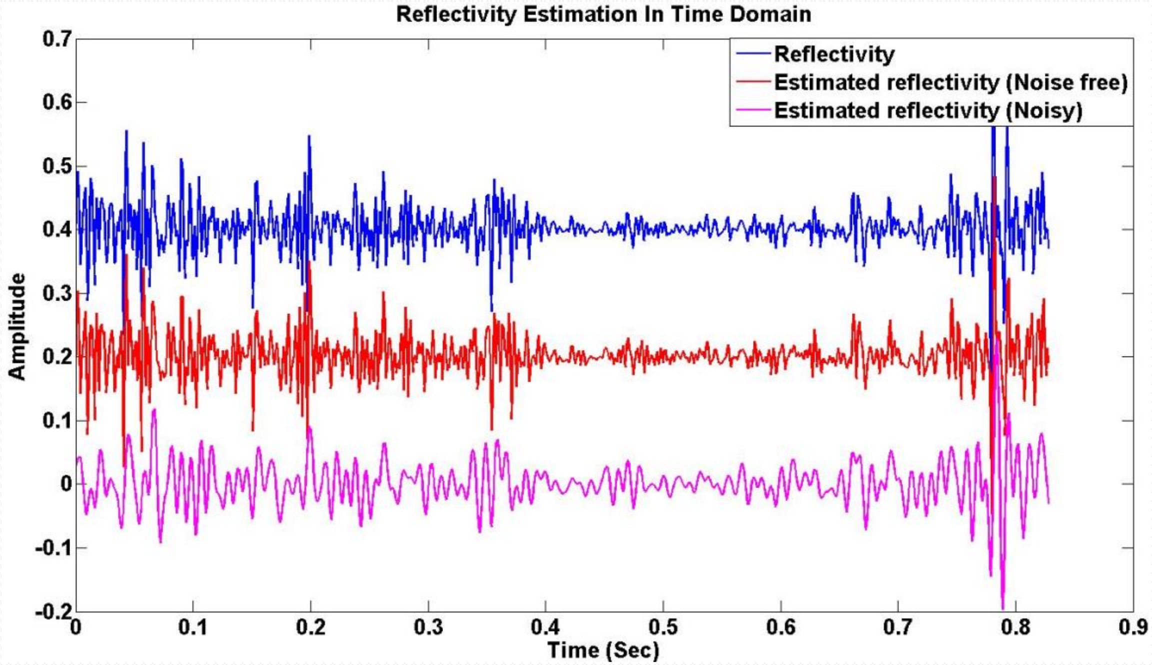
Noise free case:



Noisy case:

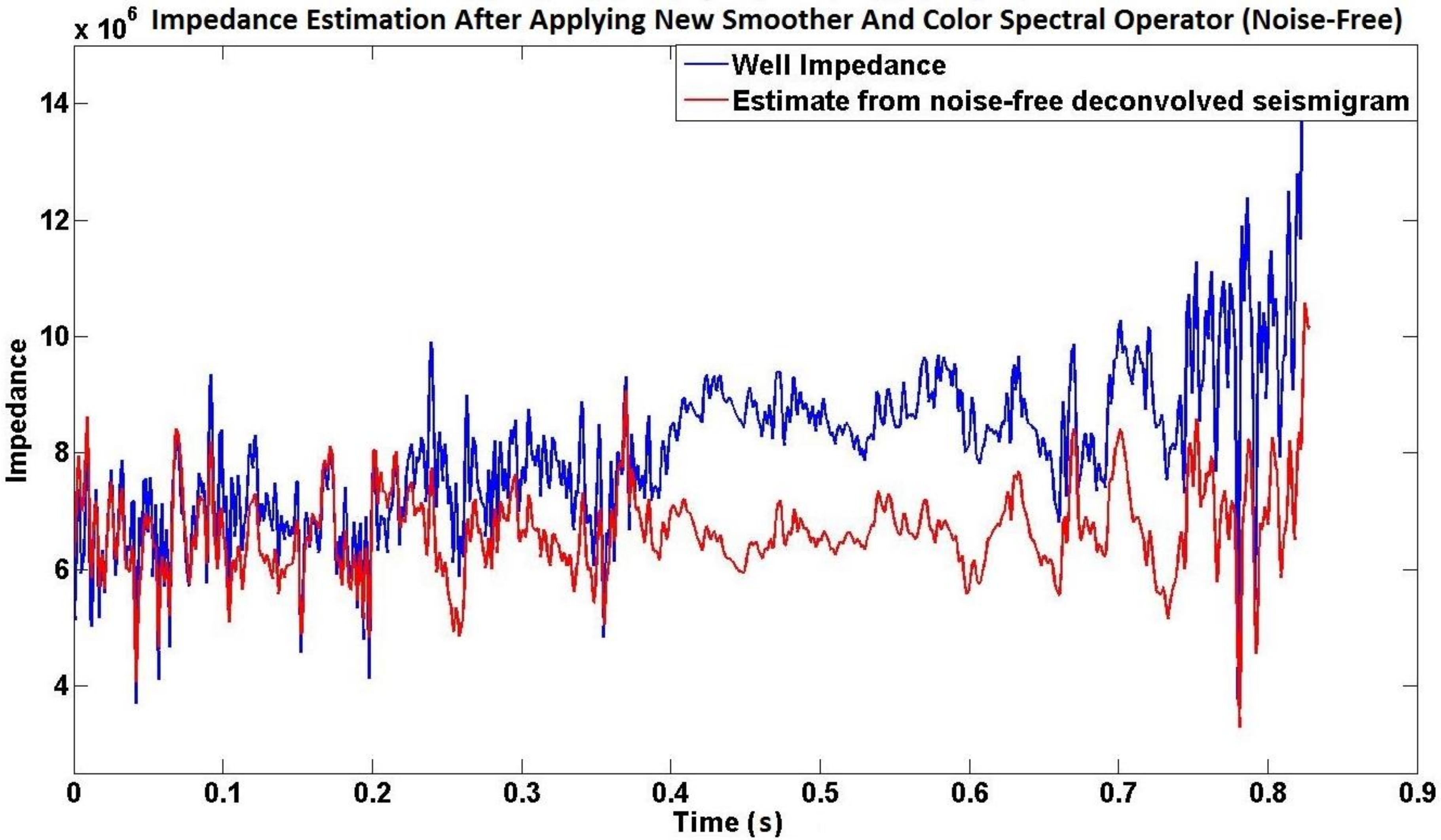


Final Results In Time Domain

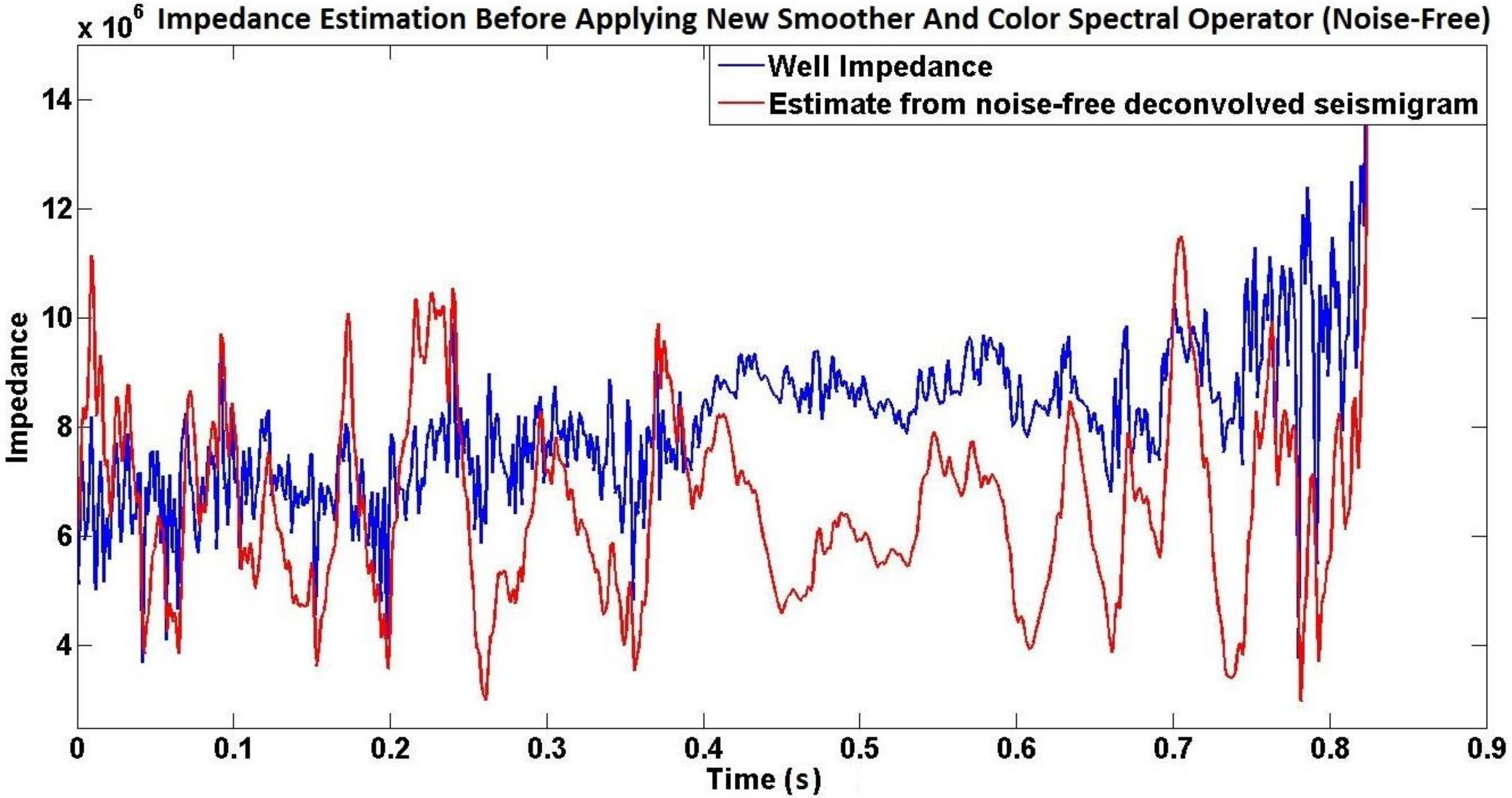


	Frequency Domain Deconvolution (Old Version)	Frequency Domain Deconvolution (New Version)
Final estimated reflectivity (Noise Free)	Maximum Correlation = 0.8668 Lag = 0.2000	Maximum Correlation = 0.8912 Lag = 0.3000
Final estimated reflectivity (Noisy)	Maximum Correlation = 0.3198 Lag = 1.4000	Maximum Correlation = 0.5698 Lag = 0.5000

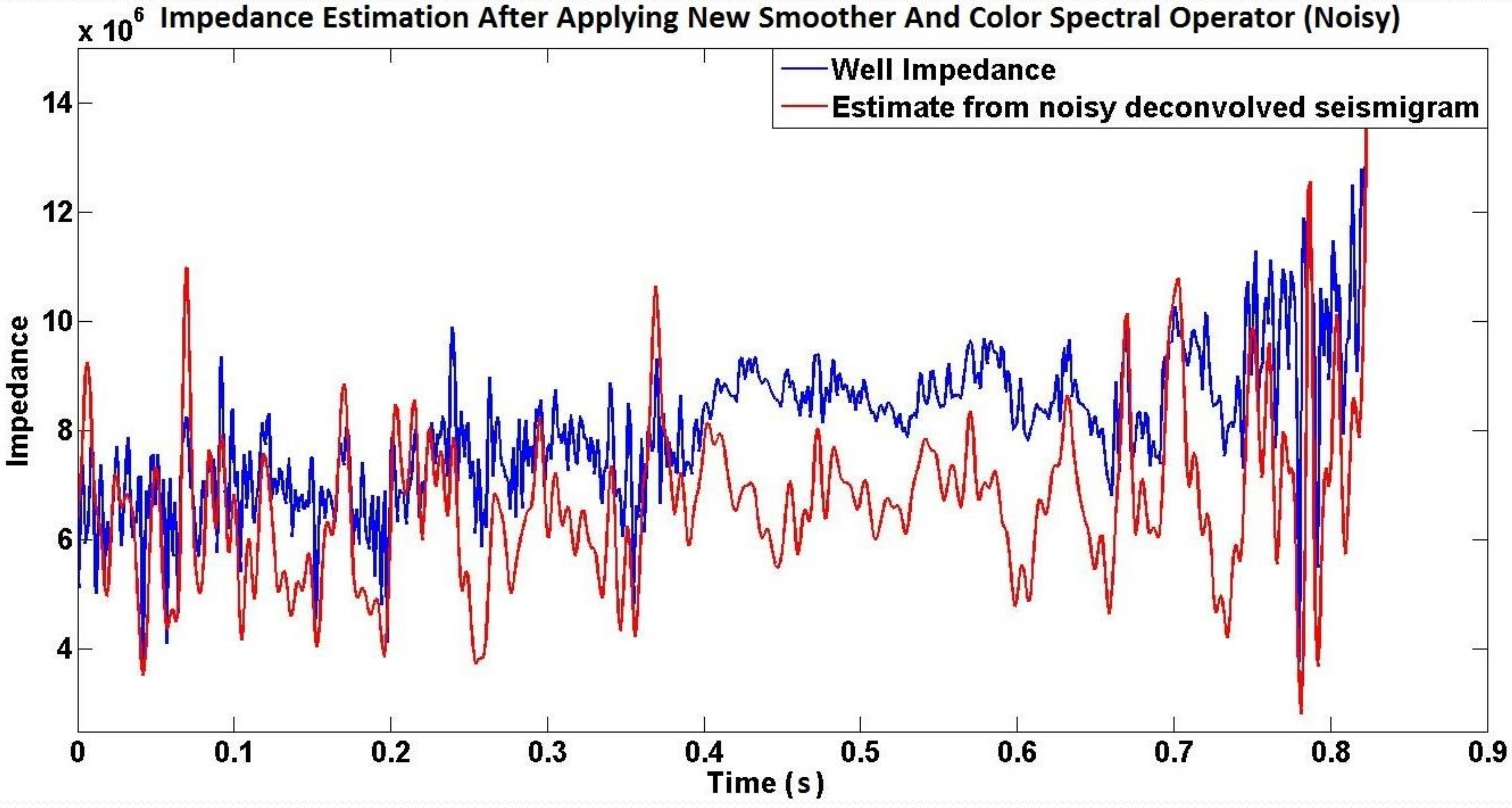
Impedance Inversion for noise-free seismogram



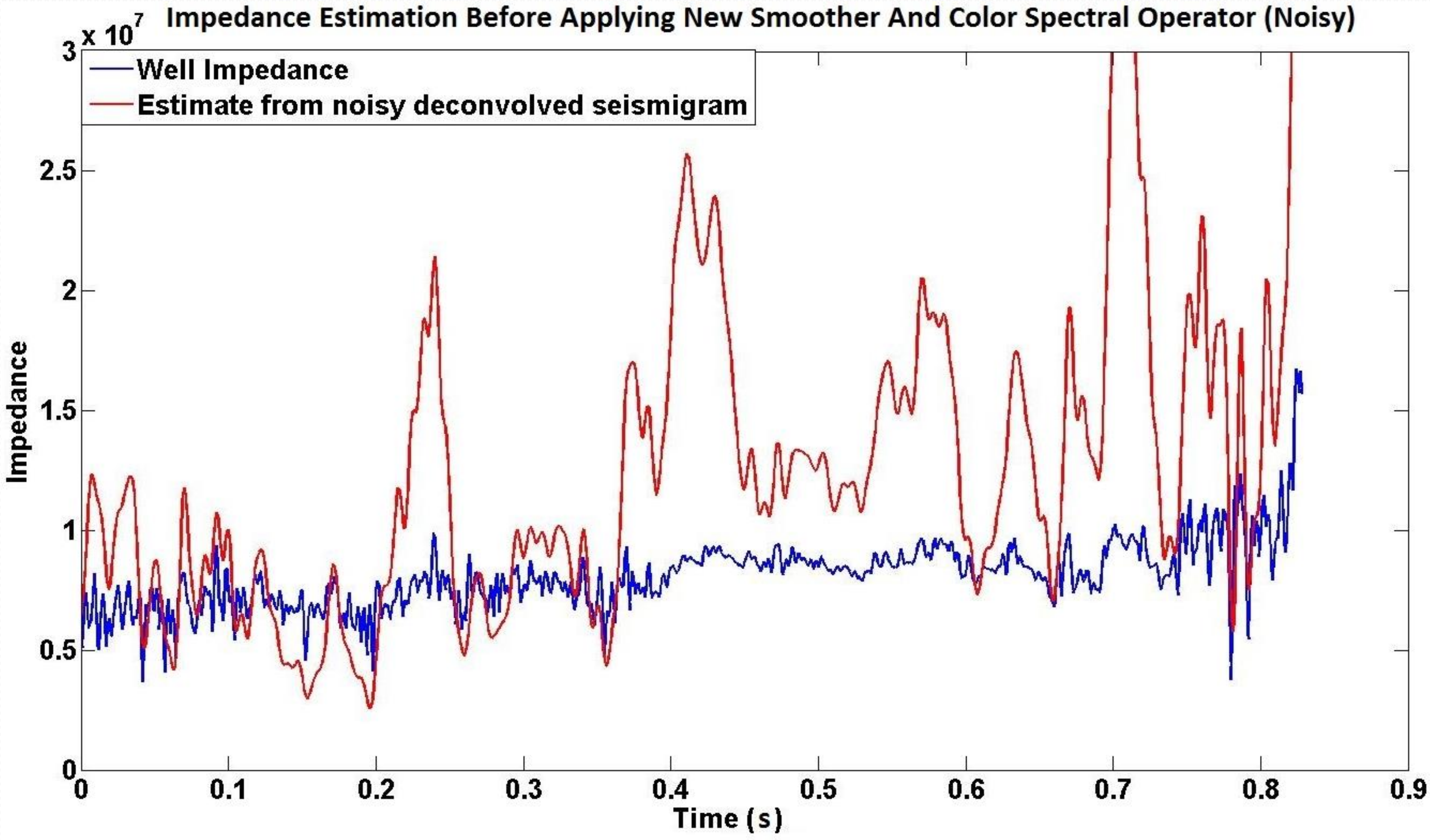
Impedance Inversion for noise-free seismogram



Impedance Inversion for noisy seismogram



Impedance Inversion for noisy seismogram



Conclusion

- The bandlimited nature of wavelet and also noise contamination causes the missing low and high frequency in recorded data.

- The better seismic data smoothing, the more realistic reflectivity estimation we can reach.

- The colored spectrum of low frequencies data could be recovered by spectral color operator.

- Acoustic impedance can be estimated much precisely if the low frequency can be recovered from seismic data.

- For the future work real recorded seismic data will be used

Acknowledgement

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- Sponsors of CREWES
- All the students and staffs in CREWES