Understanding the Evolving Seismic Wavelet

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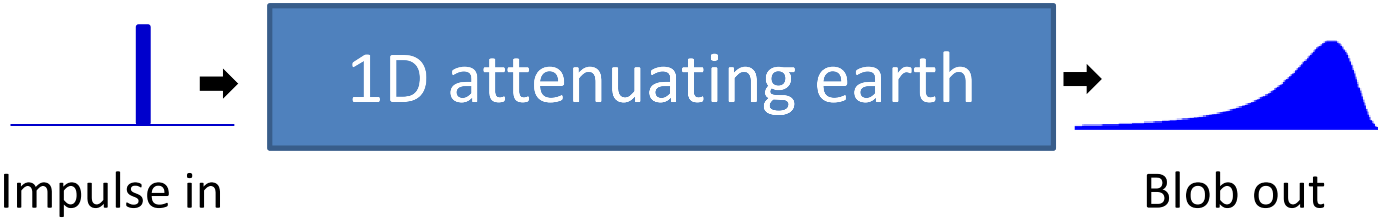
CREWES

Outline

- Anelastic attenuation based on constant Q theory
- Continuous and discrete minimum-phase wavelets
- Minimum phase and linear phase wavelets
- Conclusions

Anelastic attenuation based on constant Q theory



1D attenuating earth

Impulse in

Blob out

Amplitude

$$|W(f,x)| = |W_0(f)|\, e^{-\frac{\pi f x}{v_0 Q}}$$

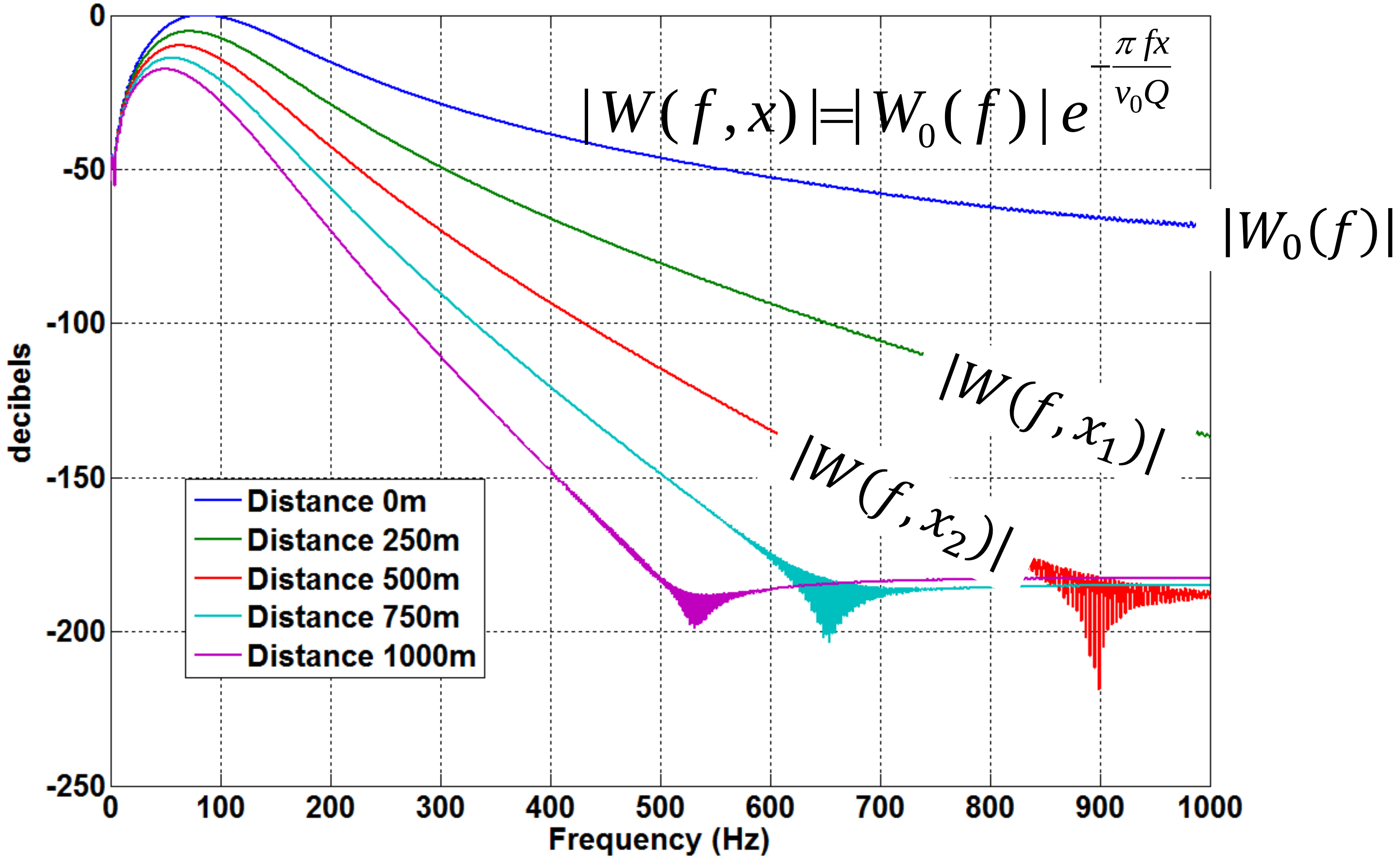
Phase

$$\varphi(f,x) = \varphi_0(f) - \frac{2\pi f x}{v(f)}$$

$$v(f) = v_0\left(1 + \frac{1}{\pi Q}\ln\frac{f}{f_0}\right)$$

Kjartanssen (1979, Geophysics)

Spectral evolution



$$|W(f,x)| = |W_0(f)|\, e^{-\frac{\pi f x}{v_0 Q}}$$

$|W_0(f)|$

$|W(f,x_1)|$

$|W(f,x_2)|$

Figure curtesy of G.F. Margrave

Anelastic attenuation based on constant Q theory

1D attenuating earth

Impulse in

Blob out

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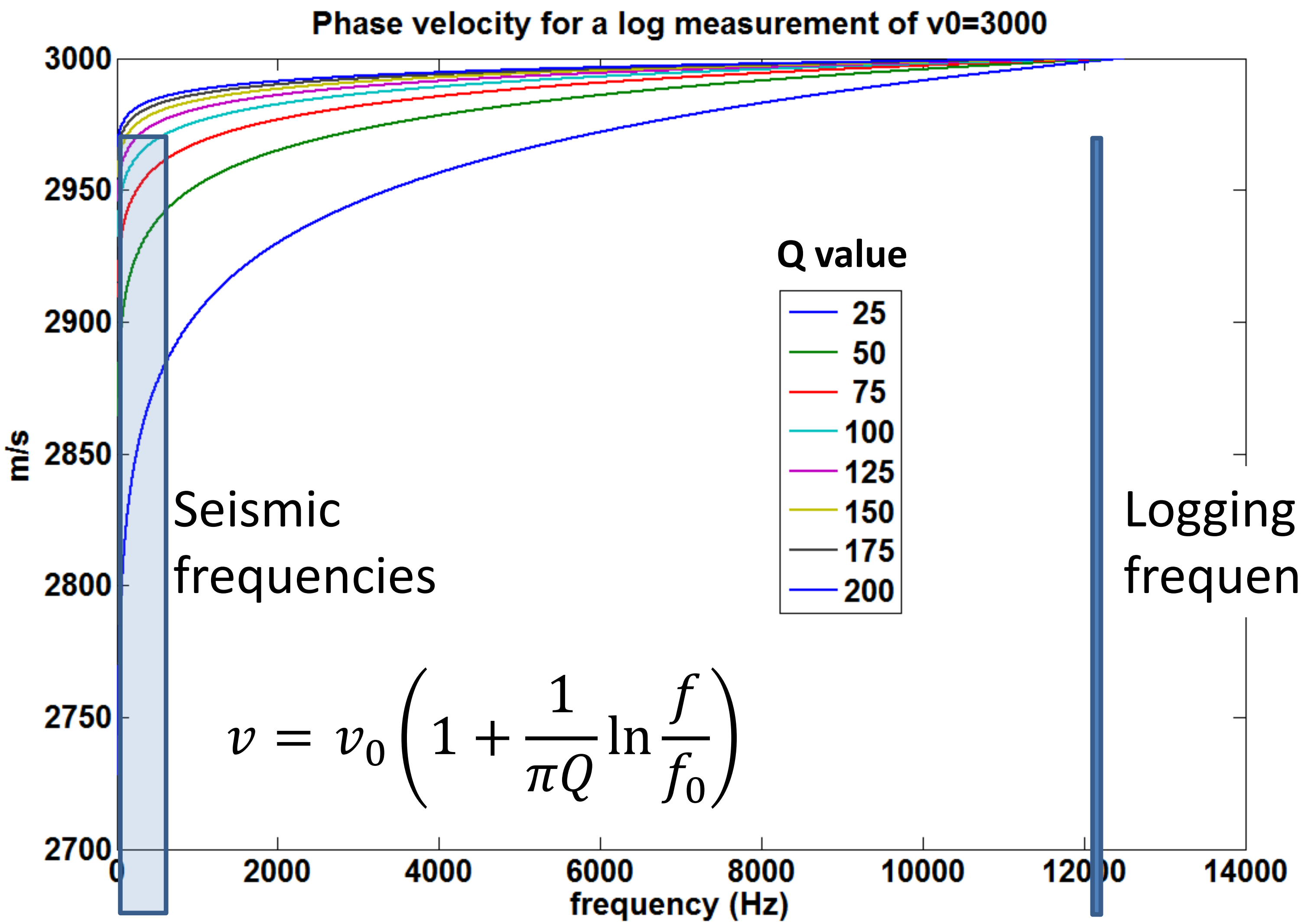
Kjartanssen (1979, Geophysics)

Frequency dependence of velocity



Seismic frequencies

Logging frequencies

$$v = v_0 \left(1 + \frac{1}{\pi Q} \ln \frac{f}{f_0} \right)$$

Figure curtesy of G.F. Margrave

Anelastic attenuation based on constant Q theory

Impulse in

1D attenuating earth

Blob out

Amplitude

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Phase

$$\varphi(f,x) = \varphi_0(f) - \frac{2\pi f x}{v(f)}$$

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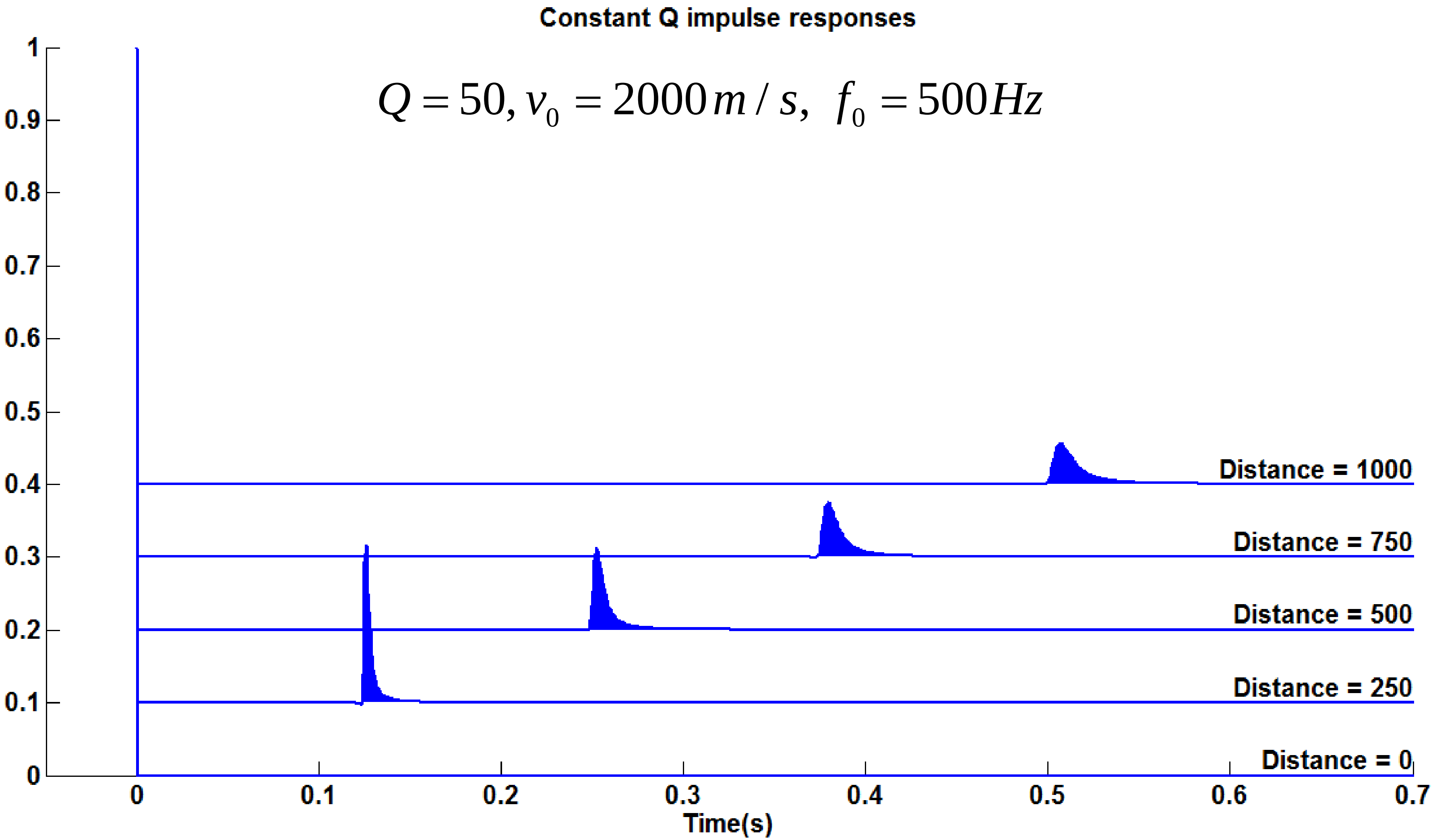
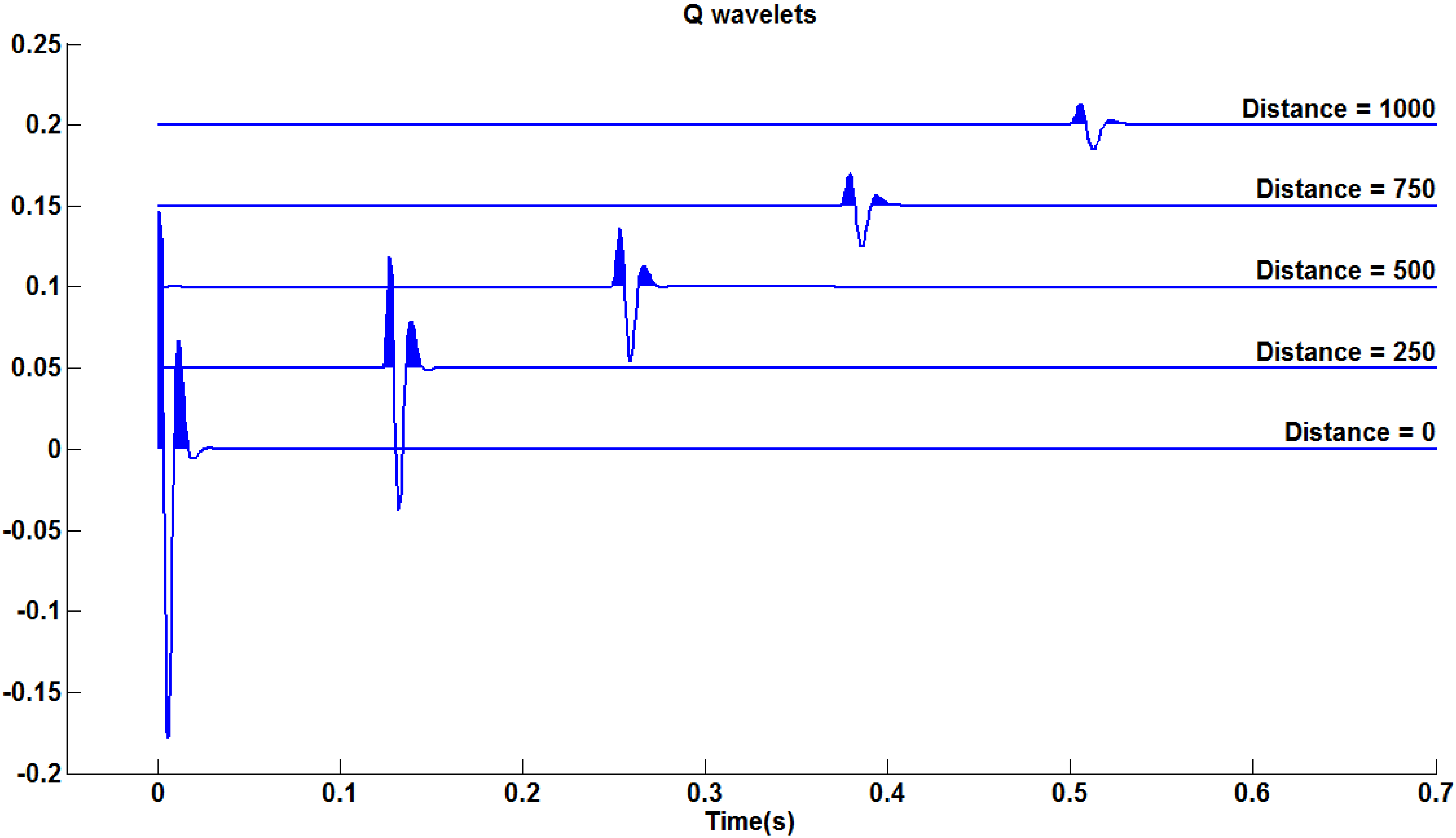
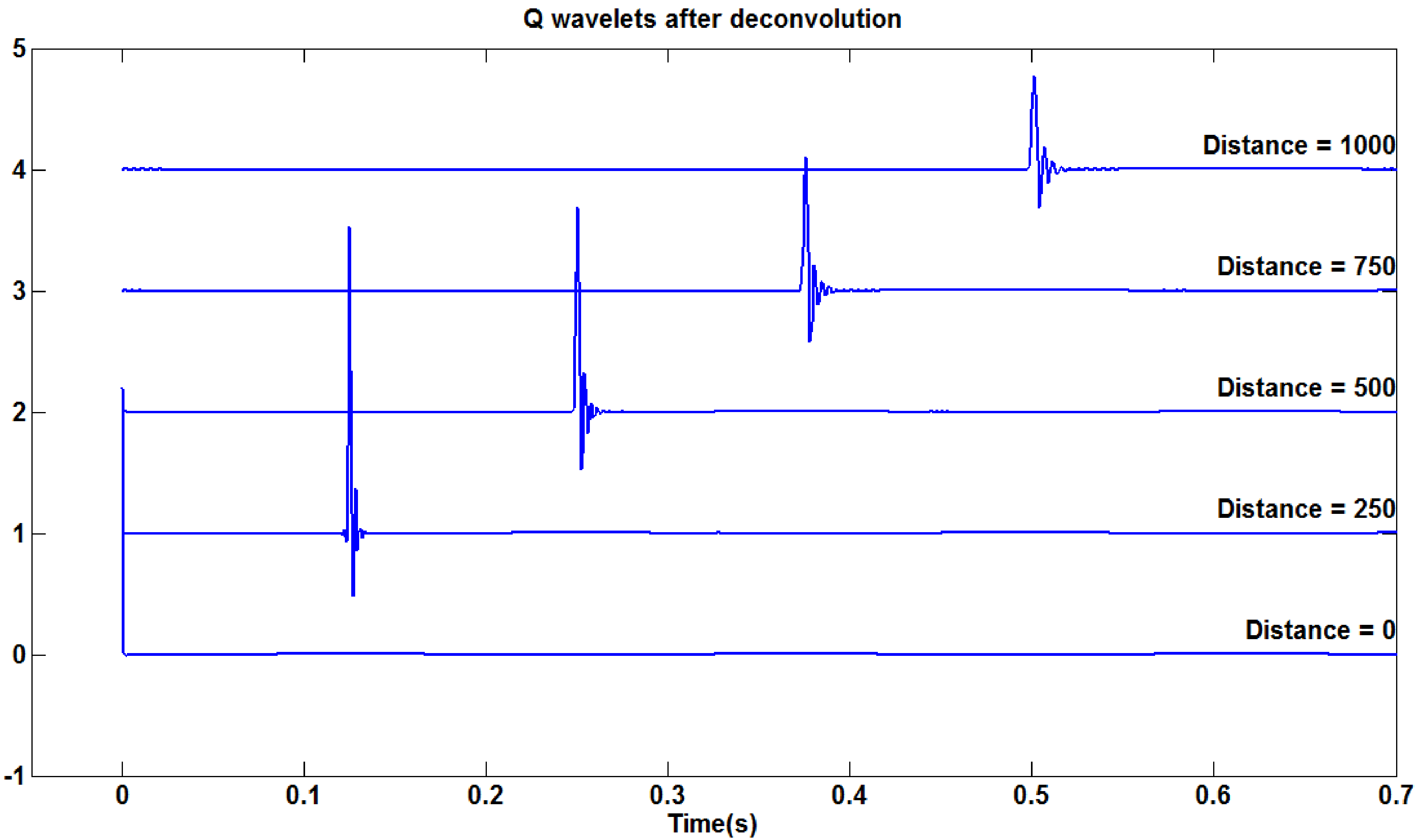
Kjartanssen (1979, Geophysics)



Constant Q impulse responses

$$Q = 50, v_0 = 2000\,m/s,\ \ f_0 = 500Hz$$


Q wavelets
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time(s)
Distance = 1000
Distance = 750
Distance = 500
Distance = 250
Distance = 0



Q wavelets after deconvolution
5
4
3
2
1
0
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time(s)
Distance = 1000
Distance = 750
Distance = 500
Distance = 250
Distance = 0

Continuous and discrete minimum-phase wavelets

Q wavelets are based on physical model. They are **continuous** minimum-phase wavelets

Deconvolution is designed for **discrete** minimum-phase wavelets

Discrete minimum-phase wavelet

A causal wavelet with a stable causal inverse is a minimum phase wavelet

$$\begin{pmatrix} w(0) & 0 & 0 \\ w(1) & w(0) & 0 \\ w(2) & w(1) & w(0) \\ 0 & w(2) & w(1) \end{pmatrix} \begin{pmatrix} w_{inv}(0) \\ w_{inv}(1) \\ w_{inv}(2) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

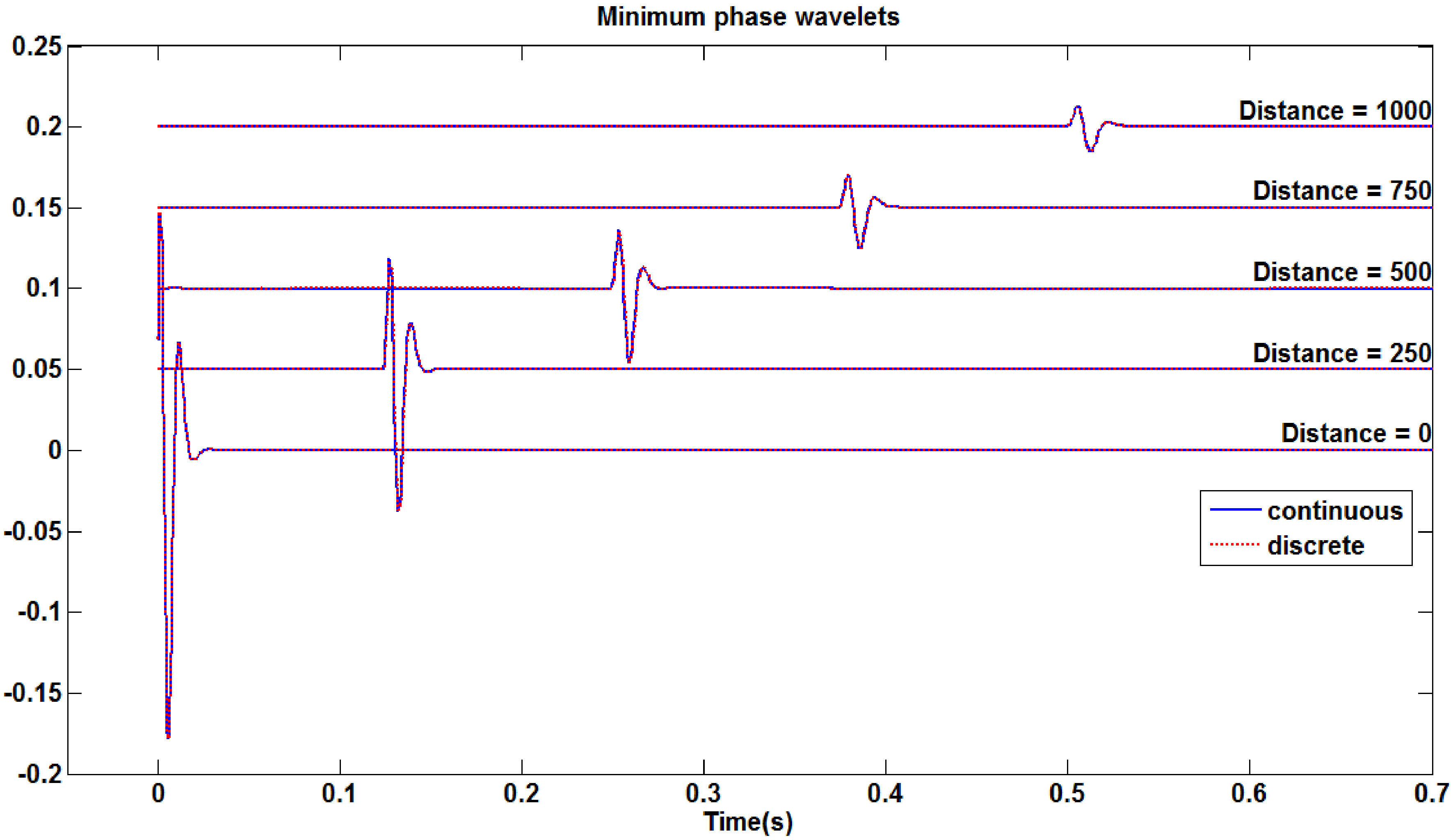
$$\begin{pmatrix} w(0) & w(1) & w(2) & 0 \\ 0 & w(0) & w(1) & w(2) \\ 0 & 0 & w(0) & w(1) \end{pmatrix} \begin{pmatrix} w(0) & 0 & 0 \\ w(1) & w(0) & 0 \\ w(2) & w(1) & w(0) \\ 0 & w(2) & w(1) \end{pmatrix} \begin{pmatrix} w_{inv}(0) \\ w_{inv}(1) \\ w_{inv}(2) \end{pmatrix} = \begin{pmatrix} w(0) & w(1) & w(2) & 0 \\ 0 & w(0) & w(1) & w(2) \\ 0 & 0 & w(0) & w(1) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

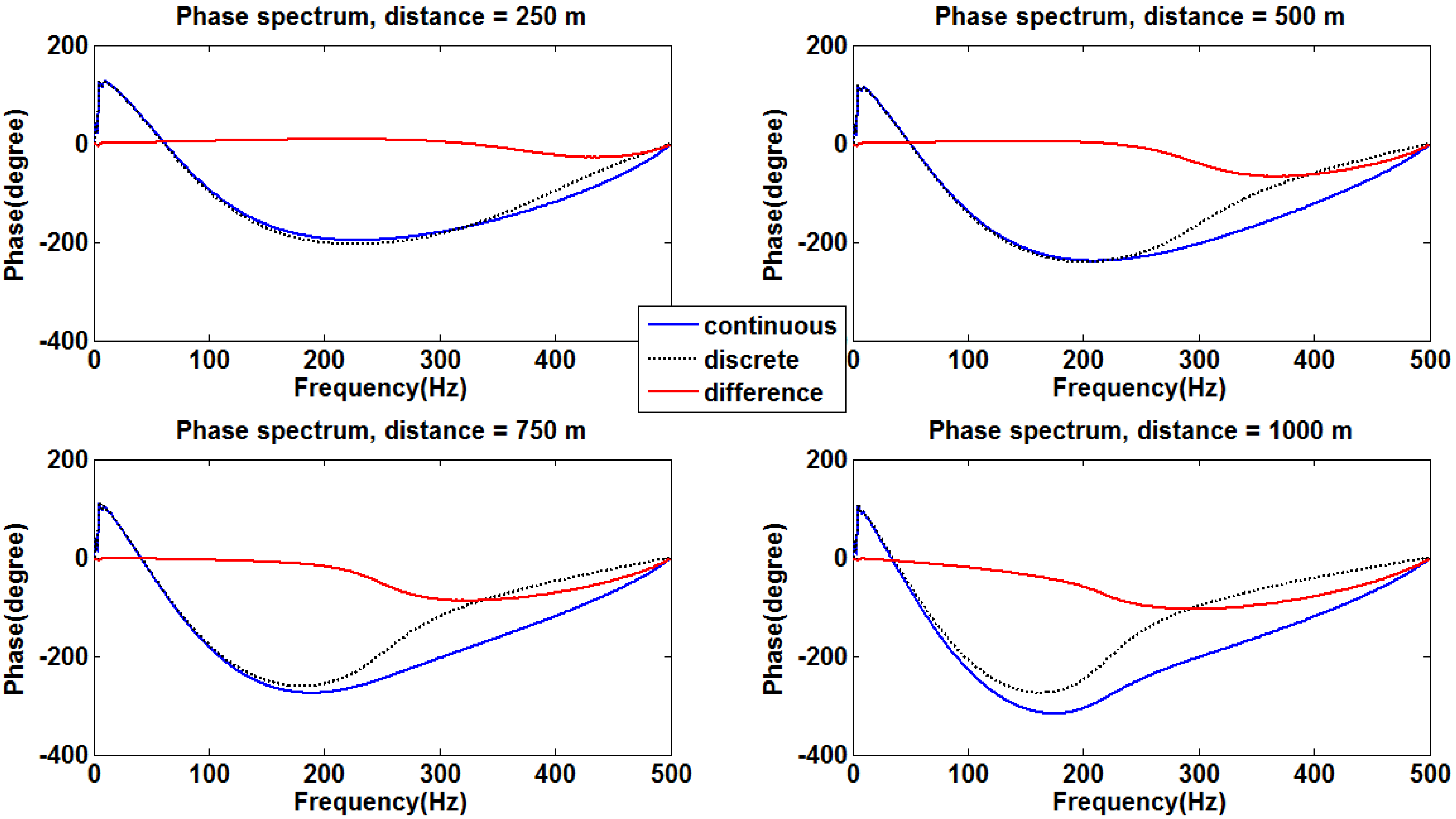
Double deconvolution method

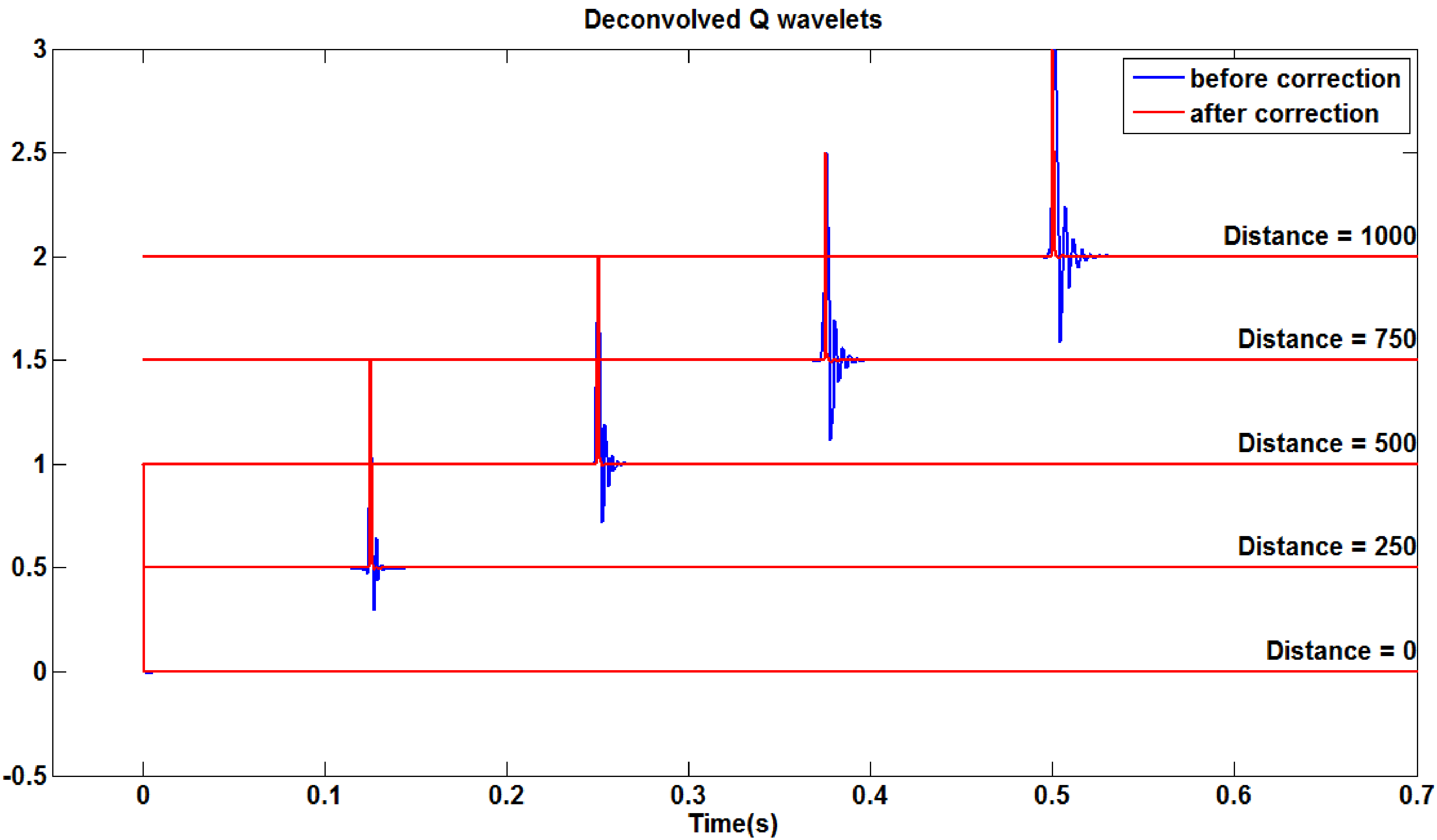
$$\begin{pmatrix} \phi_w(0) & \phi_w(1) & \phi_w(2) \\ \phi_w(1) & \phi_w(0) & \phi_w(1) \\ \phi_w(2) & \phi_w(1) & \phi_w(0) \end{pmatrix} \begin{pmatrix} w_{inv}(0) \\ w_{inv}(1) \\ w_{inv}(2) \end{pmatrix} = \begin{pmatrix} w(0) \\ 0 \\ 0 \end{pmatrix}$$

$$FT(\phi_w) = |\tilde{W}(f)|^2$$

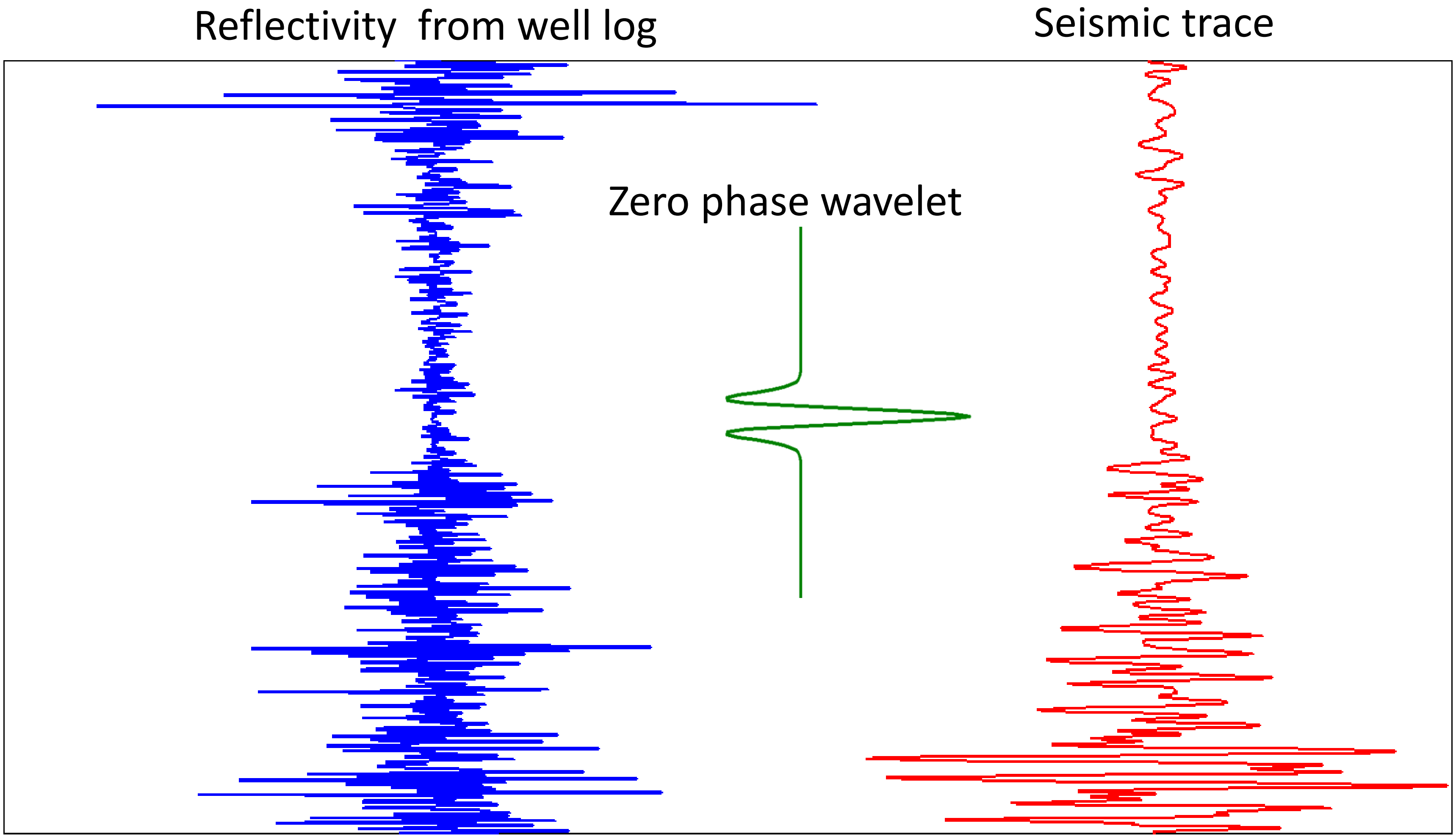
$$\begin{pmatrix} w(0) \\ w(1) \\ w(2) \end{pmatrix} = inv \begin{pmatrix} w_{inv}(0) \\ w_{inv}(1) \\ w_{inv}(2) \end{pmatrix}$$


Minimum phase wavelets
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time(s)
Distance = 1000
Distance = 750
Distance = 500
Distance = 250
Distance = 0
continuous
discrete


Phase spectrum, distance = 250 m
Phase spectrum, distance = 500 m
Phase spectrum, distance = 750 m
Phase spectrum, distance = 1000 m
Phase(degree)
Frequency(Hz)
200
0
-200
-400
0
100
200
300
400
500
continuous
discrete
difference


Deconvolved Q wavelets
before correction
after correction
Distance = 1000
Distance = 750
Distance = 500
Distance = 250
Distance = 0
3
2.5
2
1.5
1
0.5
0
-0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Time(s)

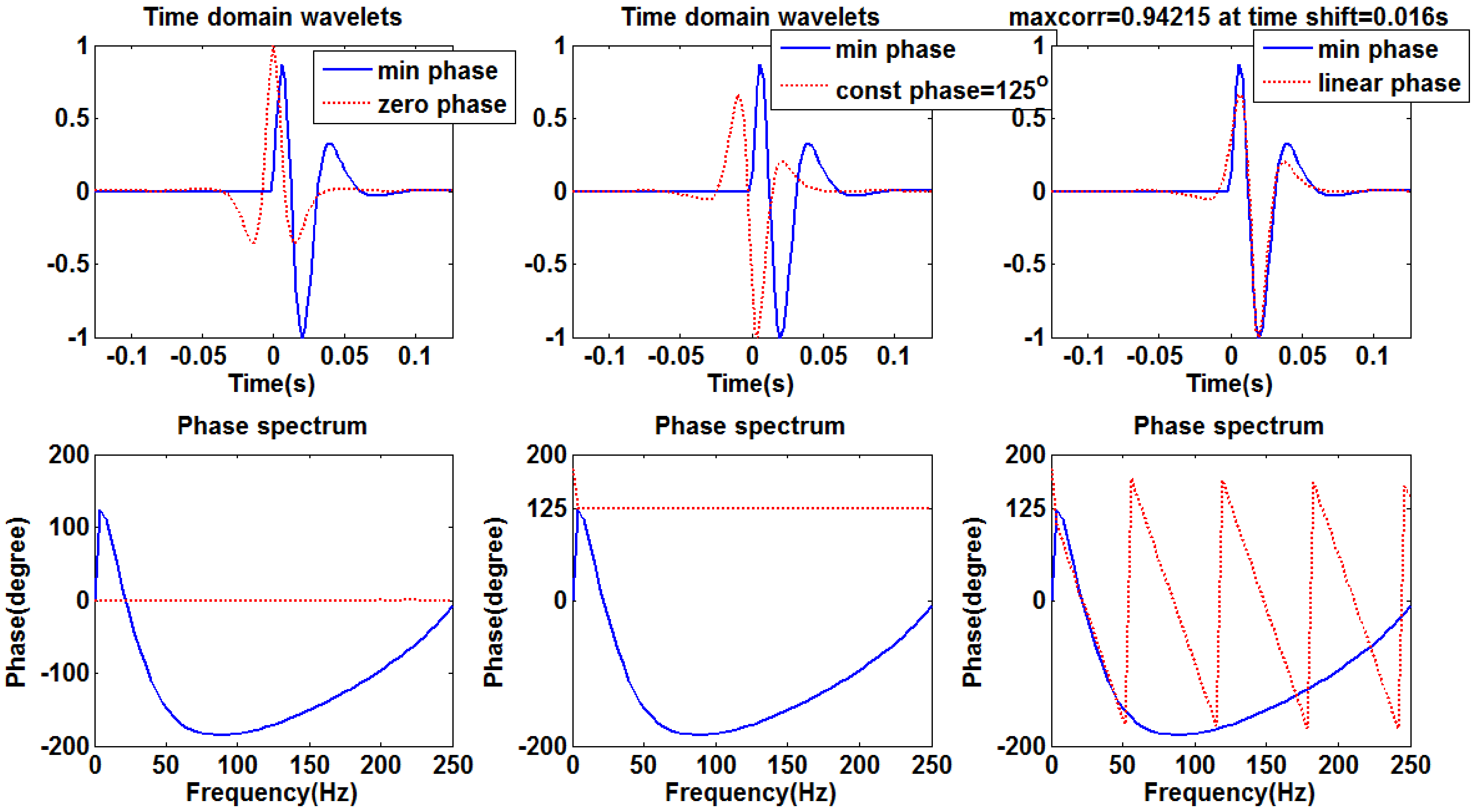
Minimum phase wavelet and linear phase wavelet

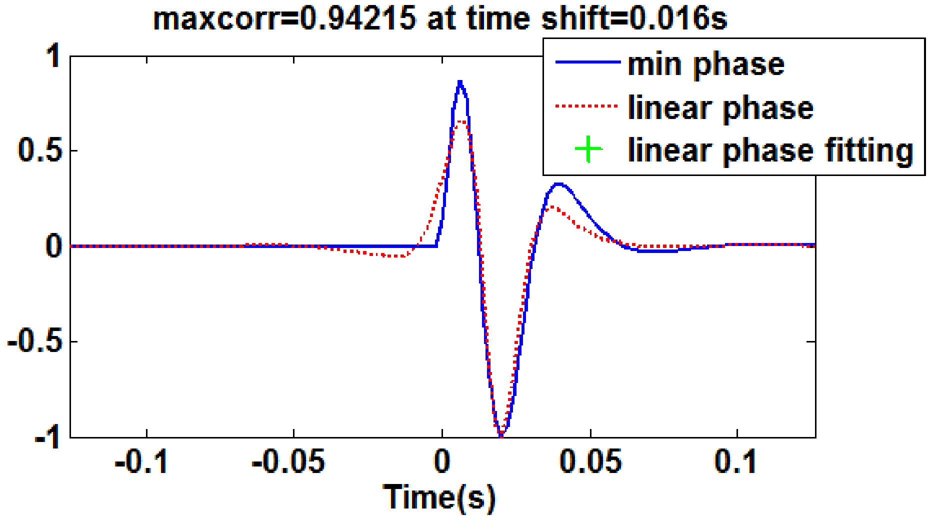


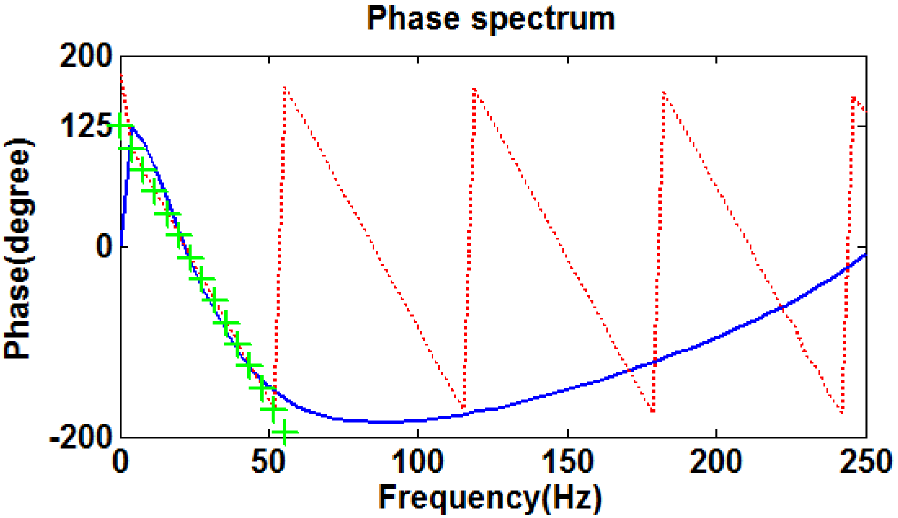
Stretch and squeeze - time shift

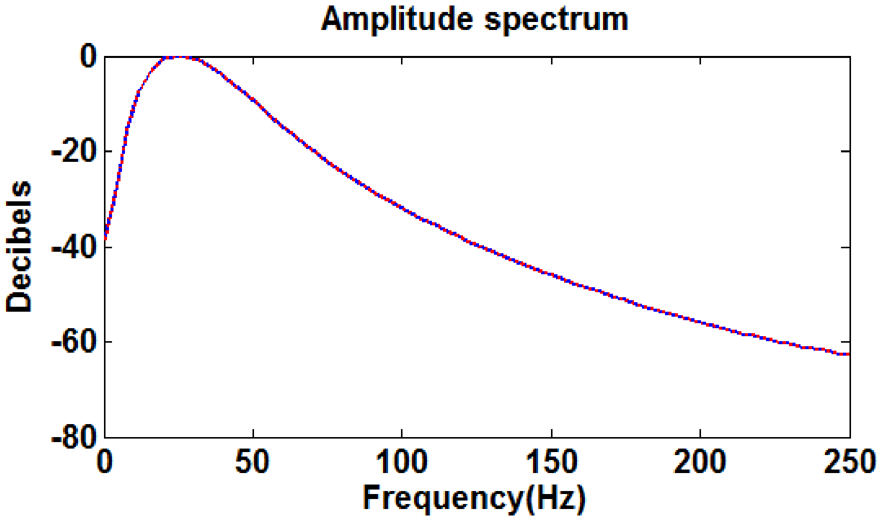
Constant phase rotation - constant phase

Finding the most similar linear phase wavelet to a minimum phase wavelet









$$spectrum = real + imag \times i$$

$$amplitude = \sqrt{real^2 + imag^2}$$

$$phase = \tan^{-1}\frac{imag}{real}$$

$$linear\ \ phase = const - 2\pi f t_0$$

$$= 125 - 2\pi f \times 0.016$$

Conclusions

- Continuous and discrete minimum-phase wavelets are different. After correcting continuous minimum-phase wavelets to discrete ones, deconvolution can get nice spikes.

- Some minimum phase wavelets can be modeled by constant phase rotation plus time shift.

Future work

- Design a time-variant operator to correct continuous minimum-phase wavelets to discrete ones
- Conduct stationary and nonstationary deconvolution on nonstationary trace； model the residual wavelets by constant phase rotation plus time shift

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Questions and Comments