Velocity-Stress Finite-Difference Modeling of Poroelastic Wave Propagation

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and

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Outline

- Introduction
- Biot’s Theory
- Staggered-Grid Finite Difference
- Numerical Examples
- Conclusion
- Acknowledgement
Introduction

- Poroelastic Medium

- Biot (1962): anelastic effects from the relative movement of the fluid.

- Biot’s theory: Important in oil and gas exploration, CO2 storage monitoring and hydrogeology.

- The Theory predicts two compressional waves and one shear wave.

(Russell et al., 2003)
Biot’s Theory (1962)

Assumptions:

- Elastic rock frame
- Connected pores
- Seismic wavelength ≫ average pore size
- Small deformations
- Statistically isotropic medium
Stress-Strain Relation For Porous Media (Biot, 1962)

Solid Stress

\[ \tau_{ij} = 2\mu e_{ij} + (\lambda_c e_{kk} + \alpha M \varepsilon_{kk}) \delta_{ij} \]

Fluid Pressure

\[ P = -\alpha M e_{kk} - M \varepsilon_{kk} \]

\[ e_{ij} = \nabla \cdot \mathbf{u} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

\[ \varepsilon_{ij} = \nabla \cdot (\mathbf{u} - \mathbf{U}) \]

\[ \alpha = 1 - \frac{K_{Dry}}{K_{Solid}} \]

\[ M = \left[ \frac{\phi}{K_{Fluid}} + \frac{(\alpha - \phi)}{K_{Solid}} \right] \]

Coupling Modulus

\[ \lambda \ & \mu : \text{ Lame Parameters of the Saturated Rock.} \]

\[ u : \text{Solid Particle Displacement} \]

\[ U : \text{Fluid Particle Displacement} \]
Equations of motion for a statistically isotropic porous media saturated with viscous fluid:

\[
(m \rho - \rho_f^2) \frac{\partial^2 u_i}{\partial t^2} = m \frac{\partial \tau_{ij}}{\partial x_j} + \rho_f b \frac{\partial w_i}{\partial t} + \rho_f \frac{\partial P}{\partial x_i}
\]

\[
(m \rho - \rho_f^2) \frac{\partial^2 w_i}{\partial t^2} = -\rho_f \frac{\partial \tau_{ij}}{\partial x_j} - \rho b \frac{\partial w_i}{\partial t} - \rho \frac{\partial P}{\partial x_i}
\]

- Effective Fluid Density: \( m = T^{\rho_f}_{\phi} \)
- Fluid Displacement Relative to the Solid: \( w = u - U \)
- \( \rho_f \) : Fluid Density
- \( \rho \) : Density of Saturated Rock
- \( b = \eta/\kappa \) : Mobility
- \( \eta \) : Viscosity
- \( \kappa \) : Permeability
Substituting $\mathbf{V} = \frac{\partial \mathbf{u}}{\partial t}$ and $\mathbf{W} = \frac{\partial \mathbf{w}}{\partial t}$ in the equations of motion and taking derivatives with respect to time from both sides of the stress-strain relationship we have:

$$(m\rho - \rho_f^2) \frac{\partial V_i}{\partial t} = m \frac{\partial \tau_{ij}}{\partial x_j} + \rho_f bW + \rho_f \frac{\partial P}{\partial x_i}$$

$$(m\rho - \rho_f^2) \frac{\partial W_i}{\partial t} = -\rho_f \frac{\partial \tau_{ij}}{\partial x_j} - \rho bW - \rho \frac{\partial P}{\partial x_i}$$

and

$$\frac{\partial \tau_{ij}}{\partial t} = 2\mu \frac{\partial e_{ij}}{\partial t} + (\lambda_c \frac{\partial e_{kk}}{\partial t} + \alpha M \frac{\partial \varepsilon_{kk}}{\partial t}) \delta_{ij}$$

$$\frac{\partial P}{\partial t} = -\alpha M \frac{\partial e_{kk}}{\partial t} - M \frac{\partial \varepsilon_{kk}}{\partial t}$$
2D case:

\[
\frac{\partial \tau_{xx}}{\partial t} = (\lambda_c + 2\mu) \frac{\partial V_x}{\partial x} + \lambda_c \left( \frac{\partial V_z}{\partial z} \right) + \alpha M \left( \frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \tag{1}
\]

\[
\frac{\partial \tau_{zz}}{\partial t} = (\lambda_c + 2\mu) \frac{\partial V_z}{\partial z} + \lambda_c \left( \frac{\partial V_x}{\partial x} \right) + \alpha M \left( \frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \tag{2}
\]

\[
\frac{\partial \tau_{xz}}{\partial t} = \mu \left( \frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z} \right) \tag{3}
\]

\[
\frac{\partial P}{\partial t} = -\alpha M \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} \right) - M \left( \frac{\partial W_x}{\partial x} + \frac{\partial W_z}{\partial z} \right) \tag{4}
\]

\[
\frac{\partial V_x}{\partial t} = A \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) + BW_x + C \frac{\partial P}{\partial x} \tag{5}
\]

\[
\frac{\partial V_z}{\partial t} = A \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) + BW_z + C \frac{\partial P}{\partial z} \tag{6}
\]

\[
\frac{\partial W_x}{\partial t} = D \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right) + EW_x + F \frac{\partial P}{\partial x} \tag{7}
\]

\[
\frac{\partial W_z}{\partial t} = D \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) + EW_z + F \frac{\partial P}{\partial z} \tag{8}
\]
Staggered-Grid Finite Difference (Levander, 1988)

- **X**: $\tau_{xx}, \tau_{zz}$ and $P$
- **Y**: $V_x$ and $W_x$
- **Z**: $V_z$ and $W_z$
- **O**: $\tau_{xz}$
Numerical Examples

Single layer model based on QUEST Project

- CO₂ storage in Basal Cambrian Sands or BCS, which is a saline aquifer within Western Canadian Sedimentary Basin (WCSB)
- Data from well SCL-8-19-59-20W4

Modified after Bachu et al., 2000.
Single Layer Model

Gassmann Fluid Substitution

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_f$</td>
<td>937 (kg/m$^3$)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2370 (kg/m$^3$)</td>
</tr>
<tr>
<td>$V_p$</td>
<td>3800 (m/s)</td>
</tr>
<tr>
<td>$V_s$</td>
<td>2400 (m/s)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>16%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1(mD)</td>
</tr>
</tbody>
</table>

BCS: 40% CO$_2$
• Fourth order in space and second order in time.
• The stability condition is the same as the one in the elastic case (Zhu:1991)

\[ \Delta t \leq \frac{h}{(V_p^2 - V_s^2)^{1/2}} \]

\[ h = 3m \quad dt = 0.2 \, ms \]

• The size of the model was 1500 m by 1500 m
• Explosive source: Ricker wavelet with dominant frequency 50 Hz
• Source location : \((x, z) = (750, 750) m\)
Vertical Particle Velocity Snapshots:

- **Solid**
- **Fluid**
Comparison with elastic algorithm
Two-Layered Model

<table>
<thead>
<tr>
<th></th>
<th>Top Layer</th>
<th>Bottom Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_f )</td>
<td>1070 ((kg/m^3))</td>
<td>937 ((kg/m^3))</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2400 ((kg/m^3))</td>
<td>2370 ((kg/m^3))</td>
</tr>
<tr>
<td>( V_p )</td>
<td>4100 ((m/s))</td>
<td>3800 ((m/s))</td>
</tr>
<tr>
<td>( V_s )</td>
<td>2390 ((m/s))</td>
<td>2400 ((m/s))</td>
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</tr>
</tbody>
</table>

Layer 1: \( \text{BCS: 100\% brine} \)

Layer 2: \( \text{BCS: 40\% CO2 + 60\% brine} \)
Conclusion and Future Goals

- The Poroelastic algorithm generates slow compressional wave as predicted by Biot’s theory.
- At a poroelastic boundary the slow P-wave is converted to a fast P-wave.
- The algorithm handles layered models and should be examined for more complex models.
- The algorithm could be used for inversion to obtain porous media properties that are ignored in elastic algorithms.
Thanks to:

- Carbon Management Canada (CMC) for financial support.
- CREWES project for extensive technical support.
- Hassan Khaniani, Peter Manning and Joe Wong from CREWES
- David Aldridge from Sandia National Laboratories.
- Shell for the data
THANKS!