Understanding and Improving Azimuthal AVO Analysis

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Azimuthal AVO

What it is:
• Analysis of incidence angle and azimuthal amplitude variations of reflection coefficients;
• Measures change in elastic properties at an interface.

Why it’s used:
• Better vertical resolution than propagation methods (e.g. VVAZ, S-wave splitting);
• Usually used to attempt to characterize natural fractures or differential stress in a reservoir.
Motivation

Want an azimuthal AVO technique that
• Works for general anisotropy (no rock physics assumptions in the initial stage);
• Has no ambiguities (solutions are unique);
• Is easy to understand.
Isotropic AVO

Shuey (1985) wrote the linearized weak contrast PP reflection coefficient in the form

\[ R_{PP}^{iso}(\theta) = A + B \sin^2 \theta + C \tan^2 \theta \sin^2 \theta \]

And Thomsen (1990) used this form to describe the effect of common seismic parameters \( V_P, V_S, \rho, \) and \( \mu \) on the reflection coefficient:

\[ R_{PP}^{iso}(\theta) = \frac{1}{2} \left[ \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\rho} \right] + \frac{1}{2} \left[ \frac{\Delta V_P}{V_P} - \left( \frac{2 V_S}{V_P} \right) \frac{\Delta \mu}{\mu} \right] \sin^2 \theta + \frac{1}{2} \left( \frac{\Delta V_P}{V_P} \right) \tan^2 \theta \sin^2 \theta \]
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$$R_{PP}^{iso}(\theta) = \frac{1}{2} \left[ \frac{\Delta V_P}{V_P} + \frac{\Delta \rho}{\bar{\rho}} \right] + \frac{1}{2} \left[ \frac{\Delta V_P}{V_P} - \left( \frac{2 \overline{V}_S}{V_P} \right) \frac{\Delta \mu}{\bar{\mu}} \right] \sin^2 \theta + \frac{1}{2} \frac{\Delta V_P}{V_P} \tan^2 \theta \sin^2 \theta$$
Anisotropic Reflection Coefficient

Thomsen (1993) derived the linearized PP reflection coefficient for small contrast weak anisotropy along a vertical plane:

\[
R_{PP}(\theta) = \frac{1}{2} \left[ \frac{\Delta Z_0}{Z_0} \right] \\
+ \frac{1}{2} \left[ \frac{\Delta V_{P0}}{V_{P0}} - \left( \frac{2V_{S0}}{V_{P0}} \right) \frac{\Delta \mu_0}{\mu_0} + (\delta_2 - \delta_1) \right] \sin^2 \theta \\
+ \frac{1}{2} \left[ \frac{\Delta V_{P0}}{V_{P0}} - (\delta_2 - \delta_1 - \epsilon_2 + \epsilon_1) \right] \tan^2 \theta \sin^2 \theta
\]

as did Vavryuk and Psencík (1998) using perturbations from background P-wave velocities, \( \alpha \), and S-wave velocities \( \beta \):

\[
R_{PP}(\theta_P) = \frac{\rho \Delta A'_{33} + 2\alpha^2 \Delta \rho}{4\rho \alpha^2} \\
+ \frac{1}{2} \left[ \frac{\Delta A'_{33}}{2\alpha^2} - \frac{4(\rho \Delta A'_{55} + \beta^2 \Delta \rho)}{\rho \alpha^2} + \Delta \delta^{**} \right] \sin^2 \theta_P \\
+ \frac{1}{2} \left( \frac{\Delta A'_{33}}{2\alpha^2} + \Delta \epsilon^{**} \right) \sin^2 \theta_P \tan^2 \theta_P
\]

Where

\[
\delta^{*(I)} = \frac{A_{13}^{(I)} + 2A_{55}^{(I)} - A_{33}^{(I)}}{\alpha^2}, \epsilon^{*(I)} = \frac{A_{11}^{(I)} - A_{33}^{(I)}}{2\alpha^2}
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+ \frac{1}{2} \left( \frac{\Delta A'_{33}}{2\alpha^2} + \Delta \epsilon^* \right) \sin^2 \theta_P \tan^2 \theta_P 

Where

\[ \delta^*(I) = \frac{A_{13}^{(I)} + 2A_{55}^{(I)} - A_{33}^{(I)}}{\alpha^2} \]

\[ \epsilon^*(I) = \frac{A_{11}^{(I)} - A_{33}^{(I)}}{2\alpha^2} \]
Curvature Term

Substituting $\Delta \varepsilon^*$ into the equation for a reflection coefficient from Vavryuk and Psencík (1998) results in a curvature of

$$\frac{1}{2} \left( \frac{\Delta A'_{33}}{2\alpha^2} + \frac{\Delta A'_{11} - \Delta A'_{33}}{2\alpha^2} \right) = \frac{\Delta A'_{11}}{4\alpha^2} \approx \frac{1}{2} \frac{\Delta V'_{P_H}}{V'_{P_H}}$$

This term
- Is only influenced by a single stiffness coefficient
- Easily relates to the isotropic AVO equations
- Is simple to understand
Curvature compared to Ruger’s Equation

The coordinate transformation for $A'_{11}$ along an arbitrary vertical plane at an angle $\phi$ from the original plane is

$$A'_{11} = A_{11} \cos^4 \phi + 4A_{16} \cos^3 \phi \sin \phi + 2(A_{12} + 2A_{66}) \cos^2 \phi \sin^2 \phi + 4A_{26} \sin^3 \phi \cos \phi + A_{22} \sin^4 \phi$$

Ruger’s curvature is

$$\frac{1}{2} \left[ \frac{\Delta \alpha}{\bar{\alpha}} + \Delta \epsilon^{(V)} \cos^4 \phi + \Delta \delta^{(V)} \sin^2 \phi \cos^2 \phi \right]$$

and inserting the weak anisotropy parameters and assuming $\alpha^2 = A_{33}$ it becomes

$$\frac{1}{2} \left[ \left( \frac{\Delta A_{33}}{2\alpha^2} \right) + \left( \frac{\Delta A_{11} - \Delta A_{33}}{2\alpha^2} \right) \cos^4 \phi + \left( \frac{\Delta A_{13} + 2\Delta A_{55} - \Delta A_{33}}{\alpha^2} \right) \sin^2 \phi \cos^2 \phi \right] =$$

$$\frac{1}{4\alpha^2} \left[ \Delta A_{11} \cos^4 \phi + 2(\Delta A_{13} + 2\Delta A_{55}) \cos^2 \phi \sin^2 \phi + \Delta A_{33} \sin^4 \phi \right]$$
Thomsen (1993) derived the linearized PP reflection coefficient for small contrast weak anisotropy along a vertical plane:

\[ R_{PP}(\theta) = \frac{1}{2} \left[ \frac{\Delta Z_0}{Z_0} \right] \]

\[ + \frac{1}{2} \left[ \frac{\Delta V_{P_0}}{V_{P_0}} - \left( \frac{2V_{S_0}}{V_{P_0}} \right) \frac{\Delta \mu_0}{\mu_0} + (\delta_2 - \delta_1) \right] \sin^2 \theta \]

\[ + \frac{1}{2} \left[ \frac{\Delta V_{P_0}}{V_{P_0}} - (\delta_2 - \delta_1 - \epsilon_2 + \epsilon_1) \right] \tan^2 \theta \sin^2 \theta \]

as did Vavryuk and Psencík (1998) using perturbations from background P-wave velocities, \( \alpha \), and S-wave velocities \( \beta \):

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\[ + \frac{1}{2} \left( \frac{\Delta A'_{33}}{2\alpha^2} + \Delta \epsilon^{**} \right) \sin^2 \theta_P \tan^2 \theta_P \]

Where

\[ \delta^{*(I)} = \frac{A_{13}^{(I)}}{\alpha^2} + 2A_{55}^{(I)} - A_{33}^{(I)} \]

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\[ + \frac{1}{2} \left( \frac{\Delta A'_{33}}{2\alpha^2} + \Delta \epsilon^{*'} \right) \sin^2 \theta_P \tan^2 \theta_P \]

Where

\[ \delta^{*(I)}' = \frac{A_{13}' + 2A_{55}' - A_{33}'}{\alpha^2} \]

\[ \epsilon^{*(I)}' = \frac{A_{11}' - A_{33}'}{2\alpha^2} \]
Gradient along a plane

Substituting the weak anisotropy parameters into Vavrycuk and Psencik’s gradient along a vertical plane results in

$$\frac{1}{2\alpha^2} \left[ -2\Delta A'_{55} - 4\beta^2 \frac{\Delta \rho}{\rho} + \Delta A'_{13} - \frac{\Delta A'_{33}}{2} \right]$$

The only terms in this gradient that change with azimuth are $\Delta A'_{55}$ and $\Delta A'_{13}$ under the following relations

$$A'_{55} = A_{55} \cos^2 \phi + 2A_{45} \cos \phi \sin \phi + A_{44} \sin^2 \phi,$$
$$A'_{13} = A_{13} \cos^2 \phi + 2A_{36} \cos \phi \sin \phi + A_{23} \sin^2 \phi.$$
90-degree Ambiguity

Ruger writes the gradient as

\[ B(\phi_k) = B^{iso} + B^{ani} \cos^2(\phi_k - \phi_{sym}) \]

which has 3 variables and is nonunique with 2 solutions.

Azimuthal AVO gradient (blue) with two possible solutions (green and red). It is unclear if minima or maxima correspond to the symmetry axis.
Ambiguity as Minima/Maxima

The azimuthal gradient is a combination of gradients along individual vertical planes so its minima and maxima are minima and maxima of the quantity

\[-2\Delta A'_{55} + \Delta A'_{13}\]

Change in anisotropic gradient with Vs/Vp ratio for aligned vertical dry (dashed line) and wet (solid line) fractures. After Bakulin et al. (2000).
Here is an example of an ambiguity that would be caused if $B_{ani}$ had a measured magnitude of 0.5. Values of 0.5 and -0.5 are both possibilities.

Change in anisotropic gradient with $V_s/V_p$ ratio for aligned vertical dry (dashed line) and wet (solid line) fractures. After Bakulin et al. (2000).
Ways to constrain gradient

• Knowledge of liquid presence or lack of liquid presence + Vp/Vs ratio
• Approximate fracture direction from another method or from geology
• Azimuthal change in critical angle
• Azimuthal AVO curvature
Faranak’s data

Faranak collected and processed azimuthal reflection data using a model with the following approximate stiffnesses:

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>$8.70 \pm 0.49$</td>
</tr>
<tr>
<td>27°</td>
<td>$4.68 \pm 0.21$</td>
</tr>
<tr>
<td>45°</td>
<td>$5.07 \pm 0.21$</td>
</tr>
<tr>
<td>63°</td>
<td>$13.25 \pm 0.49$</td>
</tr>
<tr>
<td>90°</td>
<td>$5.13 \pm 0.23$</td>
</tr>
<tr>
<td></td>
<td>$12.25 \pm 0.49$</td>
</tr>
<tr>
<td></td>
<td>$2.89 \pm 0.12$</td>
</tr>
<tr>
<td></td>
<td>$2.34 \pm 0.12$</td>
</tr>
<tr>
<td></td>
<td>$2.28 \pm 0.12$</td>
</tr>
</tbody>
</table>

Azimuthal reflection data from Mahmoudian (2013).
Intercept, Gradient, and Curvature

- Intercept, gradient, and curvature were estimated.
- Gradient is more negative at 90 degrees but this is a combination of \(-2\Delta A'_{55}\) and \(\Delta A'_{13}\) and does not indicate fast and slow directions.

Azimuthal variations in the AVO intercept, gradient, and curvature for the physical modeling dataset.
Fitting the curvature resulted in finding 0° and 90° as the azimuths having the biggest separation between $\Delta A_{11}$ and $\Delta A_{22}$.

This allows us to determine that 0° is the slow direction.

Best Fit of the coordinate transformation of $\Delta A_{11}$ to the curvature.
Need for 2\textsuperscript{nd} order reflection coefficient

Higher order terms cause error
Higher order reflection coefficient

\[ \frac{1}{4} (\delta a_{33} + 2 \delta \rho) + \]
\[ \left( C_1 \text{psqr} \delta a_{13} - C_2 \text{psqr} \delta a_{33} + \frac{1}{8} \delta a_{33}^2 - C_3 \text{psqr} \delta a_{55} - C_4 \text{psqr} \delta \rho + \frac{1}{4} \delta \rho^2 \right) + \]
\[ \left( \frac{5}{64} \delta a_{33}^3 + C_5 \text{psqr}^2 \delta \rho + C_6 \text{psqr} \delta \rho^2 + \frac{1}{8} \delta \rho^3 + C_7 \text{psqr} \delta a_{13}^2 + \right. \]
\[ C_8 \text{psqr} \delta a_{55}^2 + C_9 \text{psqr} \delta a_{33}^2 - \frac{1}{32} \delta \rho \delta a_{33}^2 + C_{10} \text{psqr}^2 \delta a_{11} + \]
\[ C_{11} \text{psqr}^2 \delta a_{55} + C_{12} \text{psqr} \delta \rho \delta a_{55} - C_{13} \text{psqr}^2 \delta a_{13} + C_{14} \text{psqr} \delta \rho \delta a_{13} + \]
\[ C_{15} \text{psqr} \delta a_{13} \delta a_{33} + C_{16} \text{psqr}^2 \delta a_{33} + C_{17} \delta a_{33} \text{psqr} \delta \rho - \frac{1}{16} \delta \rho^2 \delta a_{33} \right) \]

- C’s are coefficients containing top layer quantities \( a_{11}^{(1)}, a_{13}^{(1)}, a_{33}^{(1)}, a_{55}^{(1)}, \) and \( \rho^{(1)} \)

- Small contrasts:
  \[ \delta x = 1 - \frac{x^{(1)}}{x^{(2)}} \]
Higher order in theta?

- 4\textsuperscript{th} order (and possibly higher) in $f(\theta)$ terms
- Would need something that is somewhat linearly independent from other terms so that small noise won’t effect it negatively
- Increasing number of variables makes matrix “less overdetermined”
Conclusions

• Analyzing reflections along individual azimuths allows for a simple representation of theory
• Assumptions about symmetry don’t need to be made to analyze certain subsurface elastic properties.
• The curvature is only dependent on a single elastic stiffness, while the gradient is dependent on two, leading to ambiguity in interpretation made from the gradient.
• Curvature can be used to determine azimuthal changes in horizontal P-velocity and can be a tool to constrain anisotropy orientation estimates from the gradient.
• Higher order terms are very important for azimuthal AVO but are also very complicated. Use exact formulas?
Possible Future Work

• Analyze curvature in real datasets;
• Calculate and understand higher order terms;
• Incorporate attenuation anisotropy to determine if cracks contain liquid;
• Analyze azimuthal change in critical angles;
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References


