Towards seismic moment tensor inversion for source mechanism

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Outline

- Basic definitions of seismic moment tensor (MT)
- A linear inversion the amplitude of P- and S-wave for MT
- Proper acquisition geometry to recover full MT
- A linear waveform inversion for S(t) and MT
- Synthetics microseismic data generated using TIGER
- Conclusions
- Acknowledgments

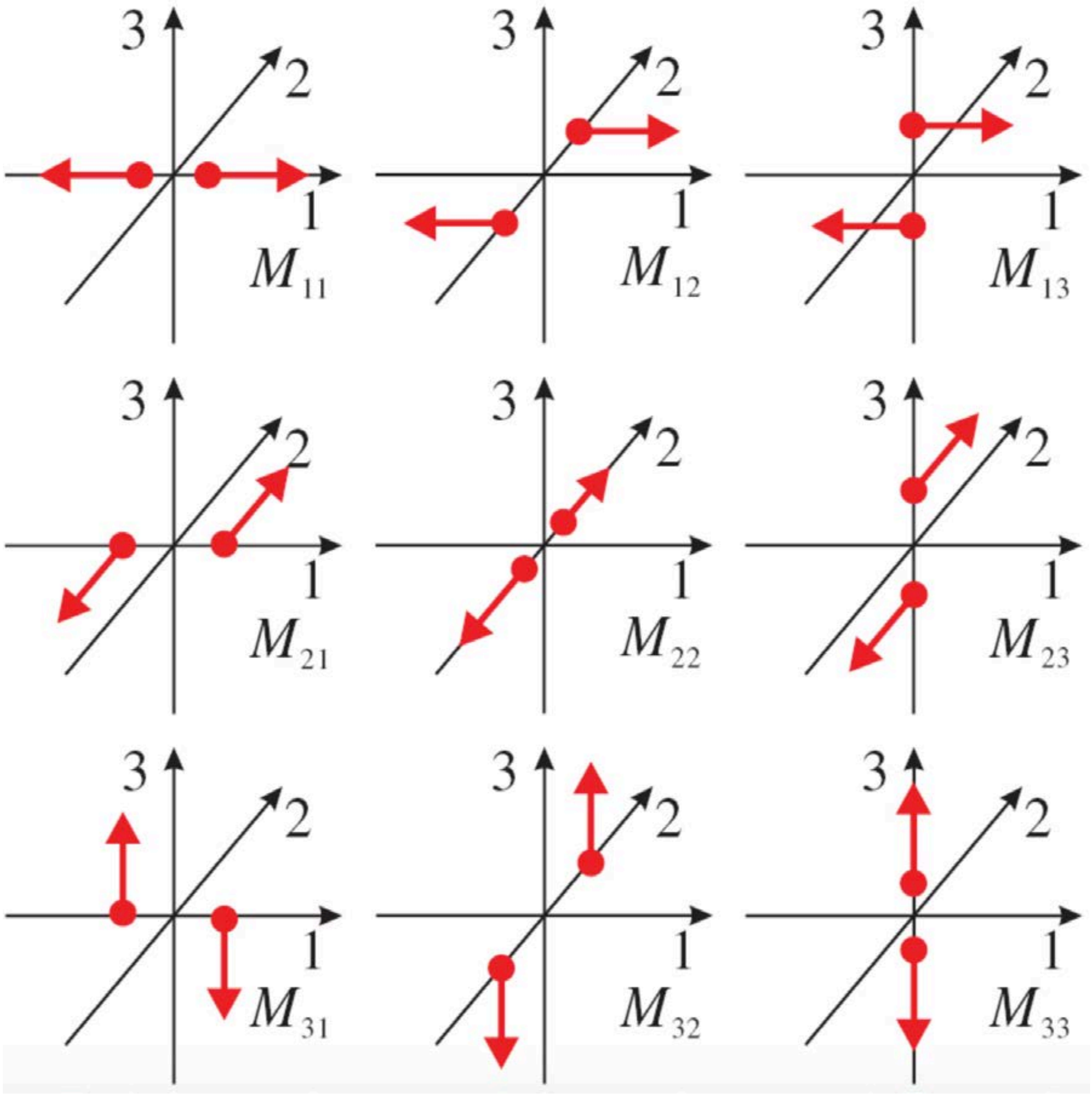
Definitions

- Representation Theorem for seismic sources

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

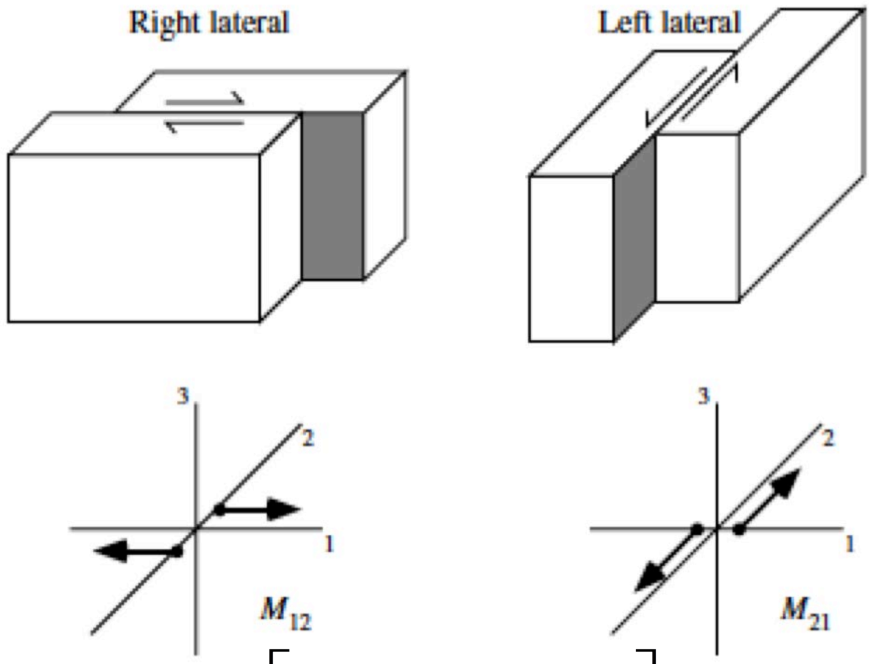
M_{pq} : force-couple

- Any source can be described by MT
- MT is symmetric ($M_{pq} = M_{qp}$)
- Six independent terms (M_{11},M_{22},M_{33},M_{23},M_{13},M_{12})



Examples

Double-couple model



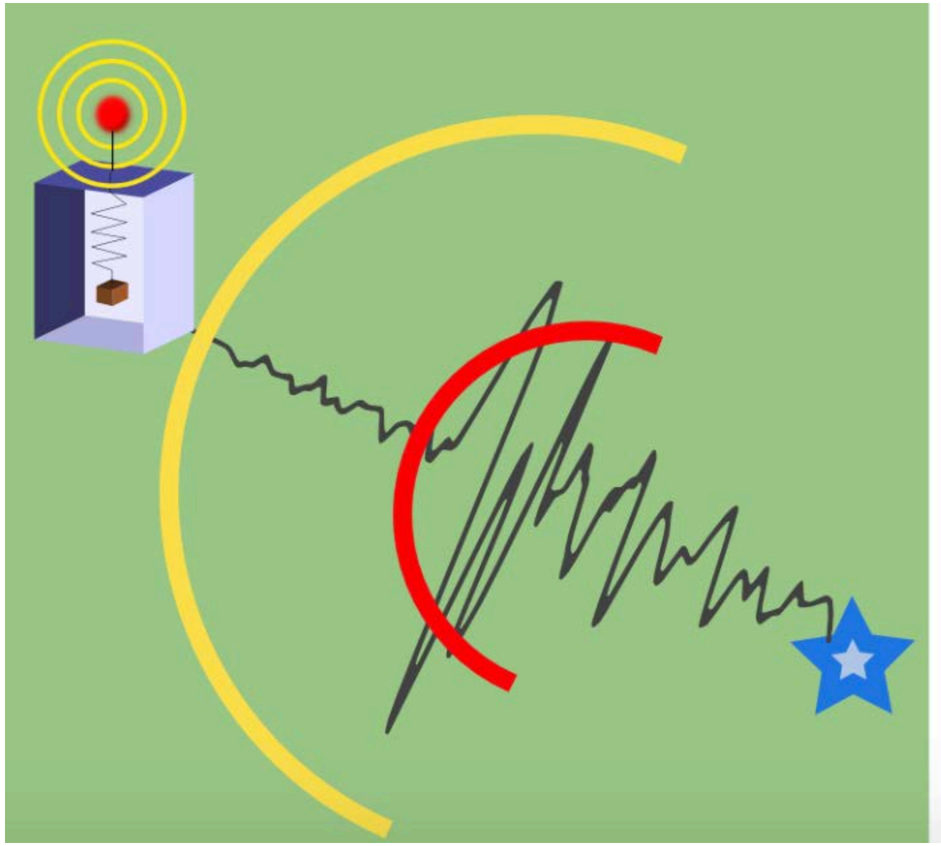
$$\begin{bmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Fundamental ambiguity: both faults has the exact same seismic displacement in the far-field

Explosive source

$$\begin{bmatrix} M_0 & 0 & 0 \\ 0 & M_0 & 0 \\ 0 & 0 & M_0 \end{bmatrix}$$

Time-dependent moment tensor



Point source:

$$m_{pq}(t) = M_{pq} S(t)$$

$$S(t) = \text{ source-time function}$$

Displacement wavefield (Aki and Richards, 2001)

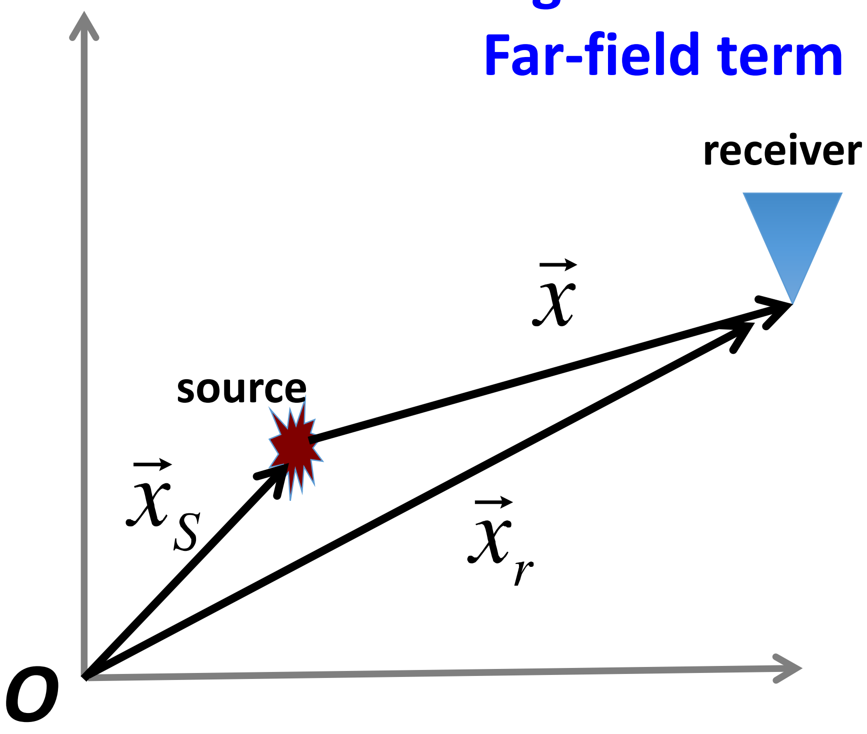
$$u_n(\vec{x},t) = g_{np,q}(\vec{x},t) * M_{pq} S(t)$$

$$n = 1,2,3$$

3C displacement wavefield observed at a geophone located at $\vec{x}_r$ due to a source at $\vec{x}_S$

Homogenous medium

Far-field term



$$g_{np,q}(\vec{x},t) = \begin{cases} \dfrac{\gamma_n \gamma_p \gamma_q}{4\pi\alpha^3 r}, & \text{for} \quad t = r/\alpha \\ \dfrac{(\delta_{np} - \gamma_n \gamma_p)\gamma_q}{4\pi\beta^3 r}, & \text{for} \quad t = r/\beta \\ 0 & \text{for other} \quad t \end{cases}$$

$$\gamma_n = \frac{(\vec{x}_r - \vec{x}_S)_n}{r}, \quad r = |\vec{x}| = |\vec{x}_r - \vec{x}_S|$$

α = P-wave velocity, β = S-wave velocity

Displacement wave-field (Aki and Richards, 2001)

$$u_n(\vec{x},t) = g_{np,q}(\vec{x},t) * M_{pq} S(t)$$

$$n = 1,2,3$$

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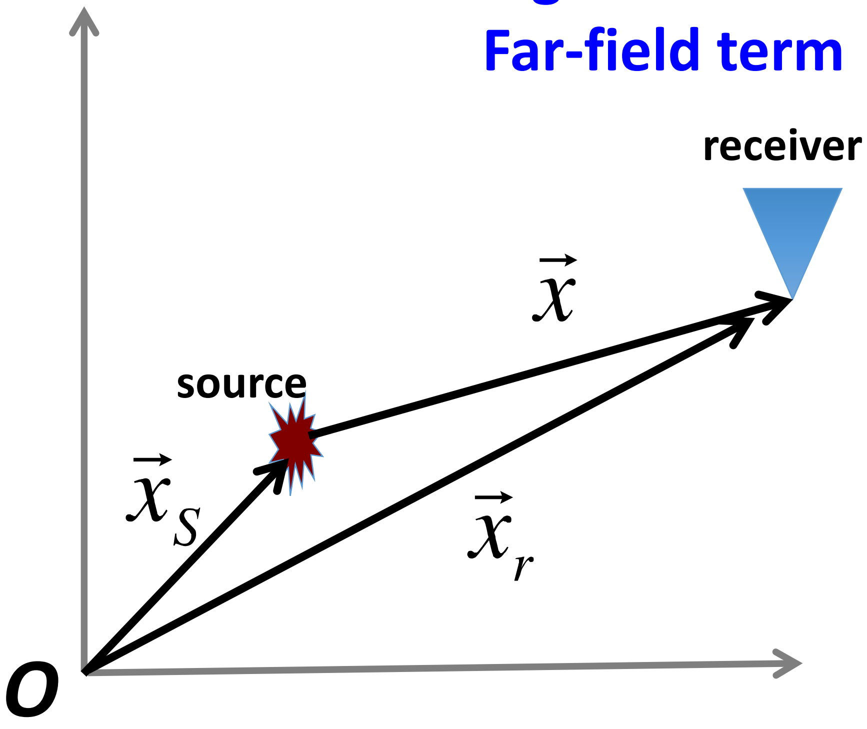


Green's function

$$u_n^P(\vec{x},t) = \sum_p \sum_q \boxed{\frac{\gamma_n \gamma_p \gamma_q}{4\pi\alpha^3 r}} M_{pq} \dot{S}\left(t - \frac{r}{\alpha}\right)$$

$$u_n^S(\vec{x},t) = \sum_p \sum_q \boxed{\frac{(\delta_{np} - \gamma_n \gamma_p)\gamma_q}{4\pi\beta^3 r}} M_{pq} \dot{S}\left(t - \frac{r}{\beta}\right)$$

$$\gamma_n = \frac{\left(\vec{x}_r - \vec{x}_S\right)_n}{r}, \quad r = \left|\vec{x}\right| = \left|\vec{x}_r - \vec{x}_S\right|$$

$$\alpha = \text{P-wave velocity}, \qquad \beta = S\text{-wave velocity}$$

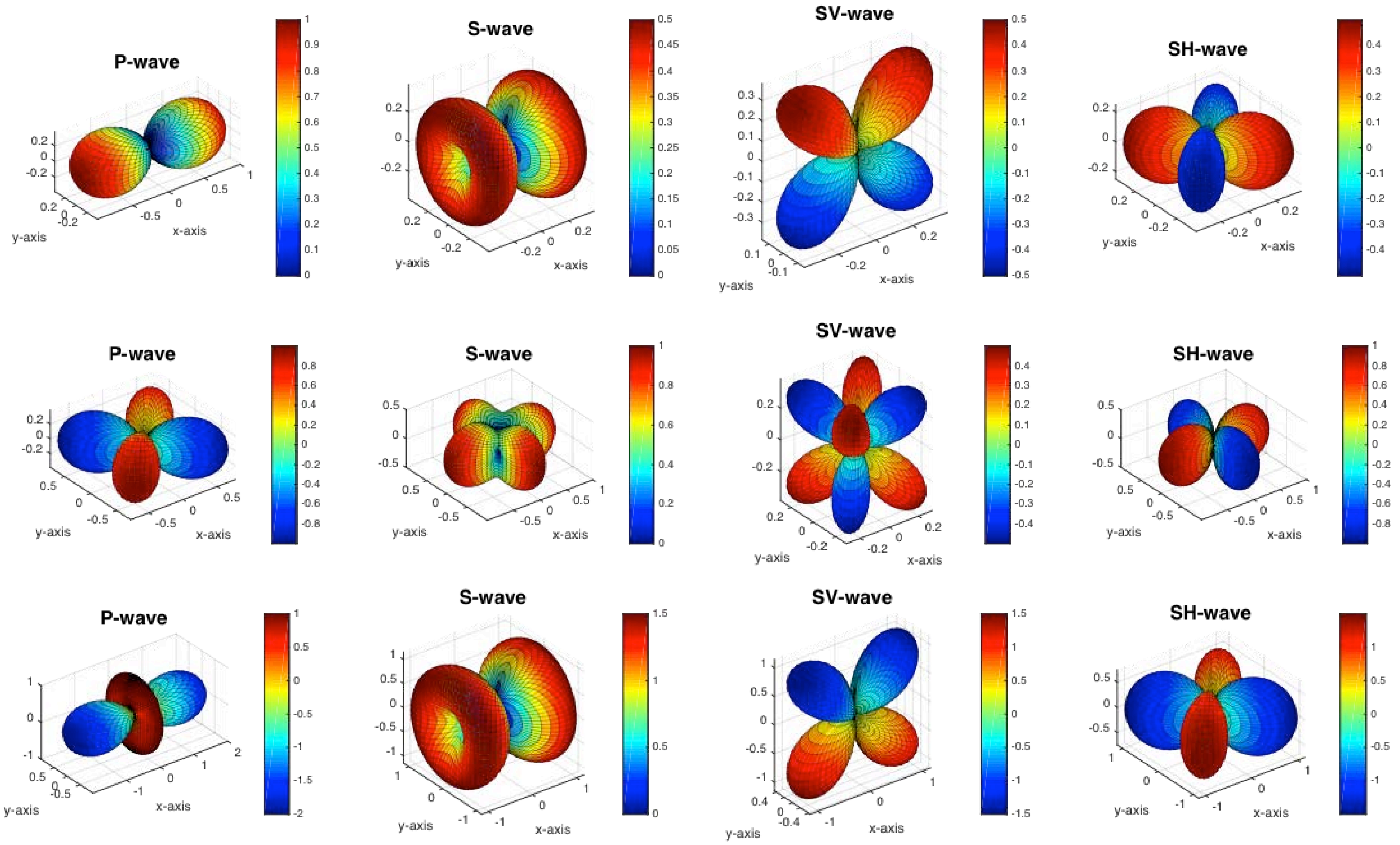
Radiation pattern

Moment tensor point source

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



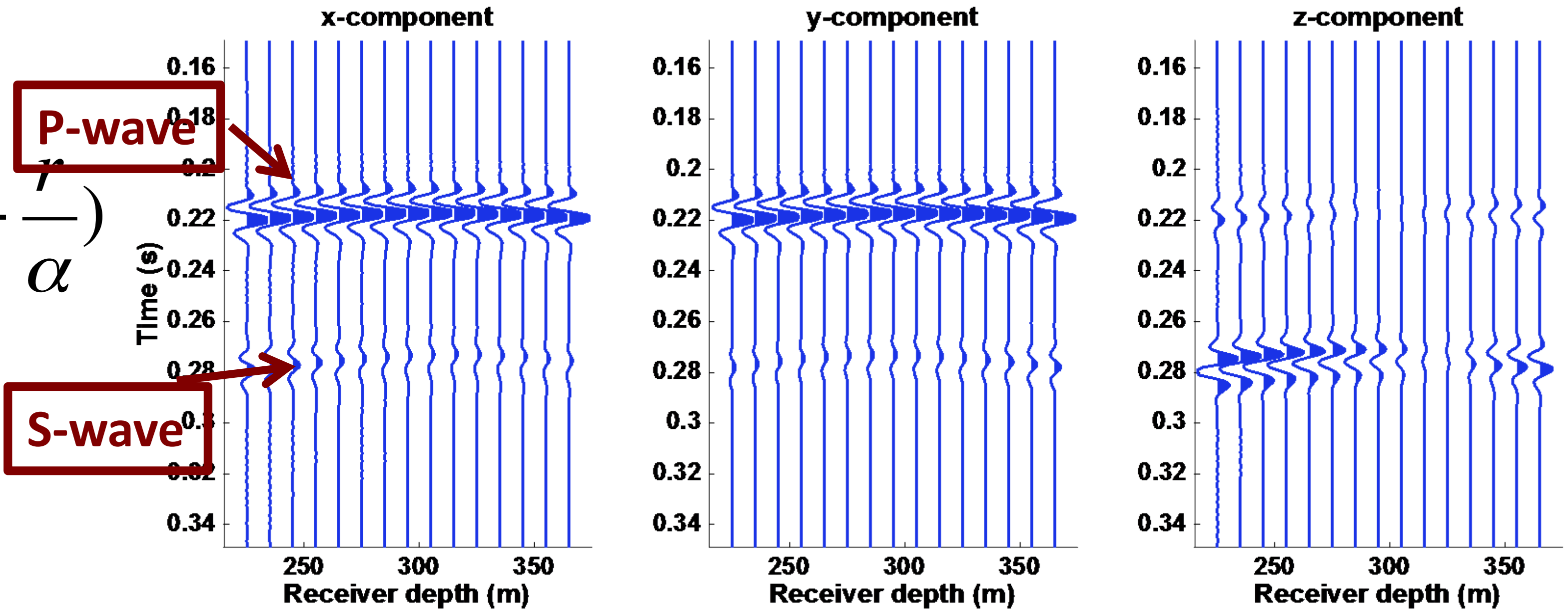
Moment tensor inversion: method 1

Data = P- and S-wave first arrival amplitude

M = **(M_{11},M_{22},M_{33},M_{23},M_{13},M_{12})**

$$u_n^P(\vec{x},t) = \sum_p \sum_q \frac{\gamma_n \gamma_p \gamma_q}{4\pi\alpha^3 r} M_{pq} \dot{S}\left(t - \frac{r}{\alpha}\right)$$

$$u_n^P(\vec{x},t) = A\gamma_n \left(\sum_p \gamma_p \sum_q M_{pq} \gamma_q \right)$$



$$\begin{bmatrix} u_1^P \\ u_2^P \\ u_3^P \end{bmatrix} = A \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$

Moment tensor inversion: method 1

$$D = GM$$

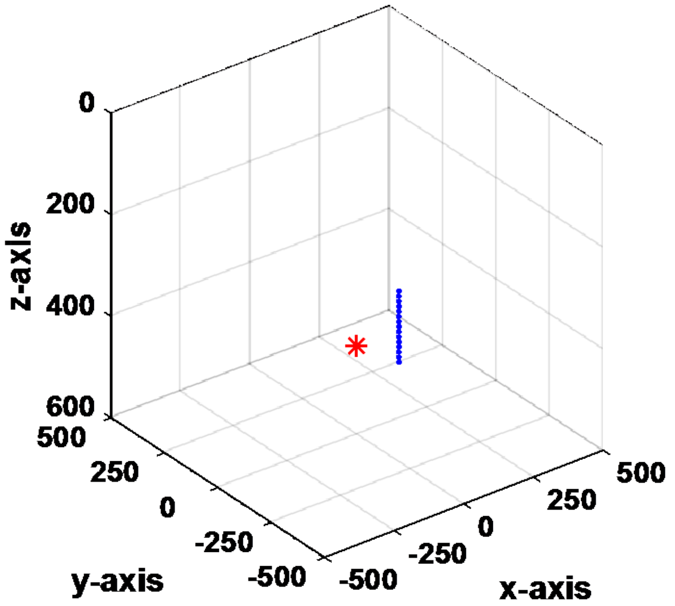
$$M = (G^T G + \mu)^{-1} G^T D$$

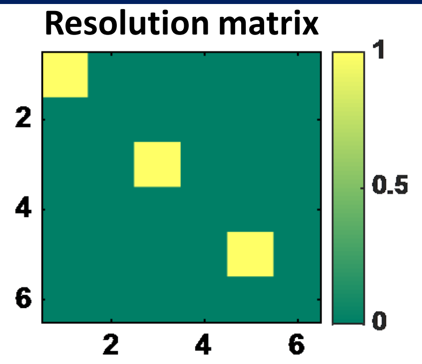
- ❑Source location
- ❑Velocity model (anisotropy)
- ❑Noise effect
- ❑Receiver geometry

$$\begin{bmatrix} u_1^P \\ u_2^P \\ u_3^P \end{bmatrix} = A \begin{bmatrix} \gamma_1^3 & \gamma_1\gamma_2^2 & \gamma_1\gamma_3^2 & 2\gamma_1\gamma_2\gamma_3 & 2\gamma_1^2\gamma_3 & 2\gamma_1^2\gamma_2 \\ \gamma_2\gamma_1^2 & \gamma_2^3 & \gamma_2\gamma_3 & 2\gamma_2^2\gamma_3 & 2\gamma_1\gamma_2\gamma_3 & 2\gamma_2^2\gamma_1 \\ \gamma_3\gamma_1^2 & \gamma_3\gamma_2^2 & \gamma_3^3 & 2\gamma_3^2\gamma_2 & 2\gamma_3^2\gamma_1 & 2\gamma_1\gamma_2\gamma_3 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}$$

$$\begin{bmatrix} u_1^S \\ u_2^S \\ u_3^S \end{bmatrix} = B \begin{bmatrix} \gamma_1 - \gamma_1^3 & -\gamma_1\gamma_2^2 & -\gamma_1\gamma_3^2 & -2\gamma_1\gamma_2\gamma_3 & \gamma_3 - 2\gamma_1^2\gamma_3 & \gamma_2 - 2\gamma_1^2\gamma_2 \\ -\gamma_2\gamma_1^2 & \gamma_2 - \gamma_2^3 & -\gamma_2\gamma_3 & \gamma_3 - 2\gamma_2^2\gamma_3 & -2\gamma_1\gamma_2\gamma_3 & \gamma_1 - 2\gamma_2^2\gamma_1 \\ -\gamma_3\gamma_1^2 & -\gamma_3\gamma_2^2 & \gamma_3 - \gamma_3^3 & \gamma_2 - 2\gamma_3^2\gamma_2 & \gamma_1 - 2\gamma_3^2\gamma_1 & -2\gamma_1\gamma_2\gamma_3 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}$$

MT **inversion**: single well and P-wave data only



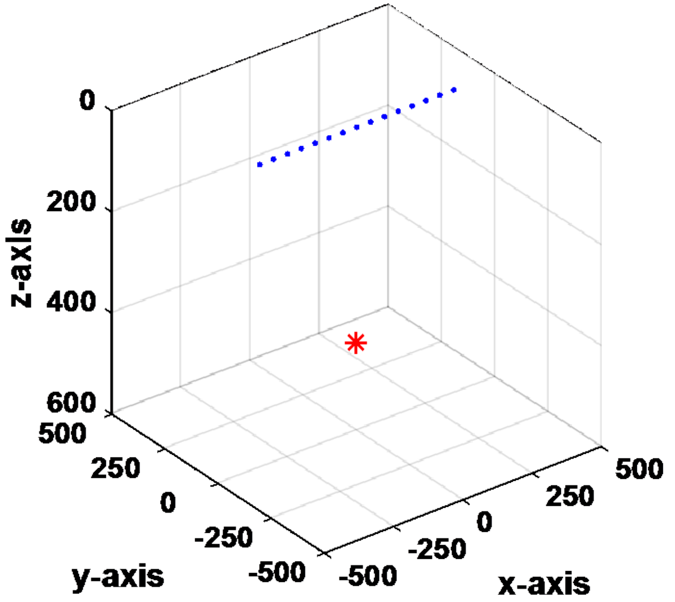


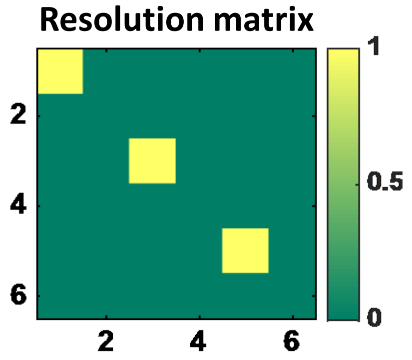
Estimated M

$$\begin{bmatrix} 1 & 0 & 0.5 \\ & 0 & 0 \\ & & 4 \end{bmatrix}$$

Singular Values

$$\begin{bmatrix} 0.0182 \\ 0.0074 \\ 0.00007 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$





Estimated M

$$\begin{bmatrix} 1 & 0 & 0.5 \\ & 0 & 0 \\ & & 4 \end{bmatrix}$$

Singular Values

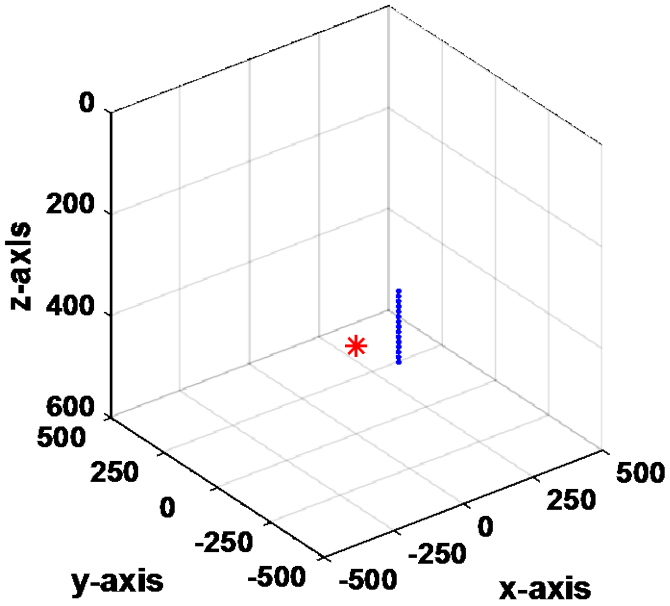
$$\begin{bmatrix} 0.0072 \\ 0.0055 \\ 0.0012 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

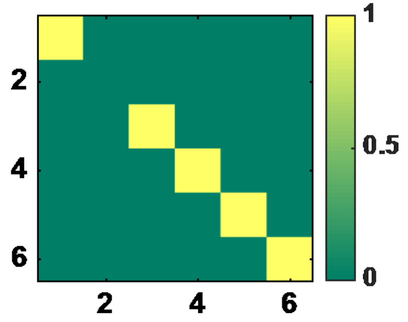
Source MT

$$M = \begin{bmatrix} 1 & 6 & 0.5 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

- 3 terms can be recovered
- 3 zero singular values
- Resolvability insensitive to the distance between array of receivers and the source

MT inversion: single well and P&S data





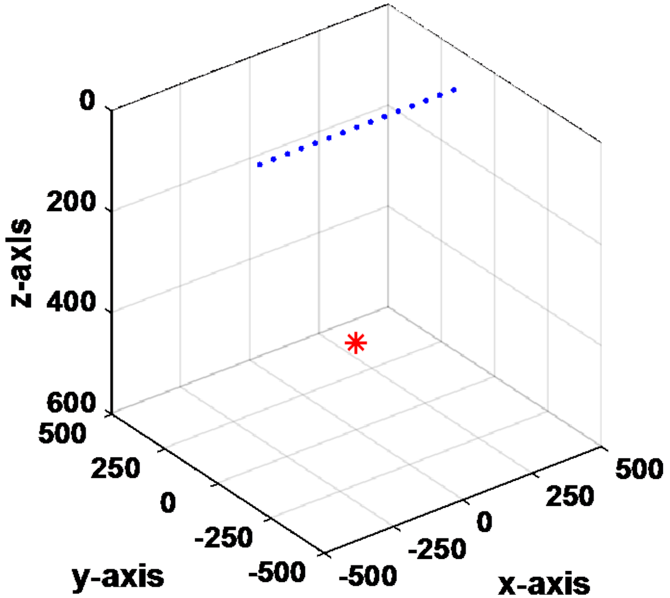
Estimated M

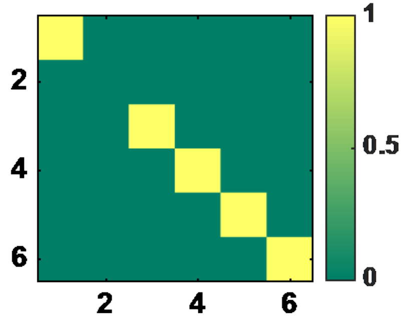
$$\begin{bmatrix} 1 & 6 & 0.5 \\ & 0 & -1 \\ & & 4 \end{bmatrix}$$

Singular Values

$$\begin{bmatrix} 0.019 \\ 0.0185 \\ 0.0185 \\ 0.0039 \\ 0.0039 \\ 0 \end{bmatrix}$$

unresolved





Estimated M

$$\begin{bmatrix} 1 & 6 & 0.5 \\ & 0 & -1 \\ & & 4 \end{bmatrix}$$

Singular Values

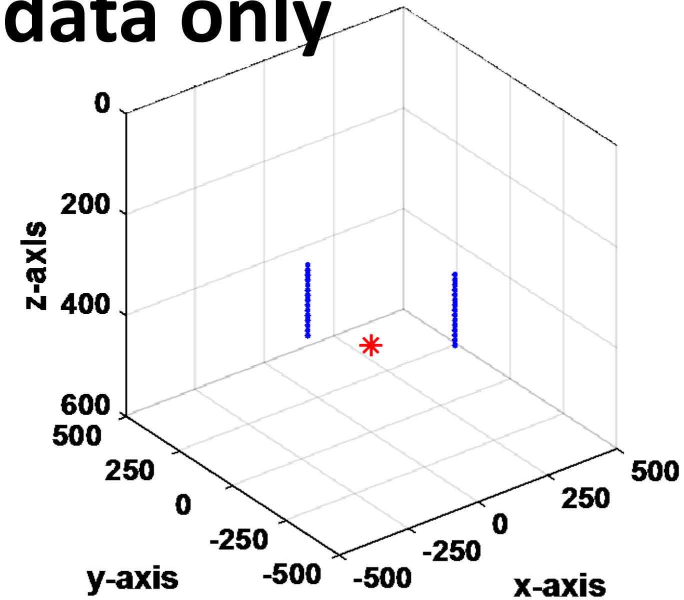
$$\begin{bmatrix} 0.0083 \\ 0.0076 \\ 0.0075 \\ 0.0032 \\ 0.0032 \\ 0 \end{bmatrix}$$

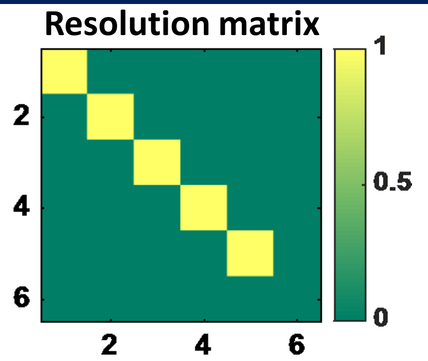
unresolved

- ❑ 5 terms can be recovered
- ❑ 1 zero singular values
- ❑ Resolvability insensitive to the distance between array of receivers and the source

MT inversion: two wells

P data only





Estimated M

$$\begin{bmatrix} 1 & 0 & 0.5 \\ & -2 & -1 \\ & & 4 \end{bmatrix}$$

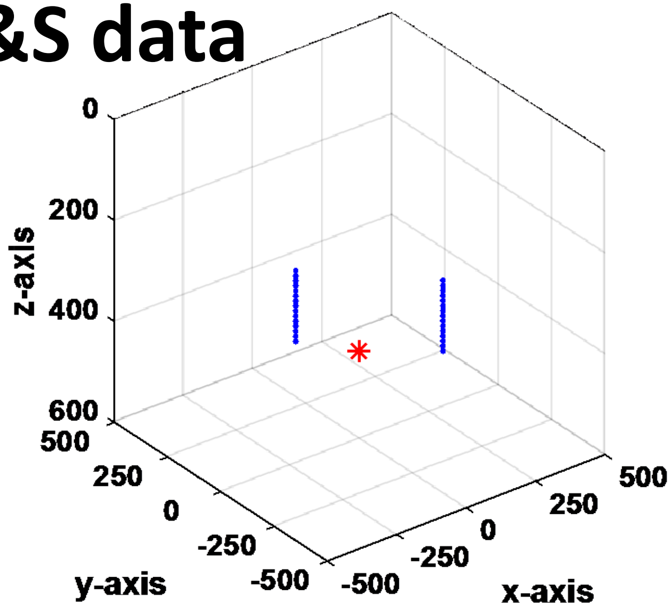
Singular Values

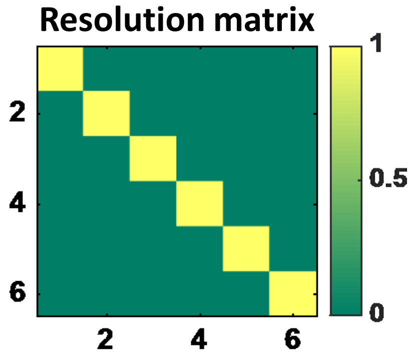
$$\begin{bmatrix} 0.018 \\ 0.00182 \\ 0.0074 \\ 0.0074 \\ 0.001 \\ 0 \end{bmatrix}$$

unresolved

- ❑ P-data, 5 terms will be resolved
- ❑ P&S data, full MT is resolvable
- ❑ Similar for two surface arrays
- ❑ Full MT from P-data, 3 wells

P&S data





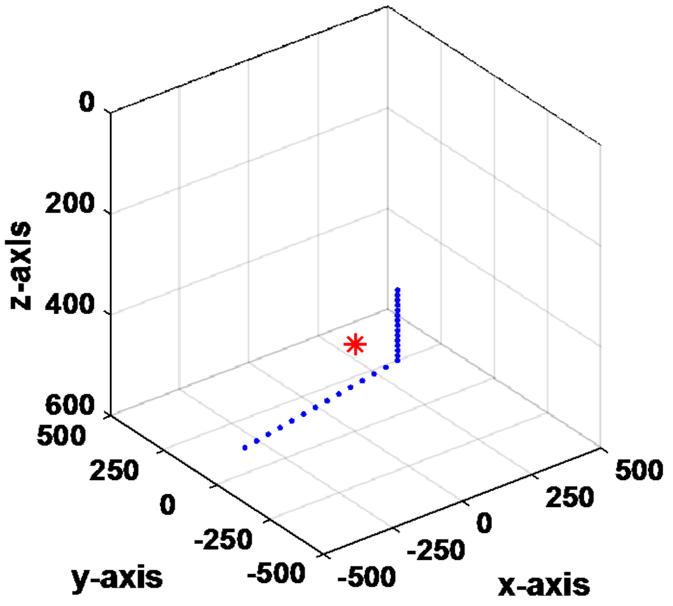
Estimated M

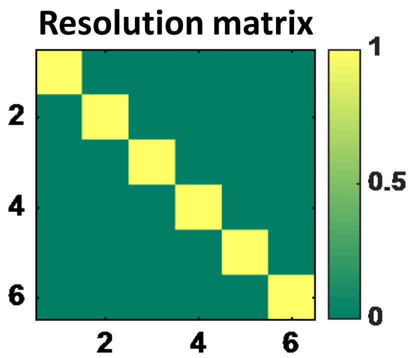
$$\begin{bmatrix} 1 & 6 & 0.5 \\ & -2 & -1 \\ & & 4 \end{bmatrix}$$

Singular Values

$$\begin{bmatrix} 0.0201 \\ 0.0192 \\ 0.0185 \\ 0.0076 \\ 0.0066 \\ 0.0039 \end{bmatrix}$$

Recover full MT from data, single well geometry





Estimated M

$$\begin{bmatrix} 1 & 6 & 0.5 \\ & -2 & -1 \\ & & 4 \end{bmatrix}$$

Singular Values $\times 10^{-3}$

$$\begin{bmatrix} 0.1174 \\ 0.1133 \\ 0.0776 \\ 0.0689 \\ 0.0337 \\ 0.0145 \end{bmatrix}$$

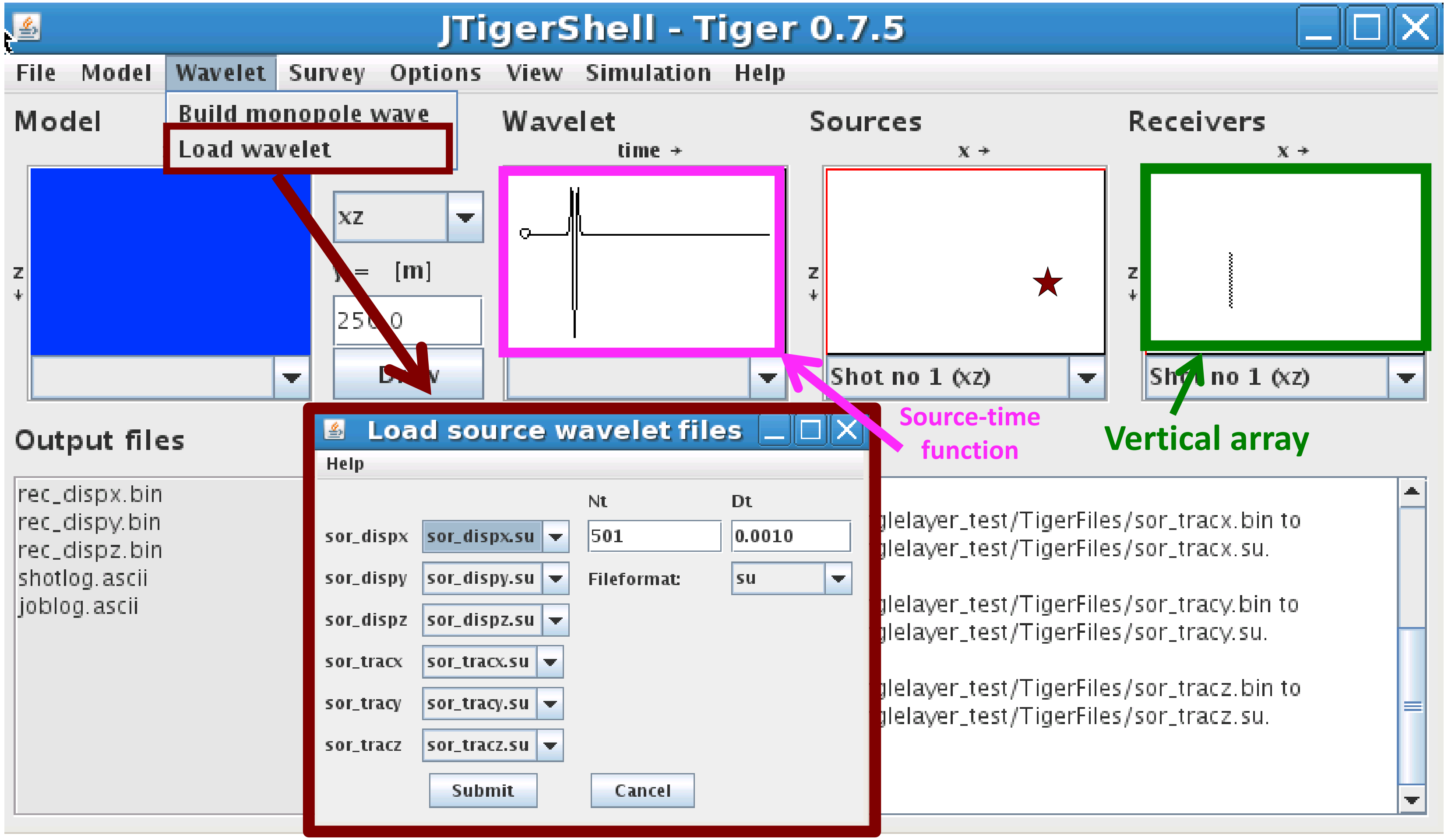
Challenges:

- ❑ P and S arrival should be picked, cumbersome task
- ❑ Unknown source-time function

Nolen-Hoeksema and Ruff (2001)
Vavryčuk (2007)
Rodriguez et al. (2011)
Eaton and Forouhideh (2011)

Microseismic data generated by TIGER software

- 3D anisotropic elastic finite-difference modeling software
- Arbitrary acquisition geometry
- Moment tensor source



Moment tensor inversion: method 2 (Vavryčuk and Kühn, 2012)

Step 1: estimate *S(t)*

$$u_n(\vec{x},t) = g_{np,q}(\vec{x},t) * M_{pq} S(t)$$

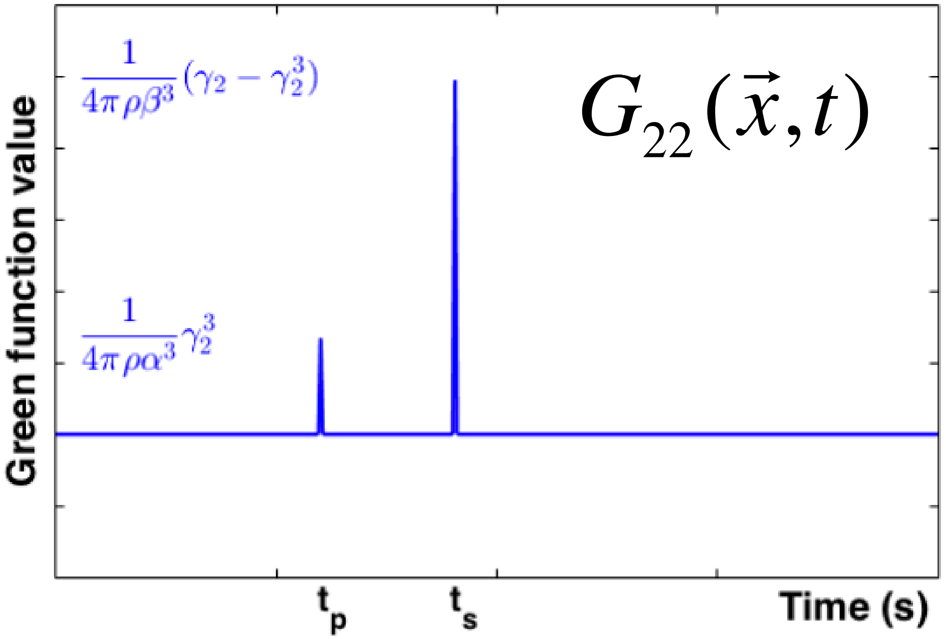
M_k = (M_{11},M_{22},M_{33},M_{23},M_{13},M_{12})
***k* =1:6**

$$G_{n1} = g_{n1,1},\ G_{n2} = g_{n2,2},\ G_{n3} = g_{n3,3},$$
$$G_{n4} = g_{n2,3} + g_{n3,2},\ G_{n5} = g_{n1,3} + g_{n3,1}, G_{n6} = g_{n1,2} + g_{n2,1}$$

$$u_n(\vec{x},t) = G_{nk}(\vec{x},t) * M_k S(t)$$

Fourier transform

$$u_n(\vec{x},\omega) = G_{nk}(\vec{x},\omega) m_k(\omega)$$



Moment tensor inversion: method 2 (Vavryčuk and Kühn, 2012)

Step 1: estimate *S(t)*

$$u_n(\vec{x},\omega) = G_{nk}(\vec{x},\omega)m_k(\omega)$$

For each ω:

$$\begin{bmatrix} u_1^{(1)}(\omega) \\ u_2^{(1)}(\omega) \\ u_3^{(1)}(\omega) \\ \vdots \\ u_1^{(N)}(\omega) \\ u_2^{(N)}(\omega) \\ u_3^{(N)}(\omega) \end{bmatrix} = \begin{bmatrix} G_{11}^{(1)}(\omega) & G_{12}^{(1)}(\omega) & G_{13}^{(1)}(\omega) & G_{14}^{(1)}(\omega) & G_{15}^{(1)}(\omega) & G_{16}^{(1)}(\omega) \\ G_{21}^{(1)}(\omega) & G_{22}^{(1)}(\omega) & G_{23}^{(1)}(\omega) & G_{24}^{(1)}(\omega) & G_{25}^{(1)}(\omega) & G_{26}^{(1)}(\omega) \\ G_{31}^{(1)}(\omega) & G_{32}^{(1)}(\omega) & G_{33}^{(1)}(\omega) & G_{34}^{(1)}(\omega) & G_{35}^{(1)}(\omega) & G_{36}^{(1)}(\omega) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{11}^{(N)}(\omega) & G_{12}^{(N)}(\omega) & G_{13}^{(N)}(\omega) & G_{14}^{(N)}(\omega) & G_{15}^{(N)}(\omega) & G_{16}^{(N)}(\omega) \\ G_{21}^{(N)}(\omega) & G_{22}^{(N)}(\omega) & G_{23}^{(N)}(\omega) & G_{24}^{(N)}(\omega) & G_{25}^{(N)}(\omega) & G_{26}^{(N)}(\omega) \\ G_{31}^{(N)}(\omega) & G_{32}^{(N)}(\omega) & G_{33}^{(N)}(\omega) & G_{34}^{(N)}(\omega) & G_{35}^{(N)}(\omega) & G_{36}^{(N)}(\omega) \end{bmatrix} \begin{bmatrix} m_1(\omega) \\ m_2(\omega) \\ m_3(\omega) \\ m_4(\omega) \\ m_5(\omega) \\ m_6(\omega) \end{bmatrix}$$

$m(\omega) = (m_1(\omega), m_2(\omega), m_3(\omega), m_4(\omega), m_5(\omega), m_6(\omega))$

Inverse Fourier Transform

$m(t) = (m_1(t), m_2(t), m_3(t), m_4(t), m_5(t), m_6(t))$

- ❑ **SVD of *m(t)***
- ❑ **Find the largest singular value, σ**
- ❑ ***S(t)*: the eigen vector associated with σ (??)**

Moment tensor inversion: method 2 (Vavryčuk and Kühn, 2012)

Step 2: estimate M_{pq}

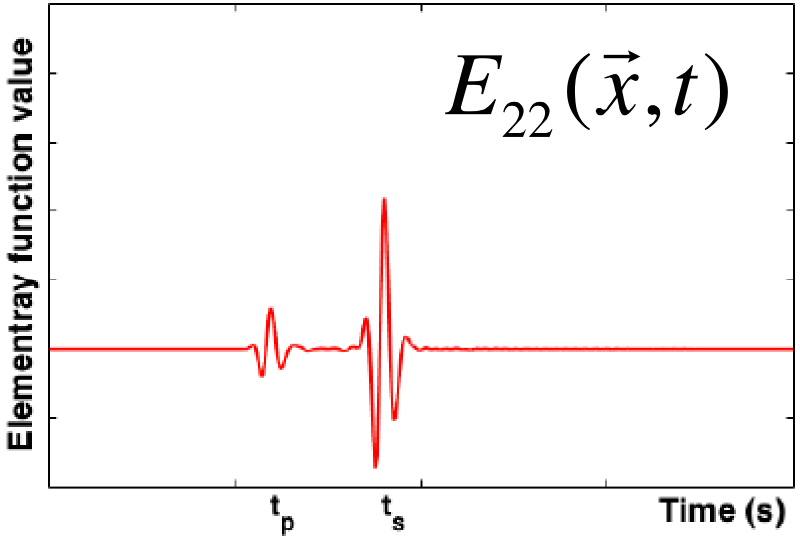
Elementary seismograms

$$E_{nk}(\vec{x},t) = G_{nk}(\vec{x},t) * S(t)$$

$$k = 1:6$$

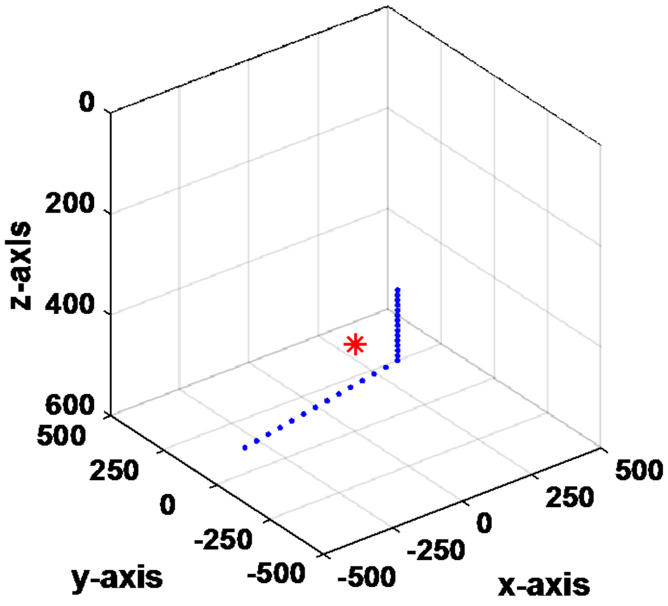
$$u_n(\vec{x},t) = E_{nk}(\vec{x},t) M_k$$

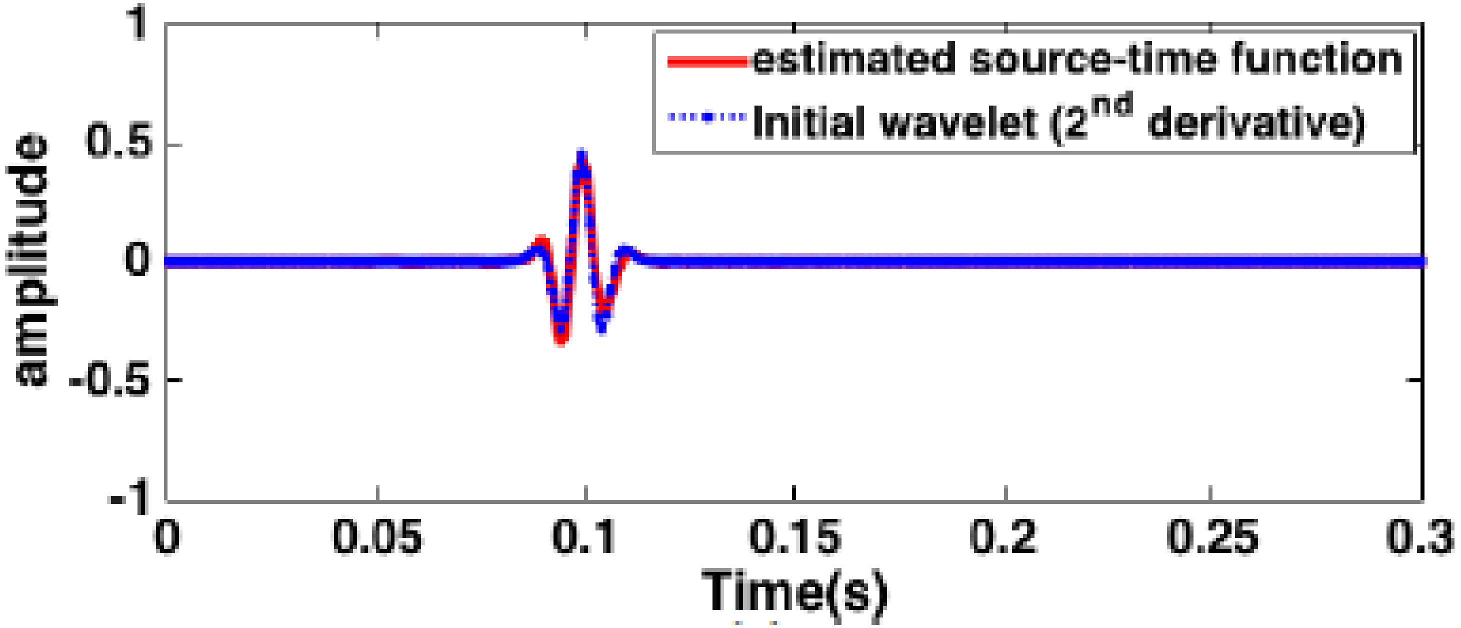
$$
\begin{bmatrix}
u_1^{(1)}(t_1) \\ u_2^{(1)}(t_1) \\ u_3^{(1)}(t_1) \\ \vdots \\ u_1^{(1)}(t_{nt}) \\ u_2^{(1)}(t_{nt}) \\ u_3^{(1)}(t_{nt}) \\ \vdots \\ u_1^{(N)}(t_1) \\ u_2^{(N)}(t_1) \\ u_3^{(N)}(t_1) \\ \vdots \\ u_1^{(N)}(t_{nt}) \\ u_2^{(N)}(t_{nt}) \\ u_3^{(N)}(t_{nt})
\end{bmatrix}
=
\begin{bmatrix}
E_{11}^{(1)}(t_1) & E_{12}^{(1)}(t_1) & E_{13}^{(1)}(t_1) & E_{14}^{(1)}(t_1) & E_{15}^{(1)}(t_1) & E_{16}^{(1)}(t_1) \\
E_{21}^{(1)}(t_1) & E_{22}^{(1)}(t_1) & E_{23}^{(1)}(t_1) & E_{24}^{(1)}(t_1) & E_{25}^{(1)}(t_1) & E_{26}^{(1)}(t_1) \\
E_{31}^{(1)}(t_1) & E_{32}^{(1)}(t_1) & E_{33}^{(1)}(t_1) & E_{34}^{(1)}(t_1) & E_{35}^{(1)}(t_1) & E_{36}^{(1)}(t_1) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
E_{11}^{(1)}(t_n) & E_{12}^{(1)}(t_n) & E_{13}^{(1)}(t_n) & E_{14}^{(1)}(t_n) & E_{15}^{(1)}(t_n) & E_{16}^{(1)}(t_n) \\
E_{21}^{(1)}(t_n) & E_{22}^{(1)}(t_n) & E_{23}^{(1)}(t_n) & E_{24}^{(1)}(t_n) & E_{25}^{(1)}(t_n) & E_{26}^{(1)}(t_n) \\
E_{31}^{(1)}(t_n) & E_{32}^{(1)}(t_n) & E_{33}^{(1)}(t_n) & E_{34}^{(1)}(t_n) & E_{35}^{(1)}(t_n) & E_{36}^{(1)}(t_n) \\
\vdots & & & & & \\
E_{11}^{(N)}(t_1) & E_{12}^{(1)}(t_1) & E_{13}^{(1)}(t_1) & E_{14}^{(1)}(t_1) & E_{15}^{(1)}(t_1) & E_{16}^{(1)}(t_1) \\
E_{21}^{(1)}(t_1) & E_{22}^{(1)}(t_1) & E_{23}^{(1)}(t_1) & E_{24}^{(1)}(t_1) & E_{25}^{(1)}(t_1) & E_{26}^{(1)}(t_1) \\
E_{31}^{(1)}(t_1) & E_{32}^{(1)}(t_1) & E_{33}^{(1)}(t_1) & E_{34}^{(1)}(t_1) & E_{35}^{(1)}(t_1) & E_{36}^{(1)}(t_1) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
E_{11}^{(N)}(t_n) & E_{12}^{(N)}(t_n) & E_{13}^{(N)}(t_n) & E_{14}^{(N)}(t_n) & E_{15}^{(N)}(t_n) & E_{16}^{(N)}(t_n) \\
E_{21}^{(N)}(t_n) & E_{22}^{(N)}(t_n) & E_{23}^{(N)}(t_n) & E_{24}^{(N)}(t_n) & E_{25}^{(N)}(t_n) & E_{26}^{(N)}(t_n) \\
E_{31}^{(N)}(t_n) & E_{32}^{(N)}(t_n) & E_{33}^{(N)}(t_n) & E_{34}^{(N)}(t_n) & E_{35}^{(N)}(t_n) & E_{36}^{(N)}(t_n)
\end{bmatrix}
\begin{bmatrix}
M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6
\end{bmatrix}
$$



(b)

MT inversion TIGER data





Conclusions

- Understanding of ***MT*** representation of seismic sources
- Seismic response of a ***MT*** source
- Proper microseismic observation geometry to recover full ***MT***
- Waveform inversion to extract **S(t)** and ***MT***
- The way forward!

Future work

- Generate accurate TIGER data!
- Estimate *S(t)* using a deconvolution method
- Investigate the effect of random noise on inversion results
- Investigate the effect of velocity model on inversion results
- Application on real data
- Simultaneously invert for velocity model and MT

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