

Turning-ray tomography/Tomostatics

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Outline

- ◆ Introduction to seismic tomography
- ◆ Travelttime tomography; reflection and refraction tomography
- ◆ Series expansion method; ART, & SIRT
- ◆ Field examples (Hussar 2D line)
- ◆ Forward modelling/ray tracing and inversion
- ◆ Results
- ◆ Conclusions



Concept of seismic tomography

- Tomography is an imaging technique which generates a cross-sectional picture (a tomogram) of an object by utilizing the object's response to the nondestructive, probing energy of an external source (Lo and Inderwiesen, 1994).
- In Seismic tomography this response or measurement such as traveltimes or amplitudes is what is used to construct an estimate of the subsurface velocity distribution.

Traveltime tomography

- Traveltime tomography constructs estimates of the subsurface velocity distribution using traveltimes. When the subsurface layer thickness is larger than the seismic wavelength then ray-based tomography can resolve this features, if not, this ray approach is inappropriate as diffraction phenomena will dominate.
- In cases such as this, diffraction tomography or full wave inversion must be used.
- My interest in this topic stems from the similarities between traveltime tomography and full waveform inversion

Traveltime tomography

- Based on ray-tracing
- Uses the Eikonal equation and it is based on the high frequency assumption.
- Can use ray tracers or finite difference approximation of the eikonal equation

$$\frac{d}{ds} \left[\frac{1}{v(\mathbf{r})} \frac{d\mathbf{r}}{ds} \right] = \nabla \left(\frac{1}{v(\mathbf{r})} \right)$$
$$\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 = \frac{1}{v^2(x, y, z)}$$

r, s, v, T, x, y & z are the position along the ray, the path length, propagation speed, traveltime and the space coordinates respectively

$$t_i = \sum_j d_{ij} / v_j = \sum_j d_{ij} s_j$$

The traveltime equation is given as

$$t_i = \sum_j d_{ij} / v_j = \sum_j d_{ij} s_j$$

Where t_i is the total travel time along the i^{th} ray-path, d_{ij} is the path length in the j^{th} cell of the velocity model for the i^{th} ray, v_j is the velocity in the j^{th} cell and s_j is the slowness in the j^{th} cell (Jones, 2009).

We can rewrite the traveltime equation above in matrix form as;

$$T = DS$$

$$S = \left(D^T D \right)^{-1} D^T T$$

$$S = \left(D^T D + \mu I \right)^{-1} D^T T$$

S is the slowness vector, D is the matrix of the lengths of rays corresponding to the Frechet derivative, T is the traveltime vector, μ is the damping factor, and I is the identity matrix

Series expansion method

Two methods are generally used in tomography; the transform method and the Series expansion method.

- Series expansion method is used in seismic traveltime tomography; it allows for curved ray paths.
- Uses the ART or SIRT technique of the Kaczmarz method

Series expansion method- ART

➤ ART; Algebraic Reconstruction technique.

Involves tracing only one ray each time, out of the total number of rays to be traced. Model update is given as;

$$\Delta s_j = d_{ij} \frac{|t_i^{observed} - t_i^{predicted}|}{\sum_j (d_{ij})^2}$$

Series expansion method- SIRT

- SIRT; Simultaneous Iterative Reconstruction technique.

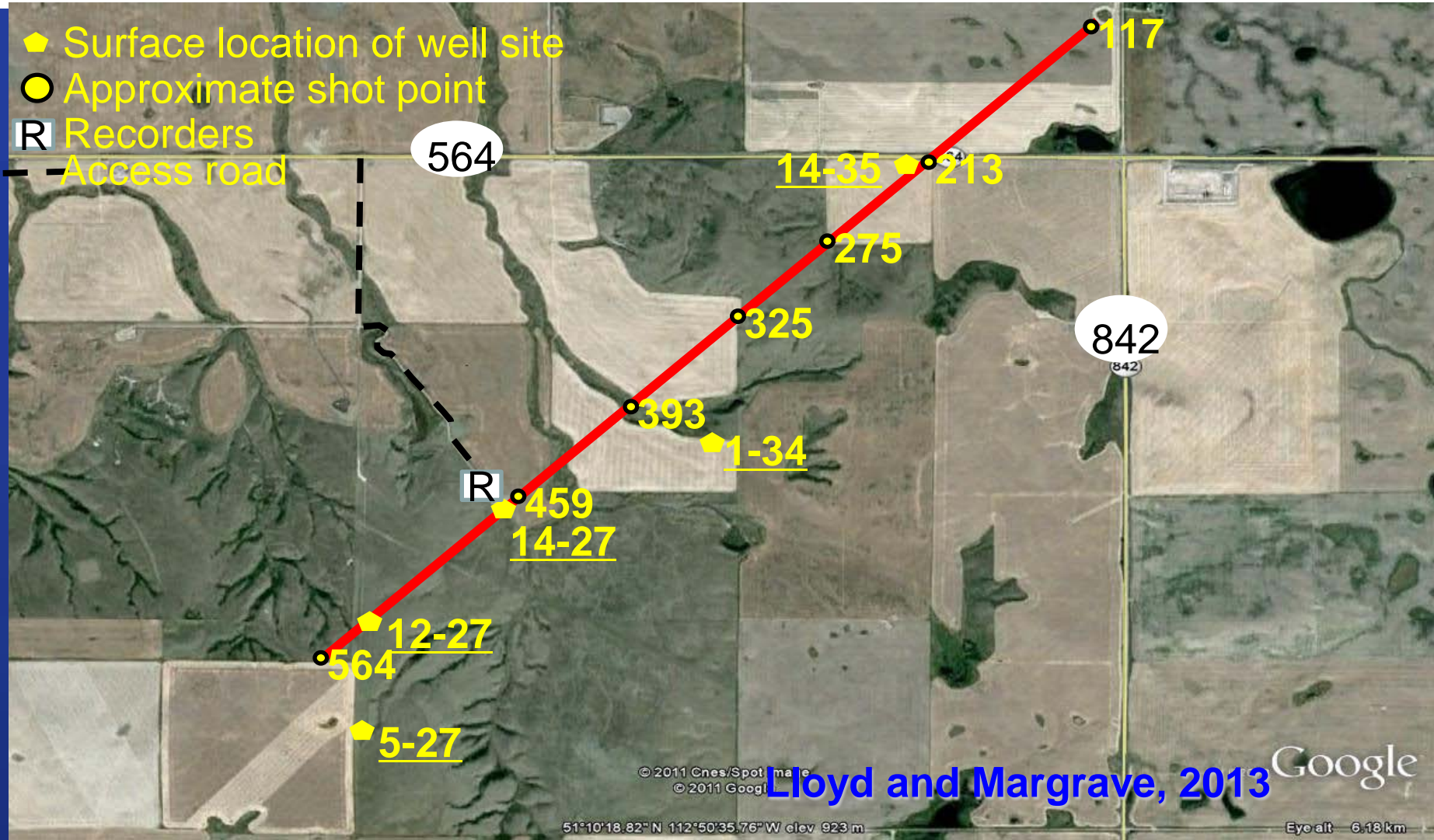
Involves tracing all rays through the model so that all corrections for all the rays are known. Model update is given as;

$$\Delta s_j = \frac{1}{W_j} \sum_{i=1}^I d_{ij} \frac{|t_i^{observed} - t_i^{predicted}|}{\sum_j (d_{ij})^2}$$

Field examples

- 2D seismic line from Hussar, central Alberta.
- About 4.5km long running from Southwest to Northeast.
- Seismic source is dynamite; shot spacing 20m
Number of shots 269.
- Number of receivers is 448 with a receiver spacing of 10m.

- ◆ Surface location of well site
- Approximate shot point
- R Recorders
- - - Access road



Lloyd and Margrave, 2013

51°10'18.82" N 112°50'35.76" W elev 923 m

Google
Eye alt 6.18 km

Field examples

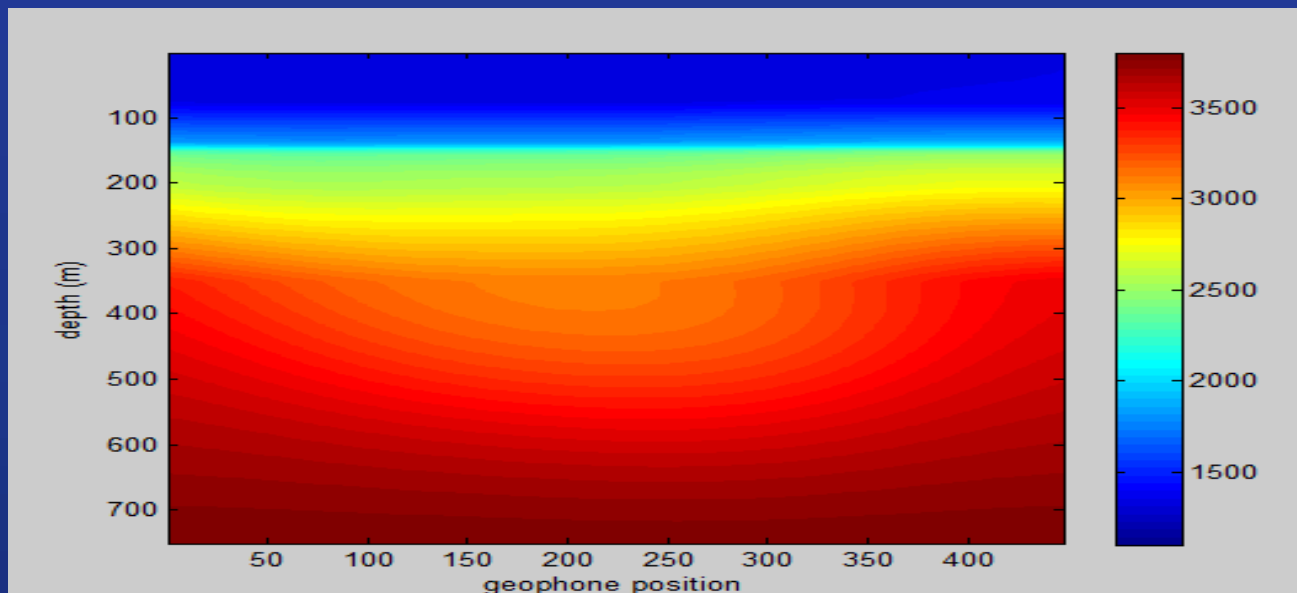
- Observed travel times is $269 \times 448 = 120512$ observed traveltimes.
- Number of unknowns are $4470 \times 1000 / 25 = 178800$
- Inverse problem is under-determined

Forward Modelling/ray tracing and inversion

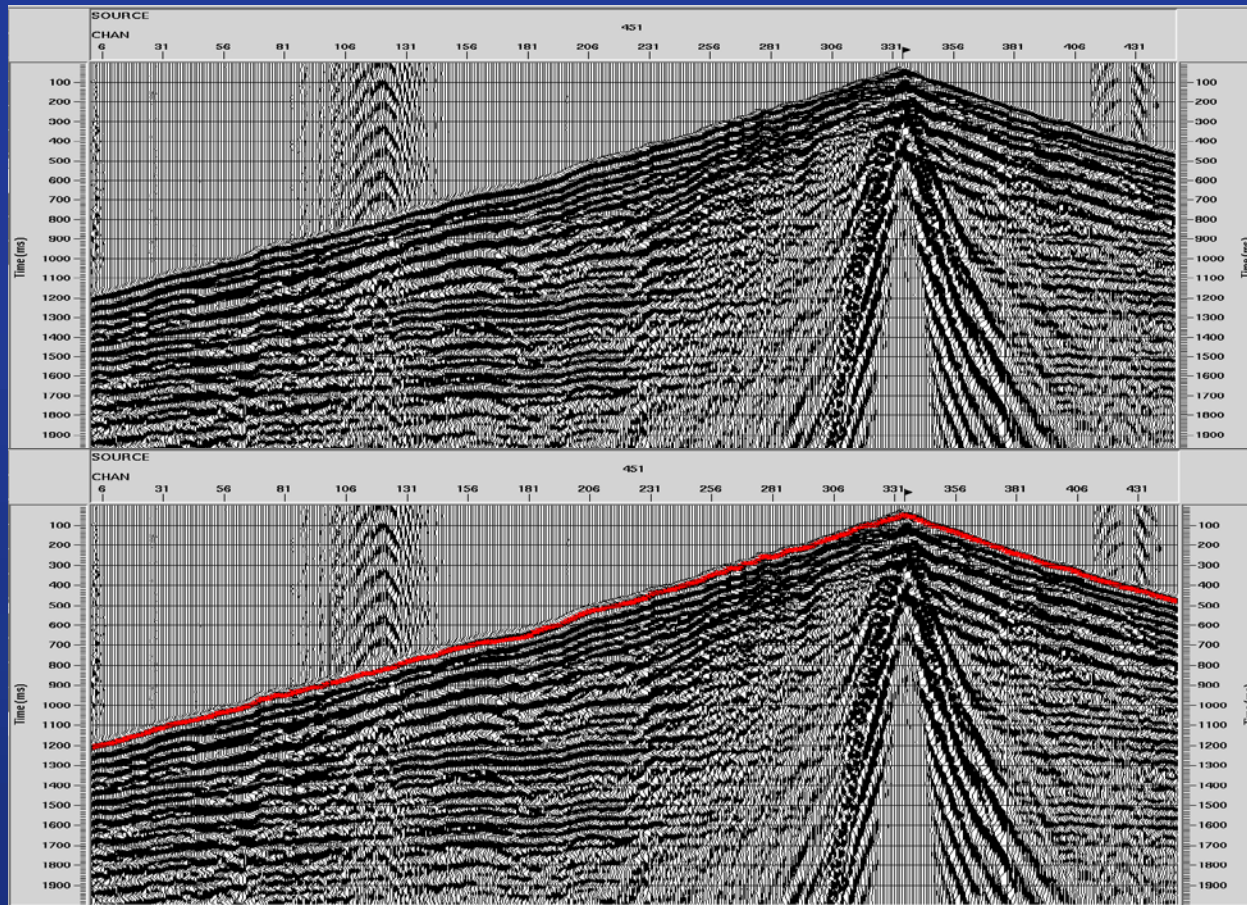
- Initial velocity model 4470 m by 1000m, grid spacing 5m by 5m.
- Initial Model is a crude velocity model from refraction method.
- Used ray-tracing of first arrival times through the model using Langan et al, (1985) approach of the two-point problem.
- Algorithm is Simultaneous Iterative Reconstruction Technique (SIRT) to update initial model

Forward Modelling/ray tracing and inversion

- Performed 50 iterations in total. Cost function at the 51st iteration was negligible (i.e. $\ll 1$ milliseconds).



Initial velocity model from refraction



A shot gather from Hussar, showing picked (observed) traveltimes

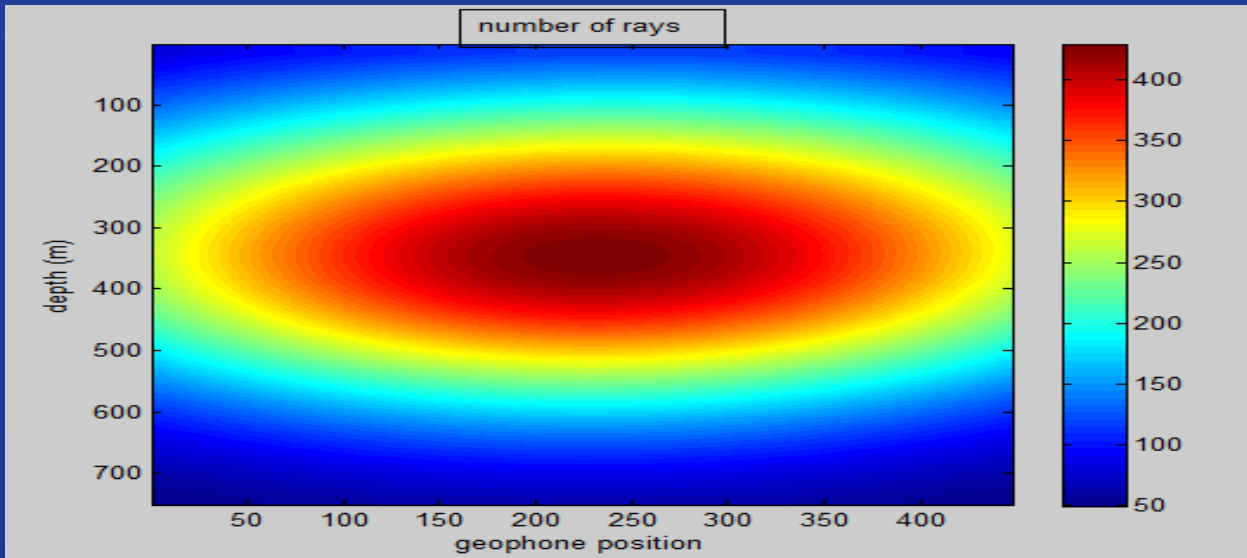
QC Methods, Zhu (2002); ray density, first arrival fitting, and stack responses.

Damping

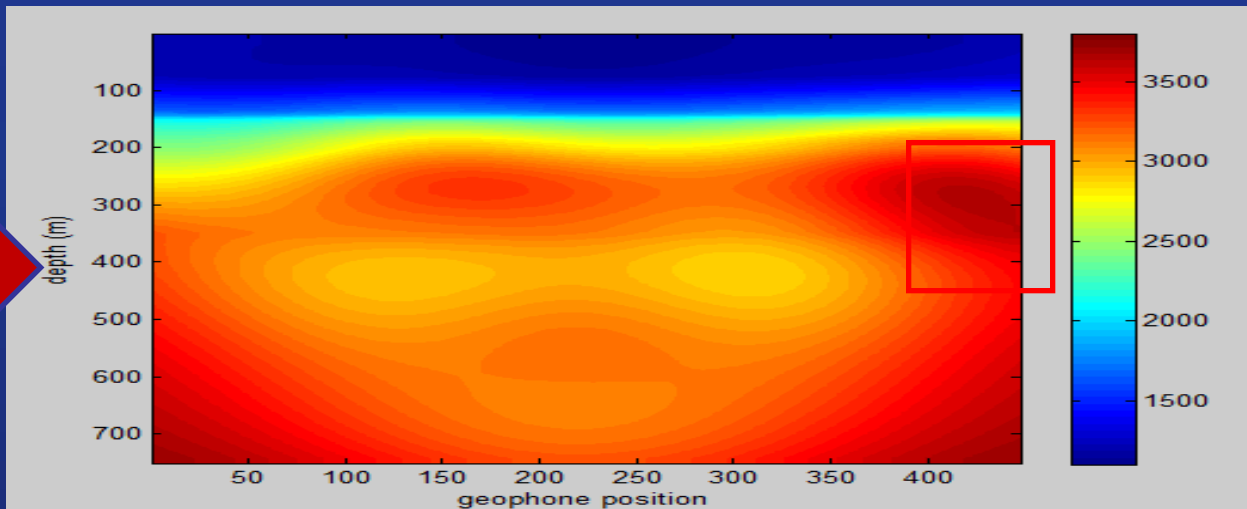
- Areas with small ray density were damped
- Smoothing laterally and in depth after each iteration

Constraints

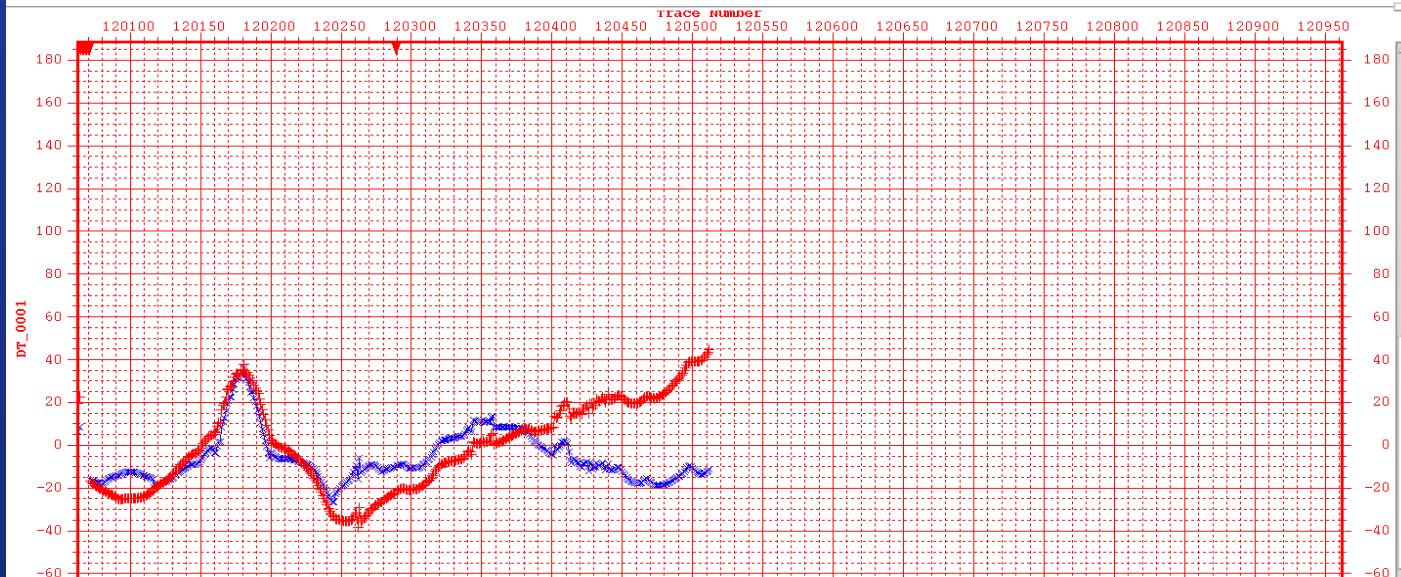
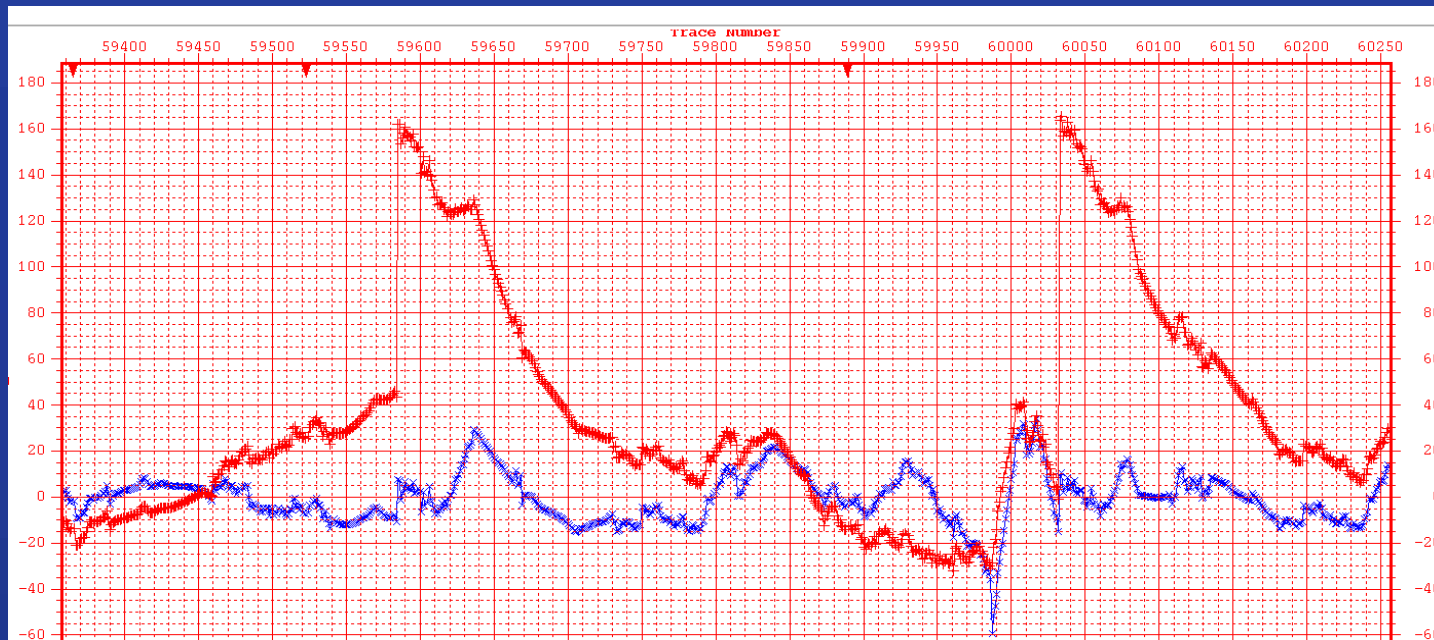
- Could use well logs.
- Maximum traveltimes & Minimum eigenvalue to consider in the inversion- Similar to the constrained damped SIRT-
cDSIRT (Zhu et al, 1992)



Low velocity layer that can make conventional refraction statics to fail

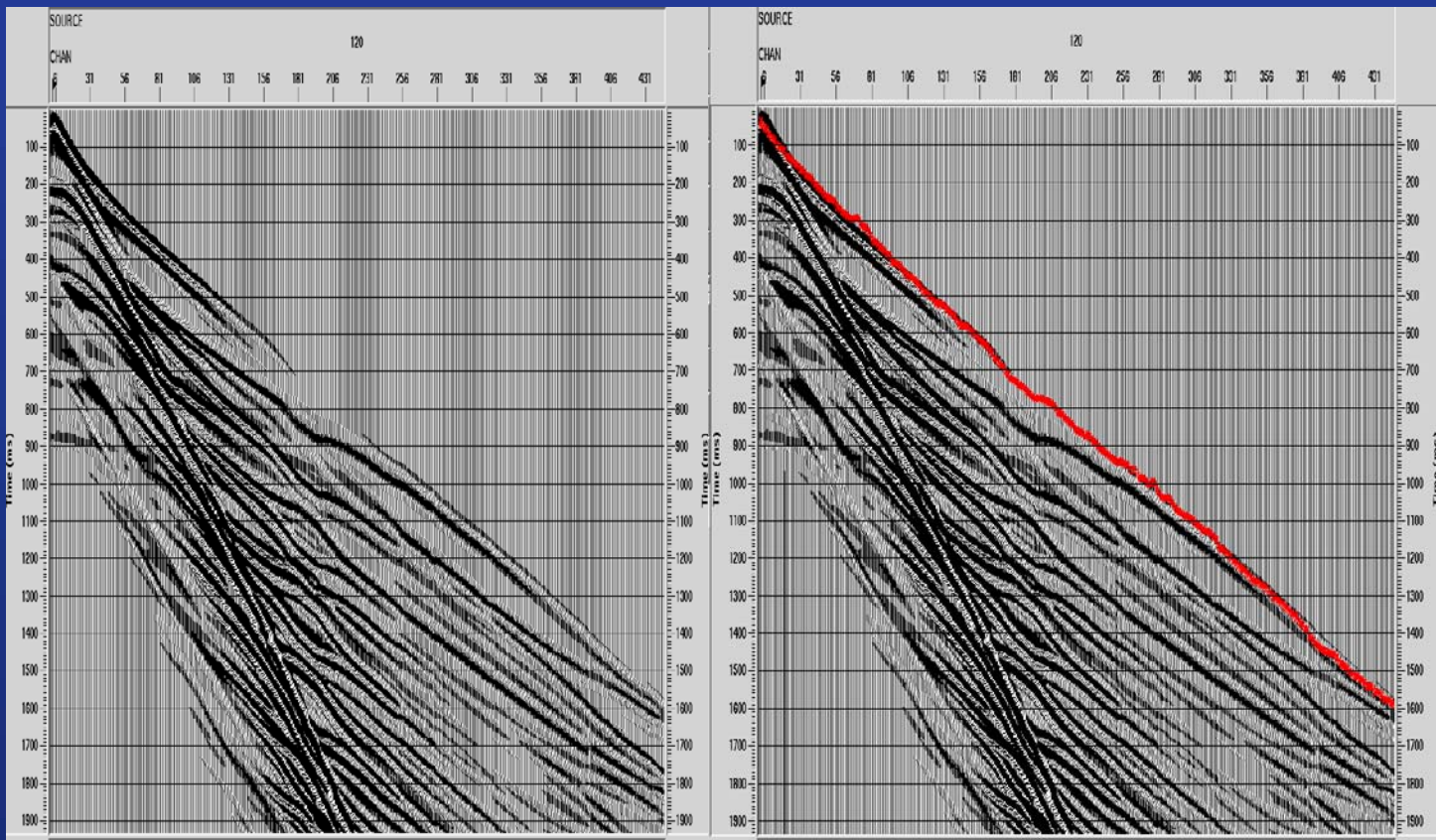


Top; ray density : bottom; velocity model after 50 iterations. Artefacts or edge effects in the red box.



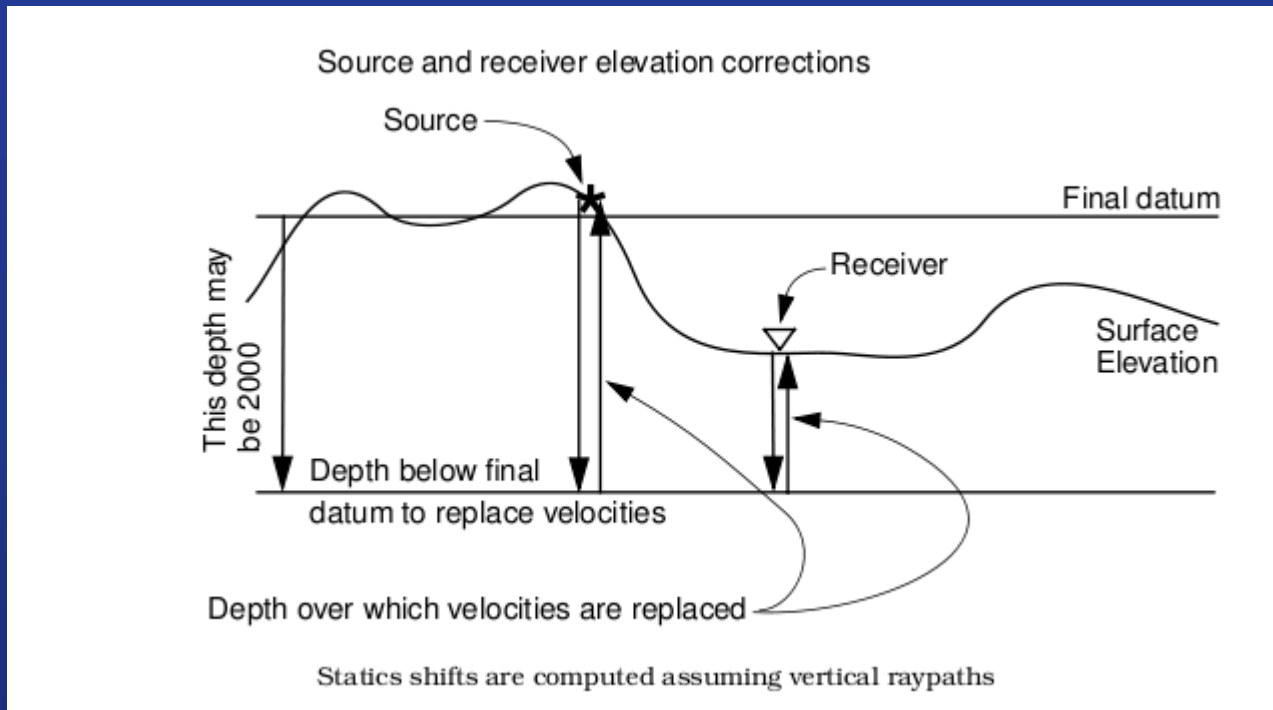
Top; cost function for shot 134 after the 1st (red) and 50th (blue) iteration : bottom last shot (269)

Synthetic data was generated using Acoustic finite difference method with a minimum phase Ricker wavelet of 25 hertz centre frequency. We superimposed the observed traveltime picks on the synthetic data.



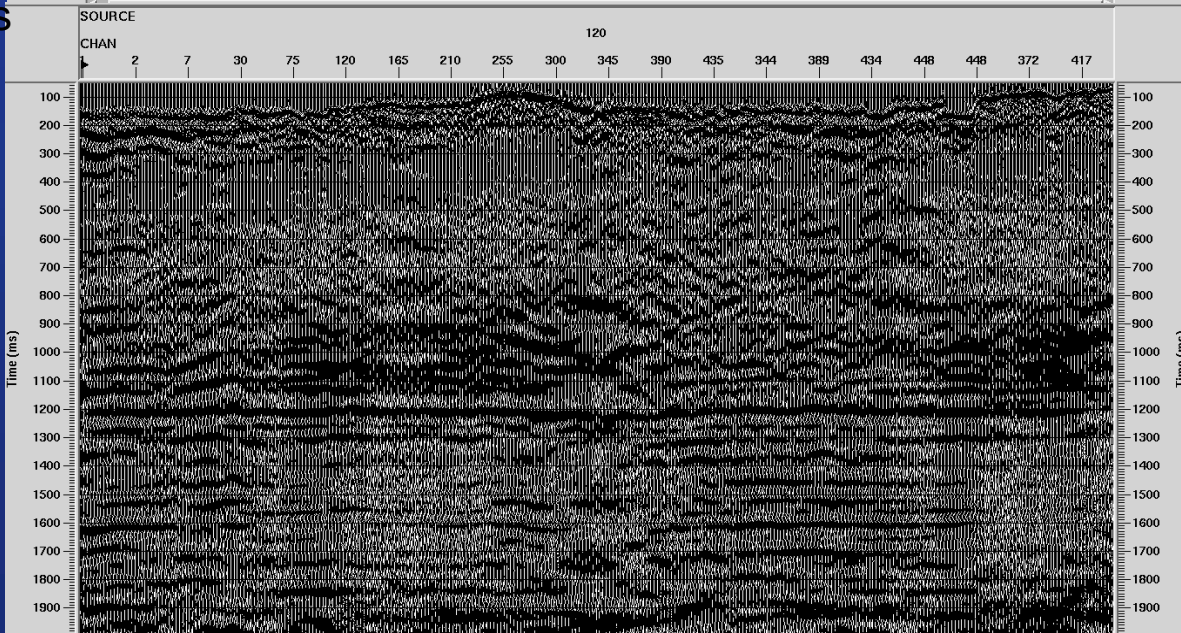
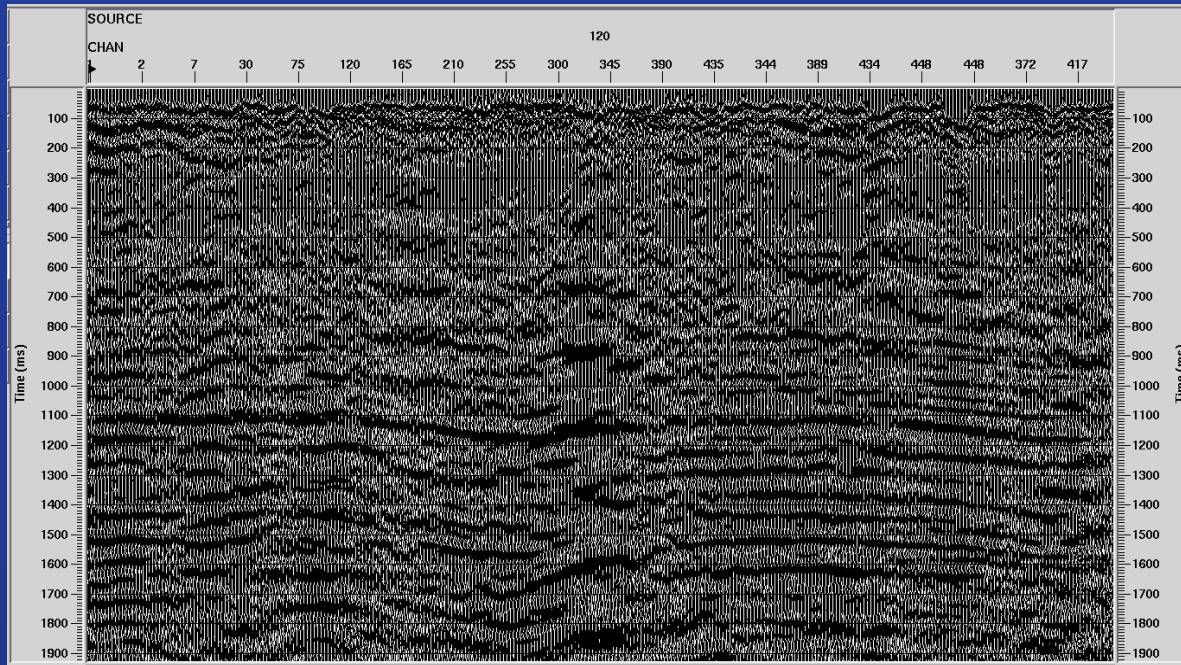
Are those the firstbreaks from the synthetic data?

We solved the statics problem using conventional refraction statics and compared with using tomostatics. The stacked sections are shown on the next slide



Promax processing guide, 1997

Results



Conclusions

- Turning-ray tomography is a viable technique for use in statics correction especially in areas where conventional refraction statics fail such as the case of a hidden layer problem as we saw in the hussar velocity model. The continuity and structure of the event at 1200ms has been improved. If we have very large offsets, then we can obtain velocity models deep enough to image shallow reservoirs
- One important advantage of TRT is that the ambiguity between reflector depth and velocity is absent.
- The constrained SIRT algorithm helps to condition the inversion and improve the result.
- The result from TRT can be used for prestack depth migration or as a starting model in FWI.

Acknowledgements

◆ Prof. Gary F. Margrave

◆ Dr. John Bancroft

◆ Landmark Graphics Corporation for Promax processing software

◆ CREWES Sponsors

◆ Helen Isaac, Rolf Maier, Dave Henley, Kevin Hall, CREWES Staff and students