Velocity model building by slope tomography

Bernard Law and Daniel Trad

Outline

- Motivation
- Slope tomography
- Numerical examples
- Synthetic example
- Conclusions

Motivation

High resolution velocity model building methods:

- Iterative depth migration velocity analysis
- Full waveform inversion

Motivation

High resolution velocity model building methods:

- Iterative depth migration velocity analysis and update
- Full waveform inversion

Both methods:

- Computationally expensive
- Require a starting model in depth

Motivation

High resolution velocity model building methods:

- Iterative depth migration velocity analysis
- Full waveform inversion

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- Computationally expensive
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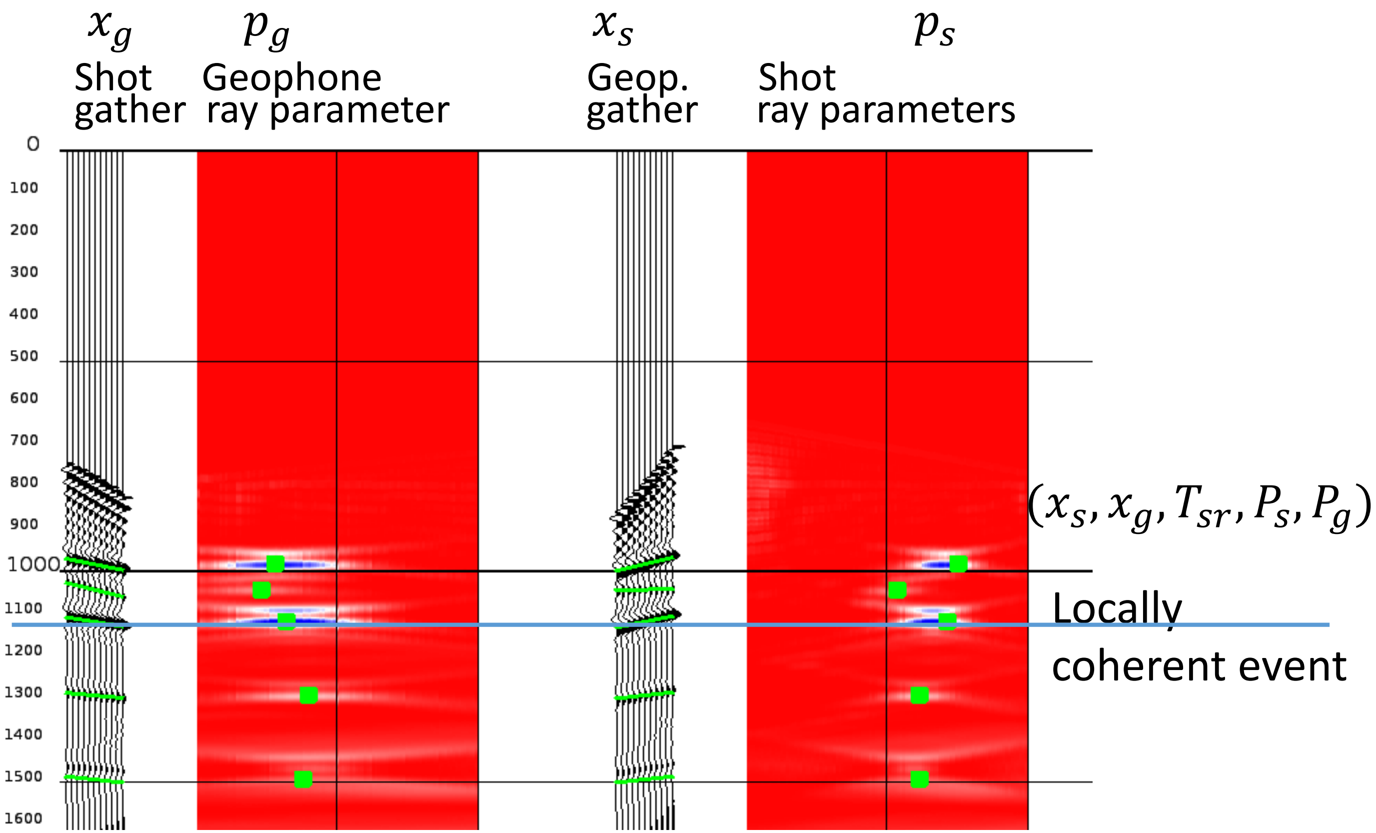
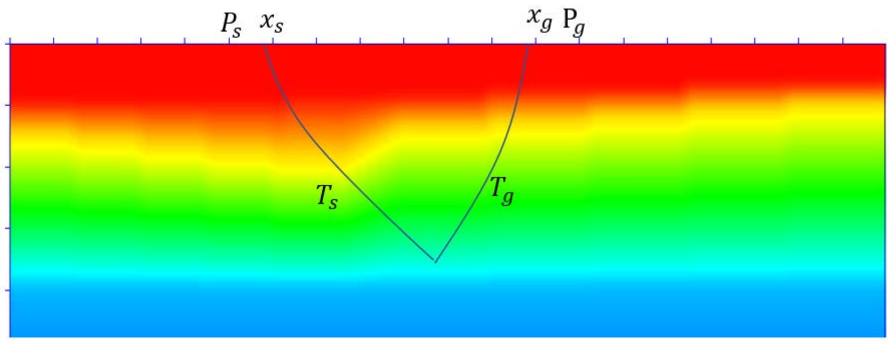
Slope tomography methods can estimate smooth velocity model:

- CDR tomography (Chuck Sword 1987)
- Stereotomography (Billette & Lambaré 1998, Chauris 2002, …)

Slope tomography

- Slope tomography characterizes a locally coherent event with the source position x_s, receiver position x_g, traveltime T_{sr} and ray parameters p_s and p_g

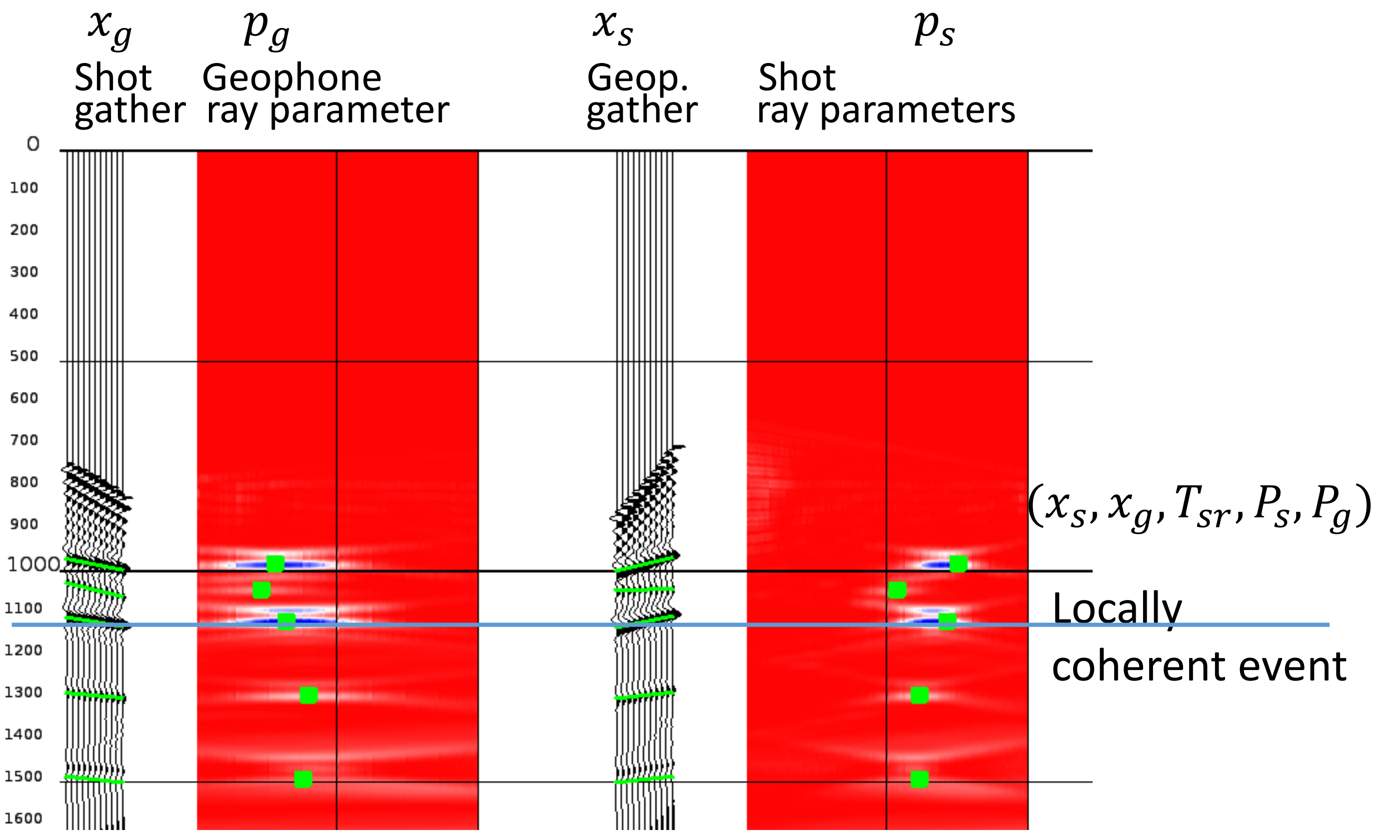


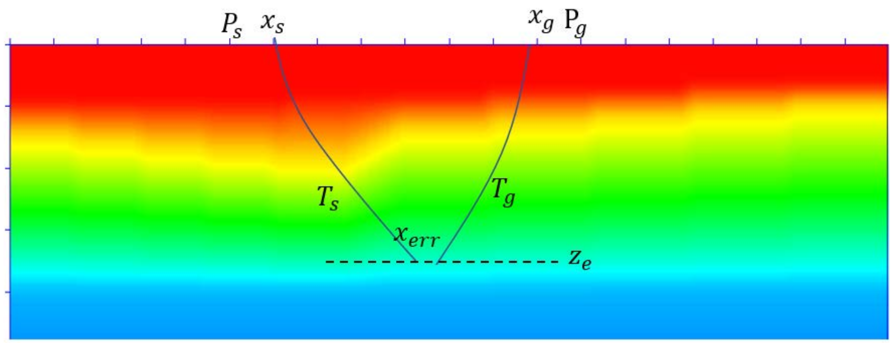


Correct velocity model

Slope tomography

- Slope tomography characterizes a locally coherent event with the source position x_s, receiver position x_g, traveltime T_{sr} and ray parameters p_s and p_g





Incorrect velocity model

Slope tomography method

Misfit: $$Xerr_j = \left(X_g - X_s\right)_j \quad (1)$$

Cost function : $$J(v) = ||\, X_{err}||^2 \quad (2)$$

Inversion: $$\begin{bmatrix} A_{11} & A_{12} & A_{13} & .. & .. & A_{1m} \\ A_{21} & A_{22} & A_{23} & .. & .. & A_{2m} \\ A_{31} & A_{32} & A_{33} & .. & .. & A_{3m} \\ .. & .. & .. & .. & .. & .. \\ A_{n1} & A_{n2} & A_{n3} & .. & .. & A_{nm} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ .. \\ \Delta V_m \end{bmatrix} = \begin{bmatrix} -X_{err\,1} \\ -X_{err\,2} \\ -X_{err\,3} \\ .. \\ -X_{err\,m} \end{bmatrix} \quad (3)$$

Fréchet derivative: $$A_{ij} = \frac{\partial X_{err\,j}}{\partial v_i} \quad (4)$$

$$i = 1\ to\ m\ velocity\ cells$$

$$j = 1\ to\ n\ ray\ pairs$$

Slope tomography method

Fréchet derivative: $A_{ij} = \frac{\partial X_{err\,j}}{\partial v_i}$ (4)

$$\frac{\partial X_{err\,j}}{\partial v_i} = \frac{\partial X_{err\,j}}{\partial \Delta X_{\,i}} \frac{\partial \Delta X_i}{\partial v_i} + \frac{\partial X_{err\,j}}{\partial \Delta t_{\,i}} \frac{\partial \Delta t_i}{\partial v_i} \quad (5)$$

$$\frac{\partial X_{err}(j)}{\partial \Delta X_{\,i}} = \begin{bmatrix} -1 & \text{for source ray path} \\ 1 & \text{for receive ray path} \end{bmatrix} \quad (6)$$

$$\frac{\partial X_{err}(j)}{\partial \Delta t_i} = -v_{se} v_{ge} \frac{\sin\theta_{ge} cos\theta_{se} - \sin\theta_{se} cos\theta_{ge}}{v_{se} cos\theta_{se} + v_{ge} cos\theta_{ge}} \quad (7)$$

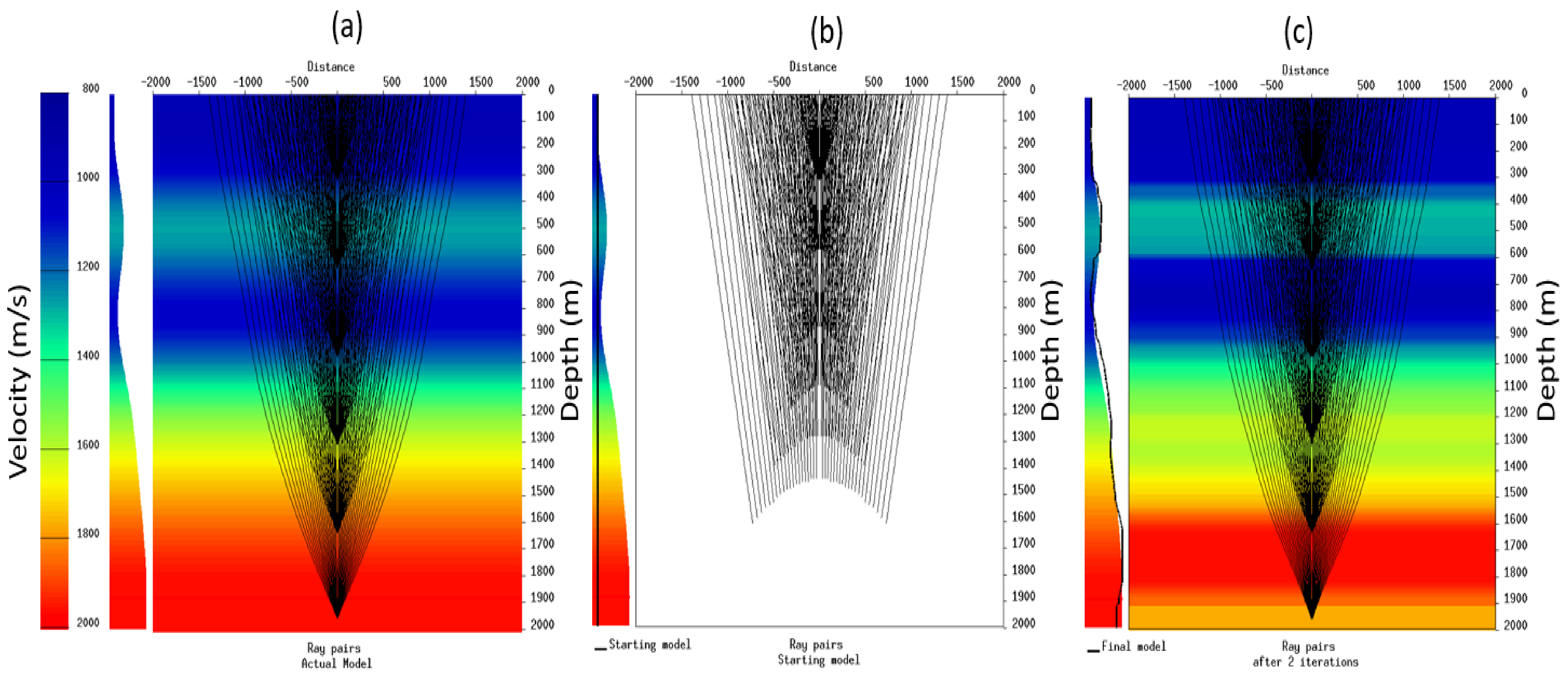
$\frac{\partial \Delta X_i}{\partial v_i}$ and $\frac{\partial \Delta t_i}{\partial v_i}$ are computed using a transfer matrix: $T_l = \begin{bmatrix} \frac{\partial x(l)}{\partial x(l-1)} & \frac{\partial x(l)}{\partial p(l-1)} & 0 \\ \frac{\partial p(l)}{\partial x(l-1)} & \frac{\partial p(l)}{\partial p(l-1)} & 0 \\ \frac{\partial \Delta t(l)}{\partial x(l-1)} & \frac{\partial \Delta t(l)}{\partial p(l-1)} & 1 \end{bmatrix}$, (8)

$$\begin{bmatrix} \frac{\partial \Delta x_i}{\partial v_i} \\ \frac{\partial \Delta p_i}{\partial v_i} \\ \frac{\partial \Delta t_i}{\partial v_i} \end{bmatrix} = T_L \cdot T_{L-1} \cdots T_{l+2} \cdot T_{l+1} \cdot \begin{bmatrix} \frac{\partial \Delta x_l}{\partial v_l} \\ \frac{\partial \Delta p_l}{\partial v_l} \\ \frac{\partial \Delta t_l}{\partial v_l} \end{bmatrix} \quad (9)$$

Sword 1987

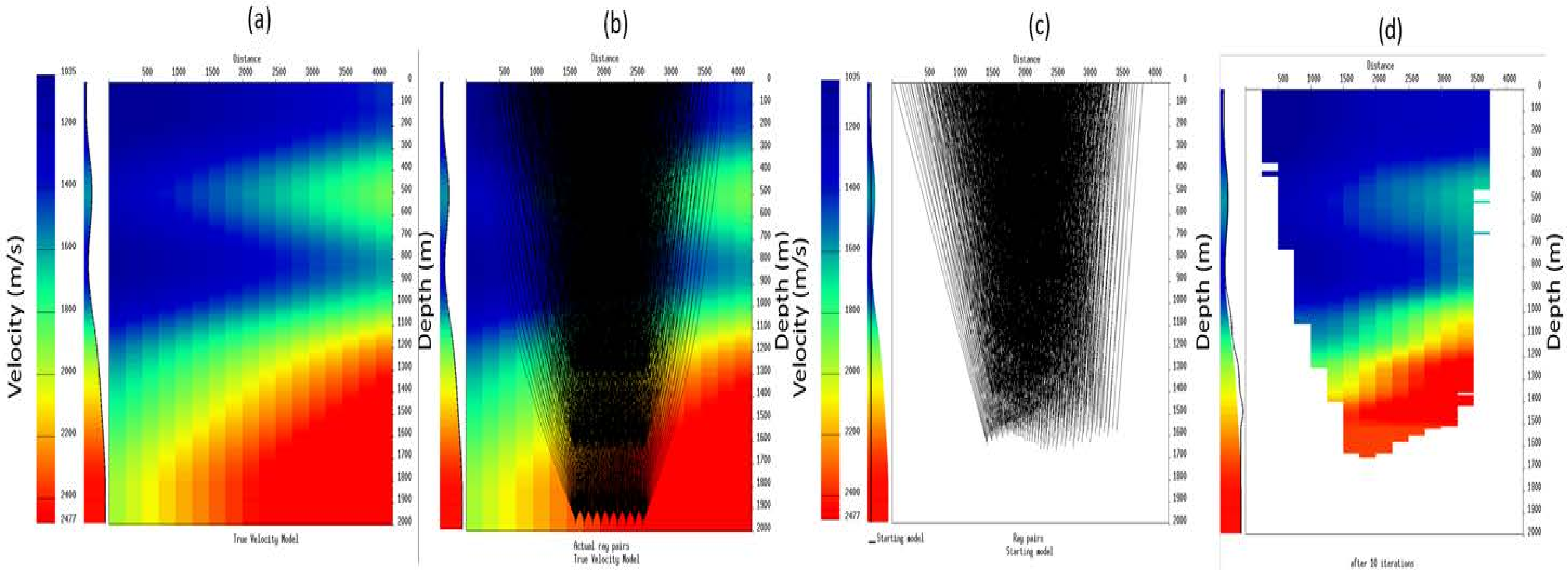
Numerical example

V(z)



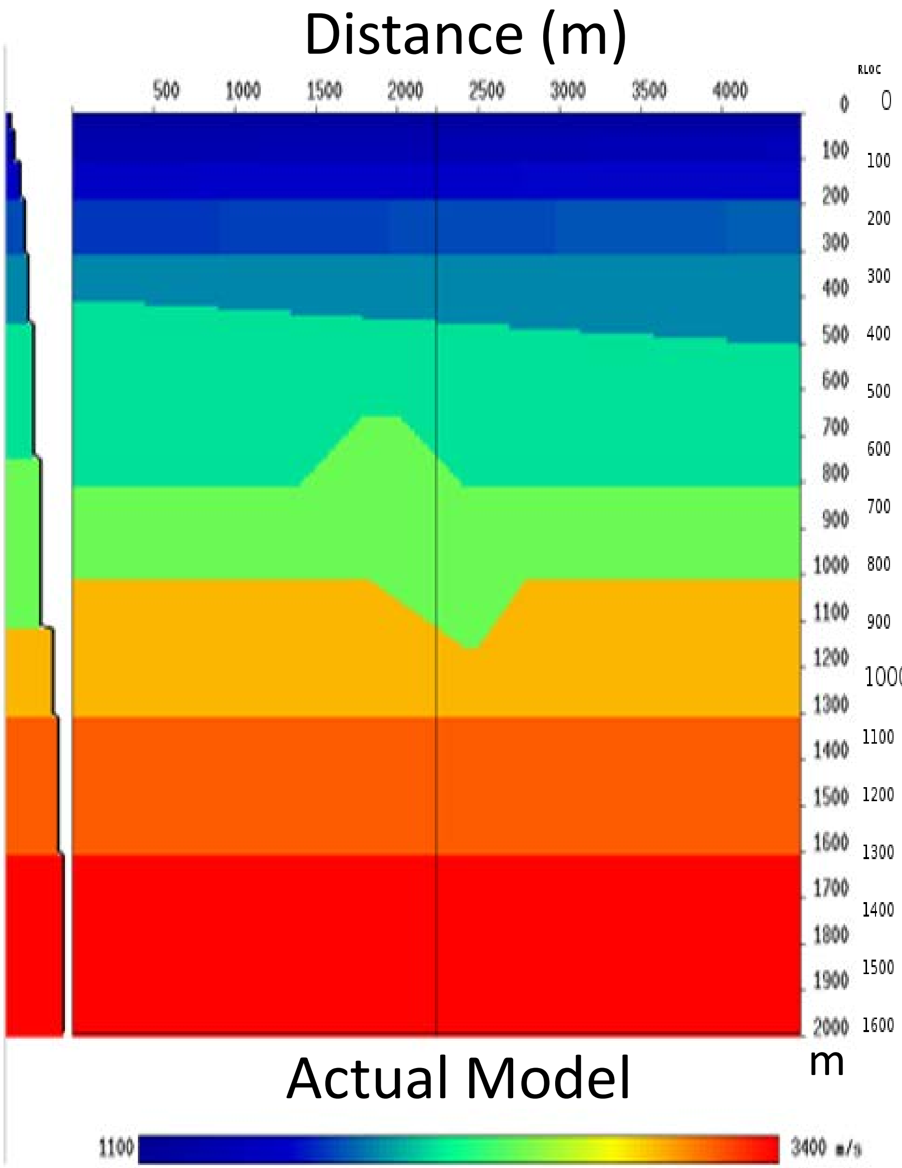
Numerical example

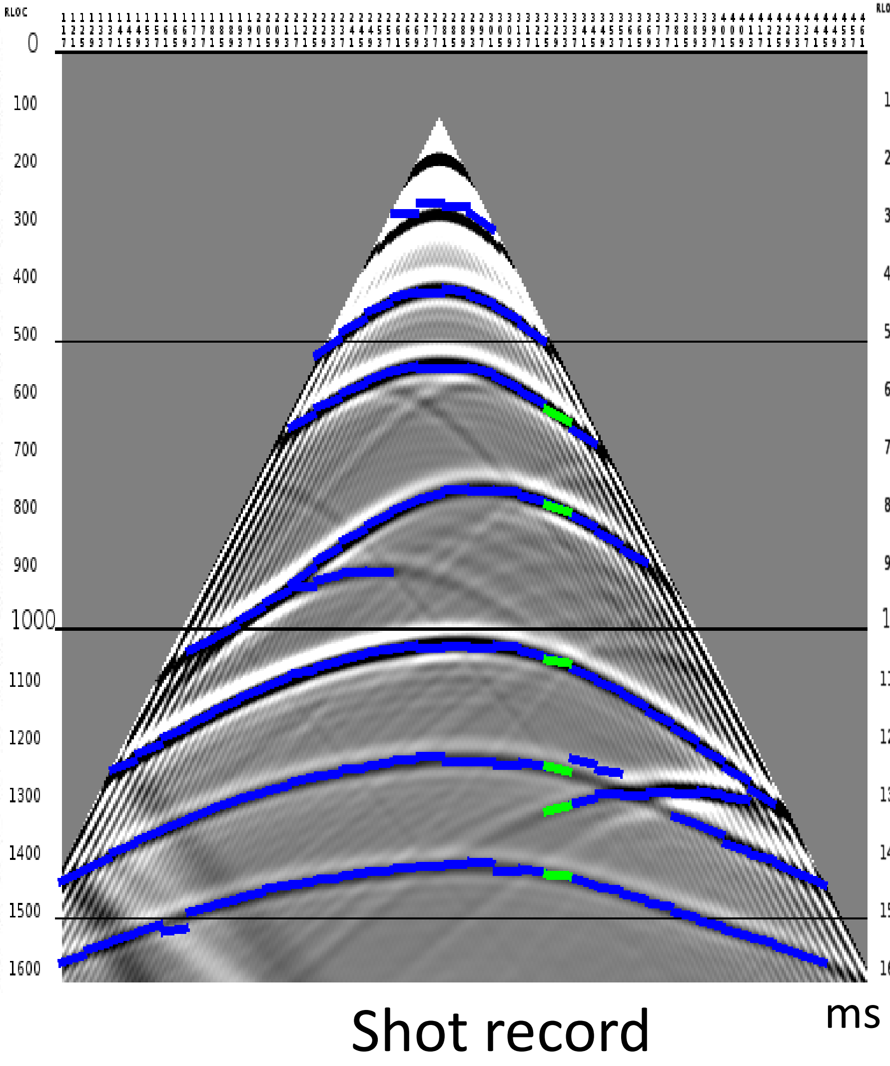
V(x,z)

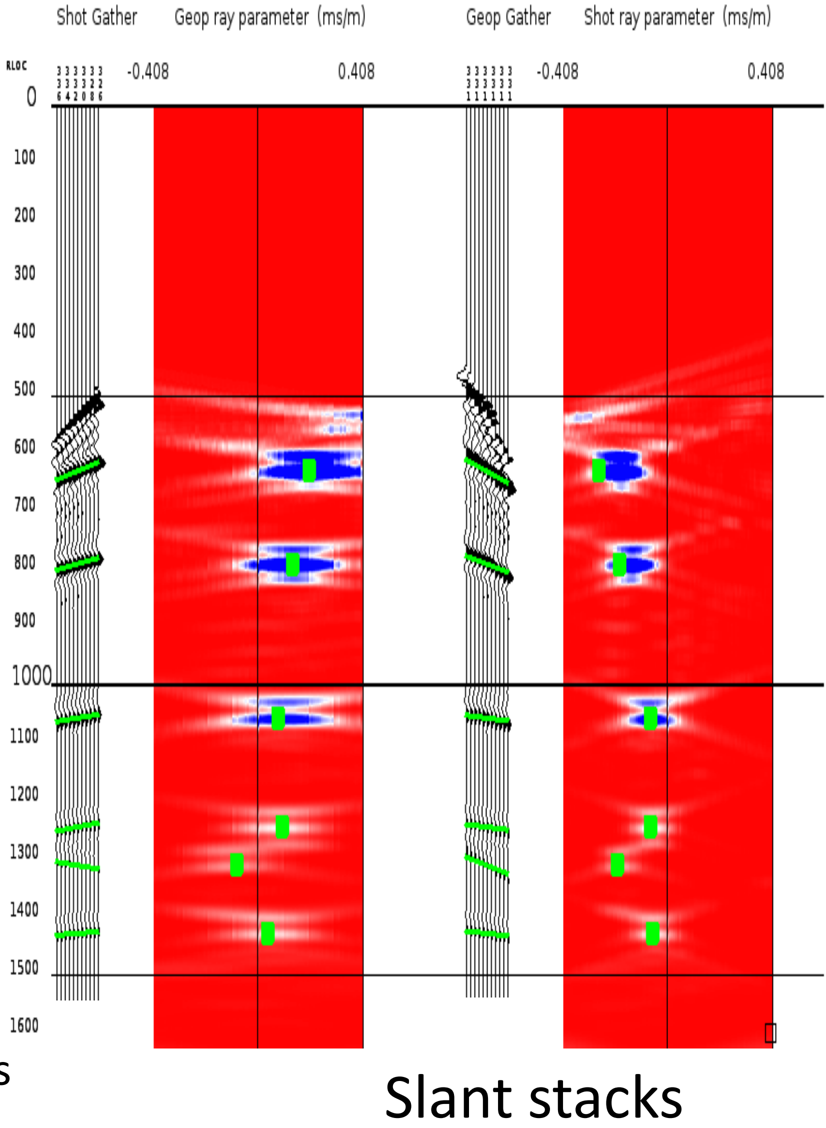


Synthetic example

V(x,z)







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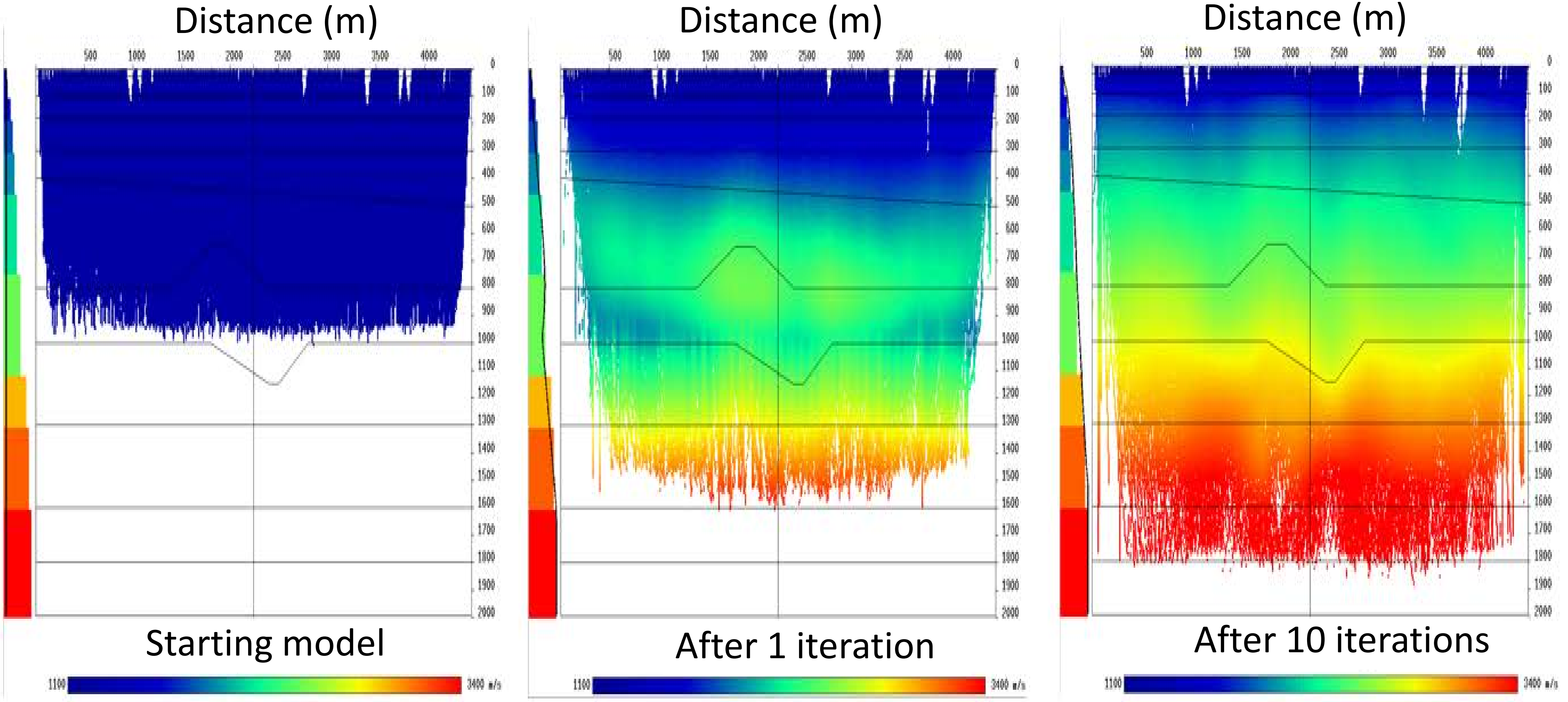
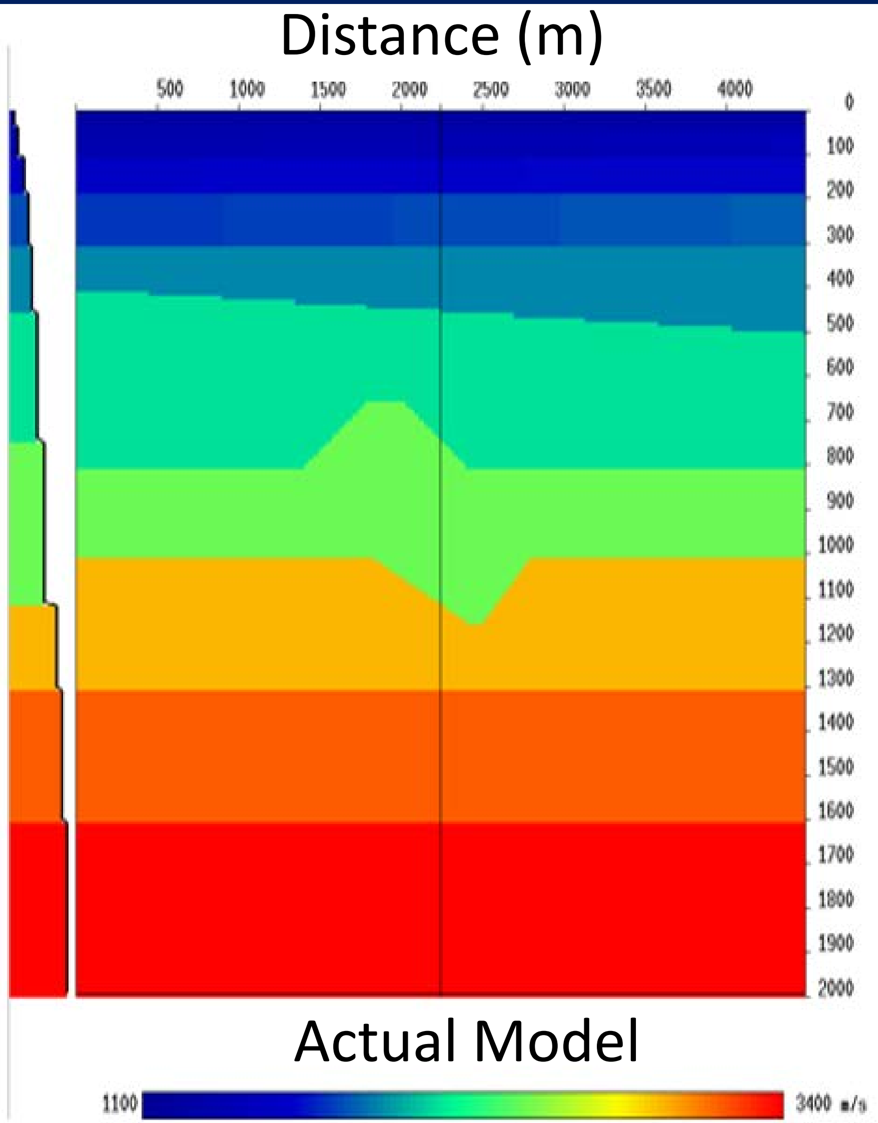
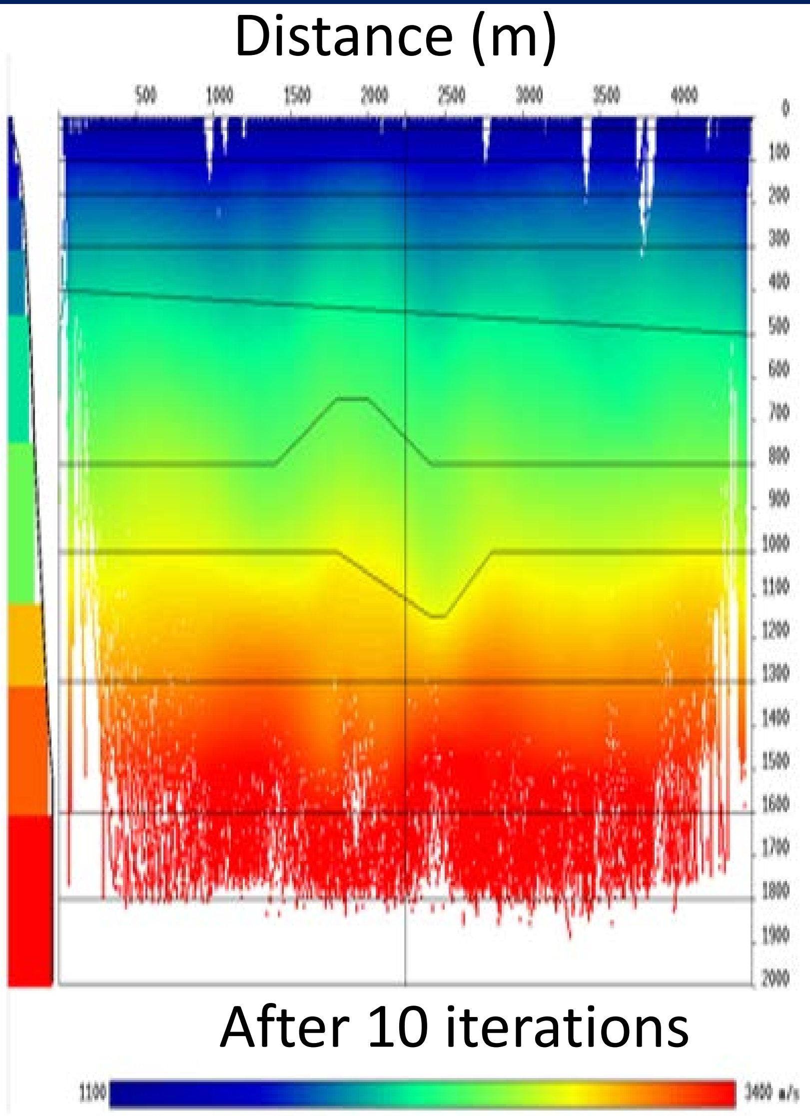


Synthetic example V(x,z)

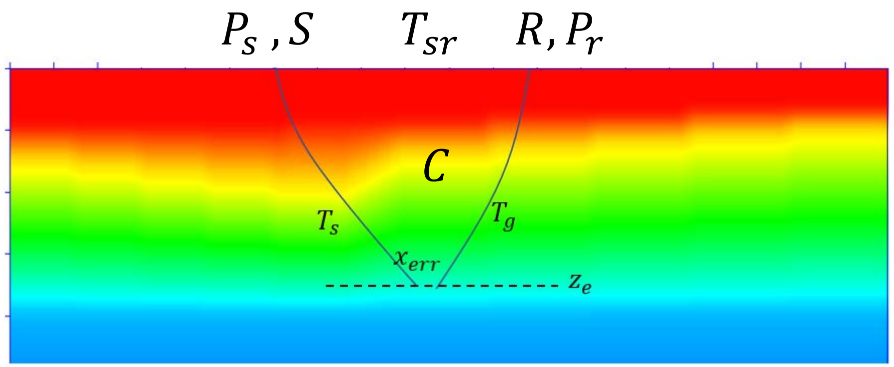


Actual Model



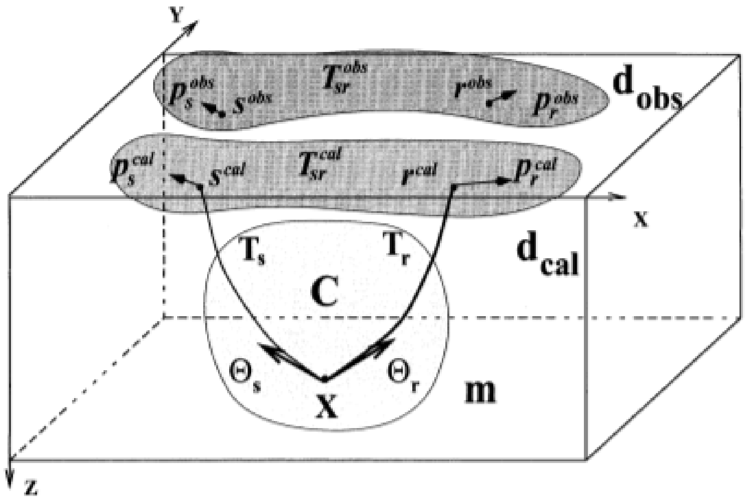
After 10 iterations

Stereotomography



Model space : $[C_i]_{i=1,M}$

Fréchet derivative: $A_{ij} = \frac{\partial X_{err\,j}}{\partial C_i}$



Stereotomography

Model space: $[(X, \Theta_s, \Theta_r, T_s, T_r)_{j=1,N}],[C_m]_{m=1,M}\,]$

Fréchet derivative: $A_{ij} = \frac{\partial(S,R,P_s,P_r,T_{sr})}{\partial(X,\Theta_s,\Theta_r,T_s,T_r,C_m)}$

Billette and Lambaré 1998

Conclusions

- We verified the slope tomography algorithm can recover velocity information.
- We demonstrated the process of picking the source and geophone ray parameters using localized slant stacks.
- We showed that we can use the ray parameters and traveltime picks to estimate a smooth velocity model.
- We will continue to investigate the stereotomography method (Billette and Lambaré, 1998).

Acknowledgments

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Thank You