Implementations of LSRTM in time and frequency domain

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Outline

- Introduction
- Time & Frequency domain LSRTM
- FWI-based LSRTM
- Summary & Future studies

Introduction

- RTM
- LSRTM
- FWI

Wavefield modeling

In the acoustic medium with constant density,

Time domain

$$\frac{\partial^2 P(\mathbf{x},t)}{\partial \mathbf{x}^2} - \sigma^2(\mathbf{x}) \frac{\partial^2 P(\mathbf{x},t)}{\partial t^2} = -S(\mathbf{x},t\,)$$

P: pressure wavefield
σ: slowness of the medium
S: source term

Frequency domain

$$[\nabla^2 + \omega^2\sigma^2(\mathbf{x})]\,\tilde{P}(\mathbf{x},\omega) = -\tilde{S}(\mathbf{x},\omega\,)$$

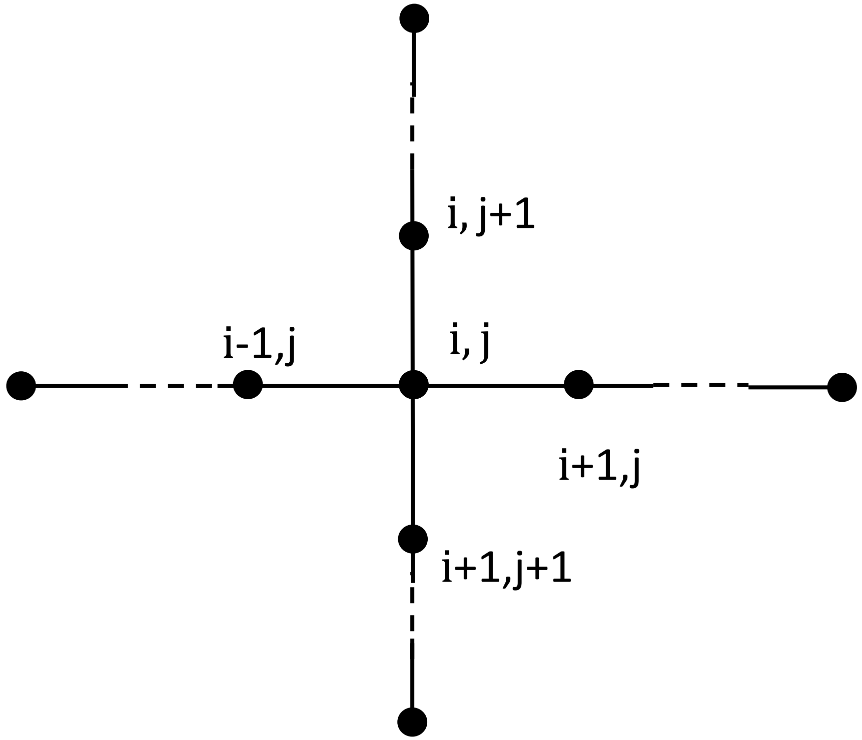
∇^2: Laplacian operator

$\frac{\partial^2}{\partial t^2} \to \omega^2$

$\tilde{P}(\mathbf{x},\omega)=\int_{-\infty}^{+\infty} P(\mathbf{x},t)e^{-i\omega t}d\omega$

$\tilde{S}(\mathbf{x},\omega)=\int_{-\infty}^{+\infty} S(\mathbf{x},t)e^{-i\omega t}d\omega$

Wavefield modeling



$$\nabla^2\tilde{P}$$

$$\left.\frac{\partial^2\tilde{P}}{\partial x^2}\right|_{i,j} = \frac{\tilde{P}_{i+1,j} - 2\tilde{P}_{i,j} + \tilde{P}_{i-1,j}}{\Delta x^2} + O(h^2)$$

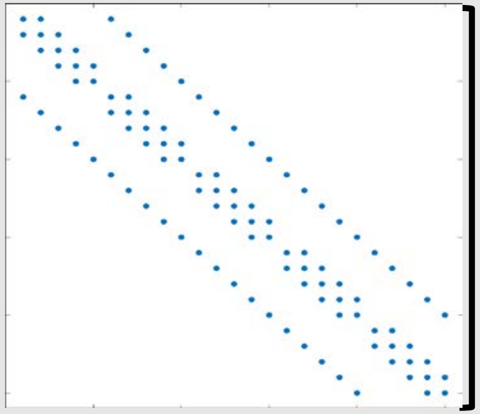
$$\left.\frac{\partial^2\tilde{P}}{\partial z^2}\right|_{i,j} = \frac{\tilde{P}_{i,j+1} - 2\tilde{P}_{i,j} + \tilde{P}_{i,j-1}}{\Delta z^2} + O(h^2)$$

Wavefield modeling

Frequency domain

$$[\nabla^2 + \omega^2\sigma^2(\mathbf{x})]\,\tilde{\mathrm{P}}(\mathbf{x},\omega) = -\tilde{\mathrm{S}}(\mathbf{x},\omega)$$

Single frequency



$$\begin{bmatrix} \tilde{\mathrm{P}}_1 \\ \tilde{\mathrm{P}}_2 \\ \vdots \\ \tilde{\mathrm{P}}_{N+1} \end{bmatrix} = \begin{bmatrix} \tilde{\mathrm{S}}_1 \\ \tilde{\mathrm{S}}_2 \\ \vdots \\ \tilde{\mathrm{S}}_{N+1} \end{bmatrix}$$

$$\mathrm{A} \quad \tilde{\mathrm{P}} = \tilde{\mathrm{S}}$$

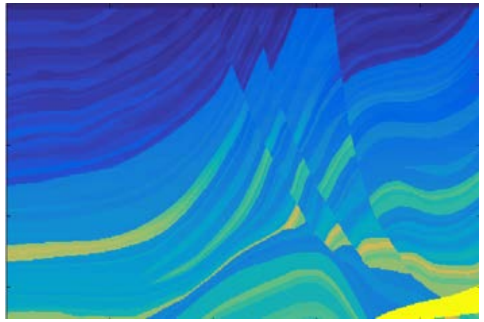
Impedance matrix

When the source is a delta function,

$$\mathrm{G} = \mathrm{A}^{-1}$$

Born modeling & Adjoint operator

$$\sigma^2 = \sigma_0^2 + \delta(\sigma^2)$$



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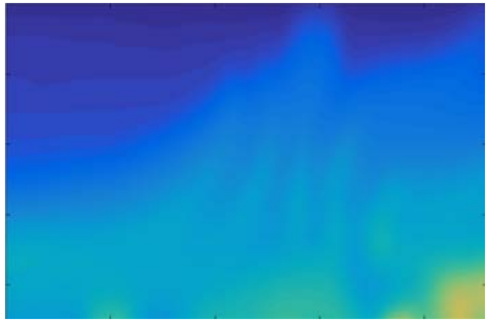

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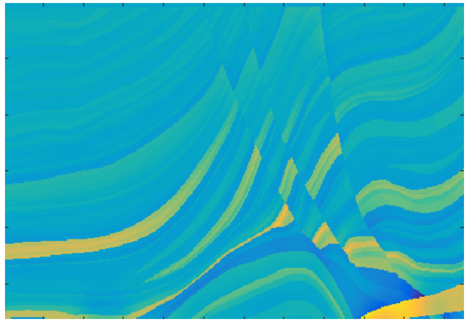
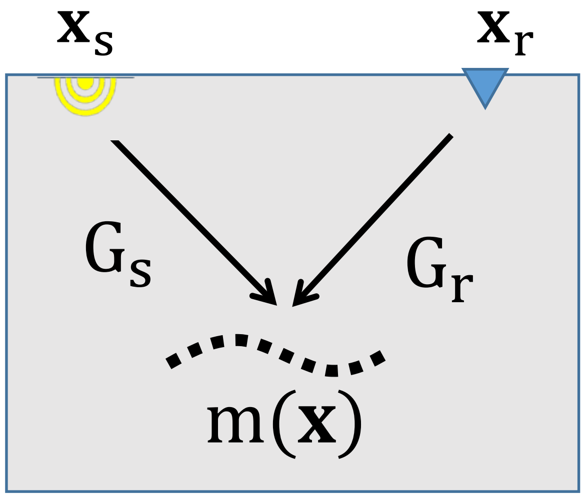


$$P \approx P_0 + \delta P$$

Assume the scattered wavefield is weak

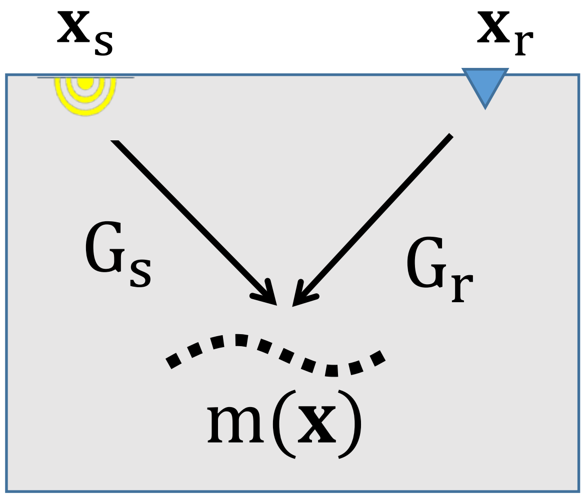
Born modeling & Adjoint operator



$$m(\mathbf{x}) = \sum_{n_\omega} \int \boxed{\omega^2 G_s^T(\mathbf{x}|\mathbf{x}_s,\omega) G_r^T(\mathbf{x}|\mathbf{x}_r,\omega)}\, d(\mathbf{x}_r,\mathbf{x}_s,\omega) d\mathbf{x}$$

$$\underbrace{m}_{\text{(reflector)}} = \underbrace{L^T \quad d}_{\text{(Migration operator)} \quad \text{(data)}}$$

Born modeling & Adjoint operator



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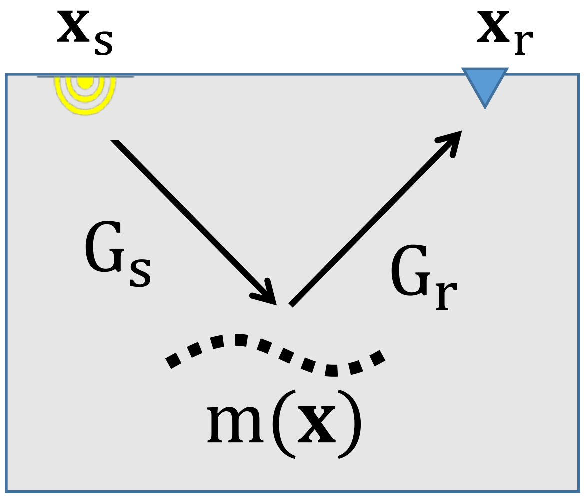
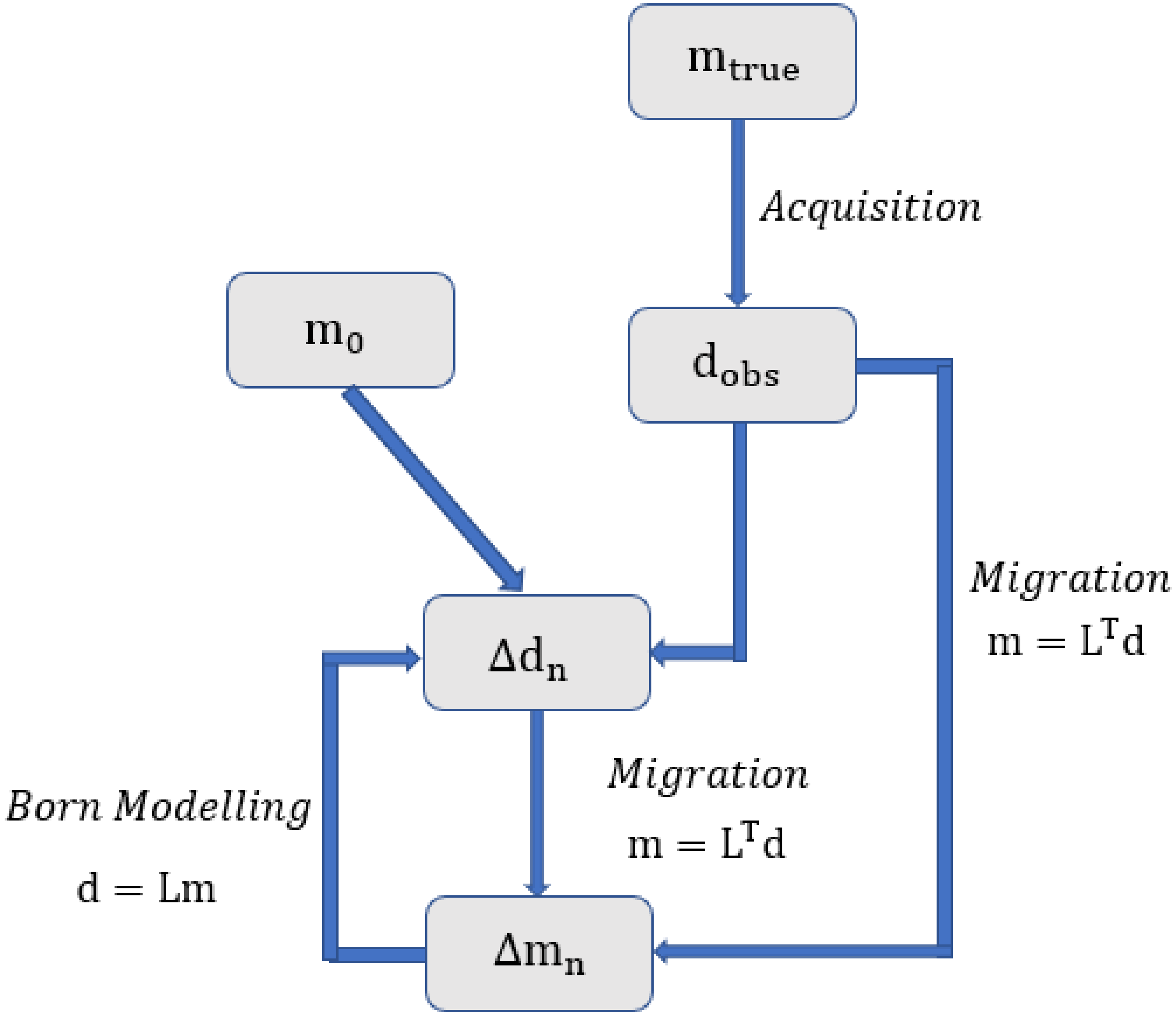
$$\underbrace{m}_{\text{(reflector)}} = \underbrace{L^T \qquad d}_{\text{(Migration operator)} \qquad \text{(data)}}$$



$$d(\mathbf{x}_r,\mathbf{x}_s,\omega) = \int \boxed{\omega^2 G_r(\mathbf{x}_r|\mathbf{x},\omega) G_s(\mathbf{x}|\mathbf{x}_s,\omega)}\, m(\mathbf{x}) d\mathbf{x}$$

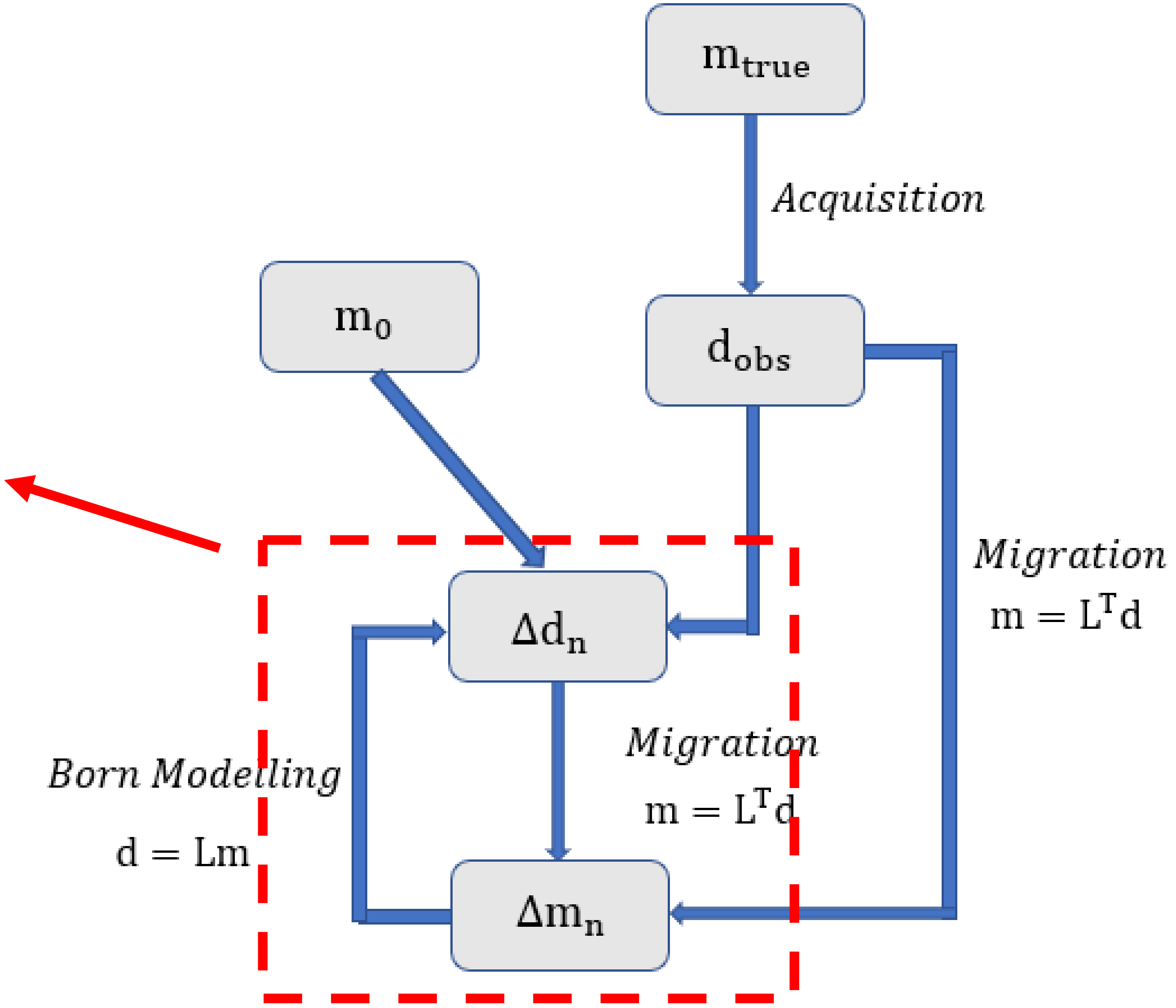
$$\underbrace{d}_{\text{(data)}} = \underbrace{L \qquad m}_{\text{(Modeling operator)} \qquad \text{(reflector)}}$$

Work Flow: Conventional LSRTM

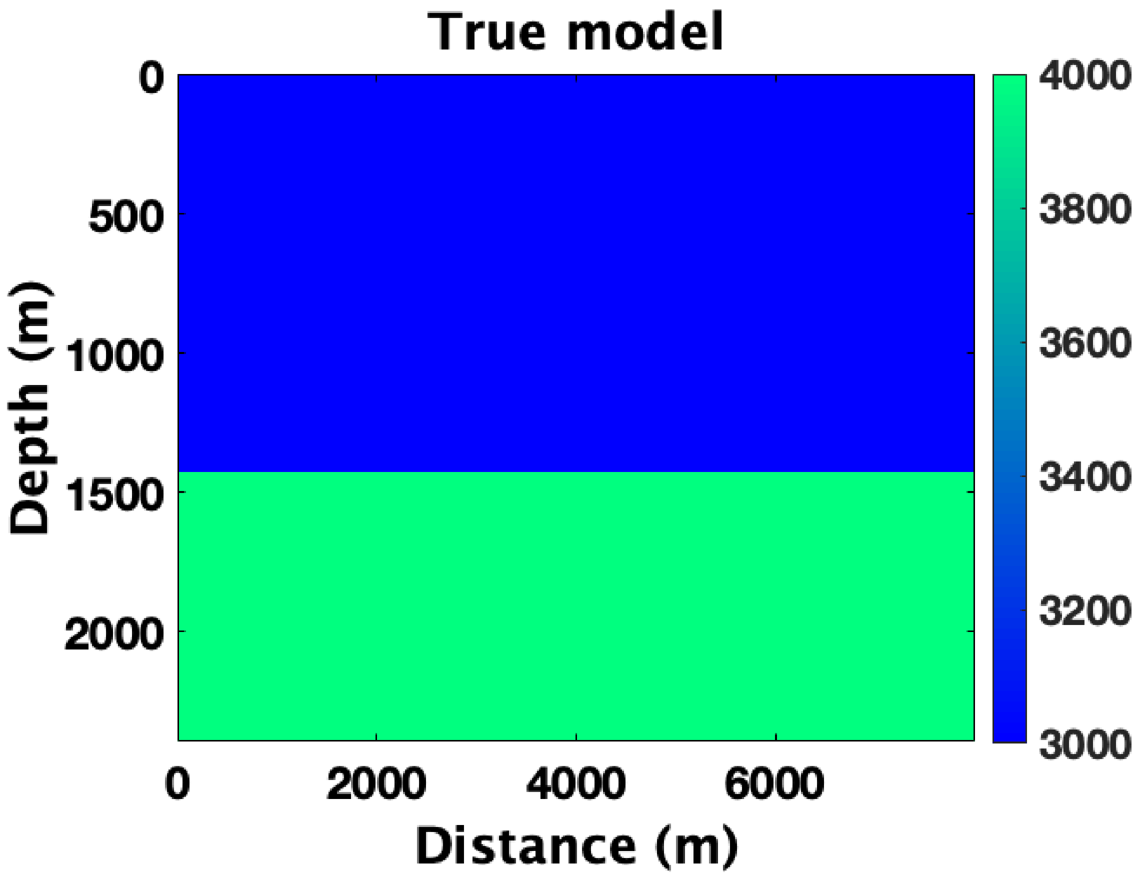


Work Flow: Conventional LSRTM

$$m = \left(L^T L\right)^{-1} L^T d$$

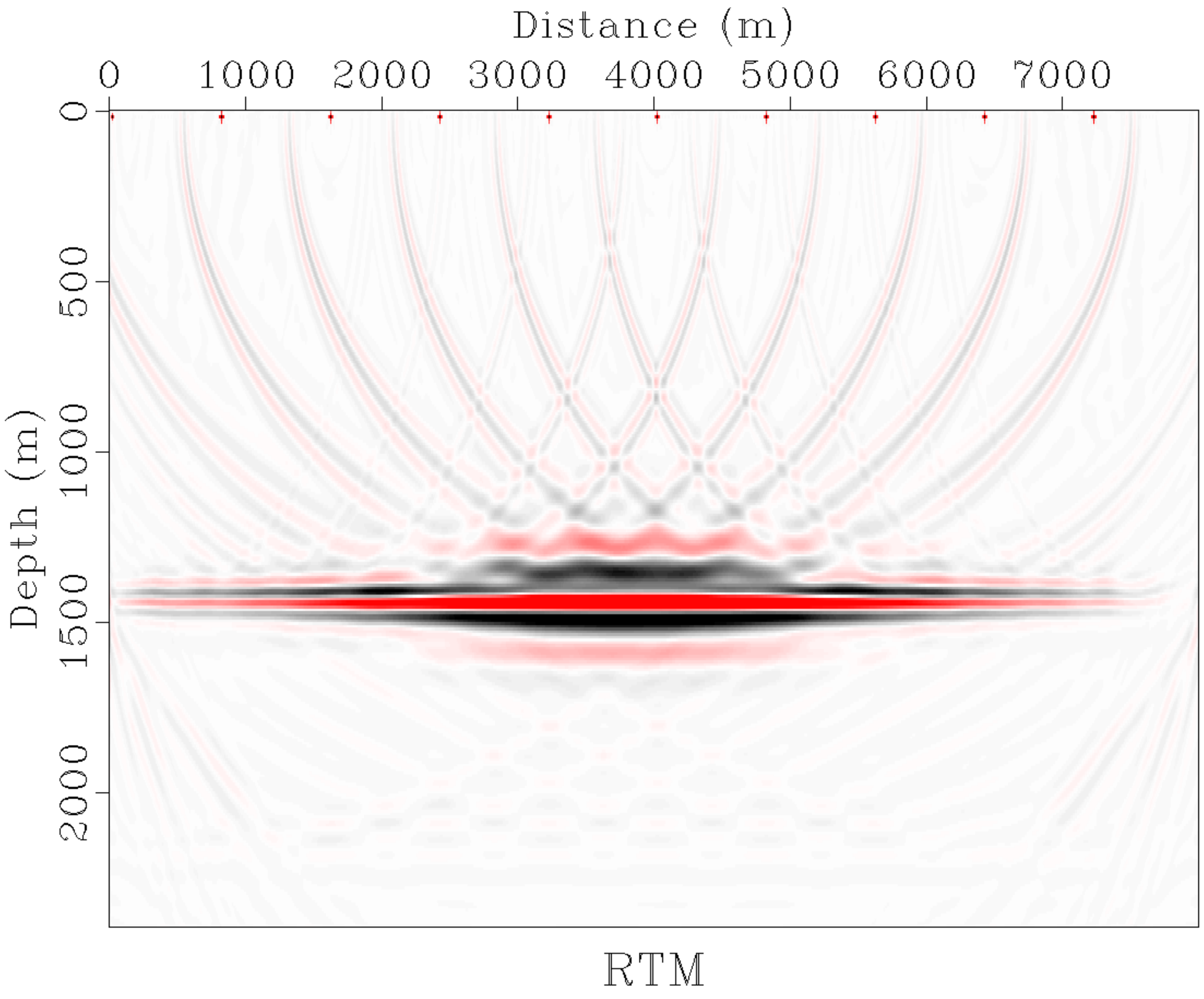


Examples: 2 layers model



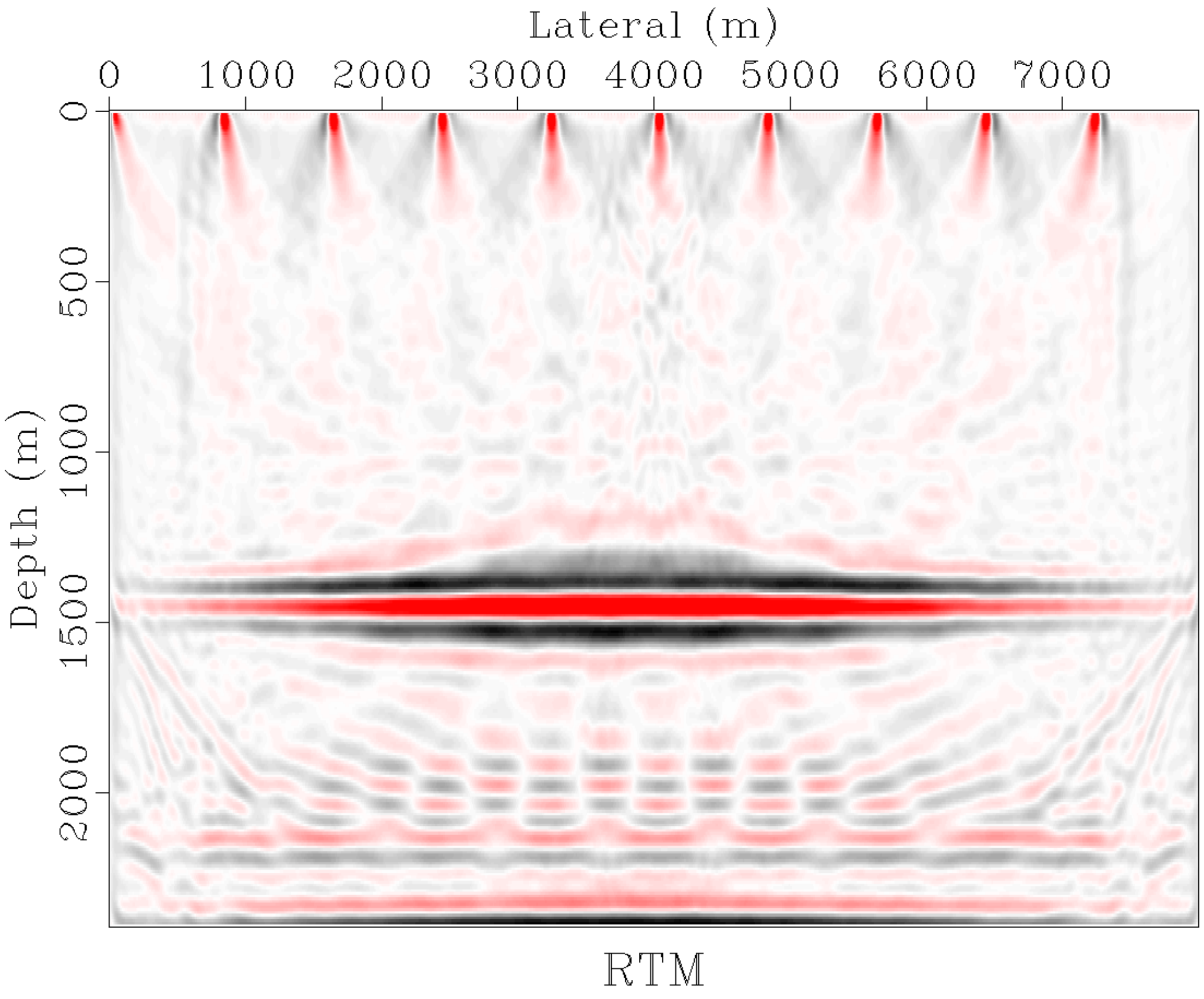
Nz: 300
Nx:1000
dx: 8m
dz: 8m
Nr: 334
Ns: 10

Examples: 2 layers model RTM



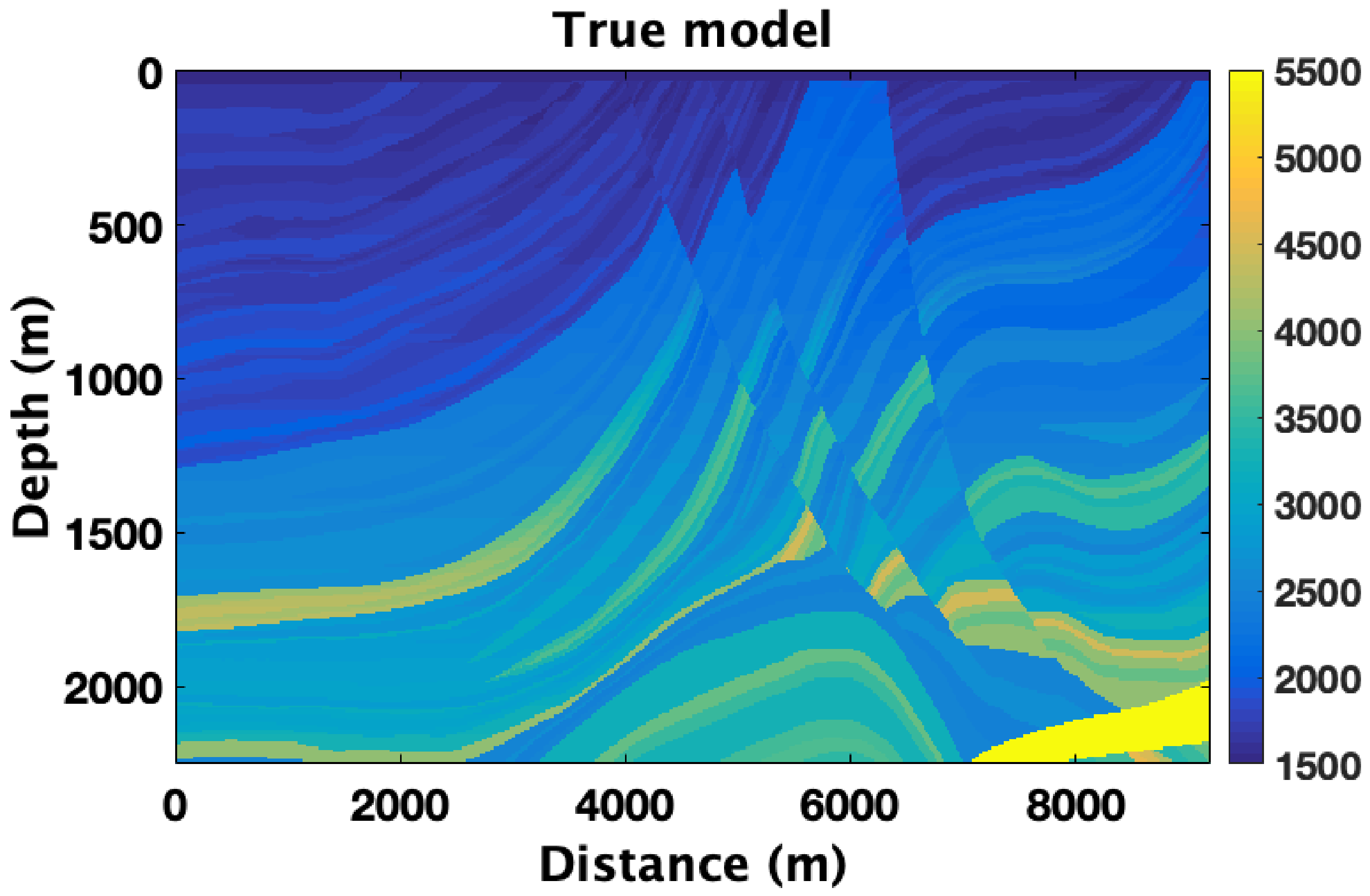
Time domain

Examples: 2 layers model RTM



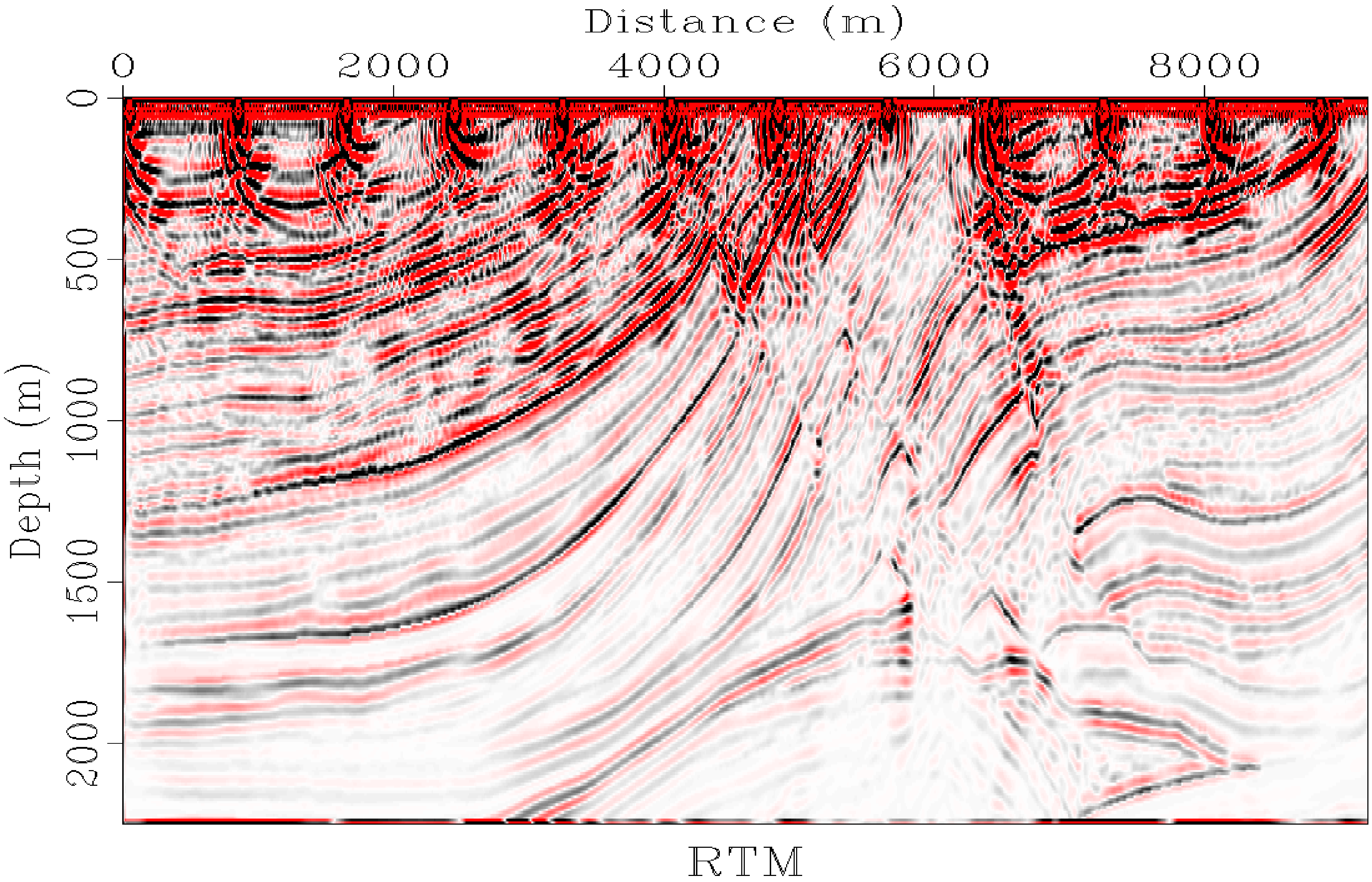
Frequency domain

Examples: Marmousi

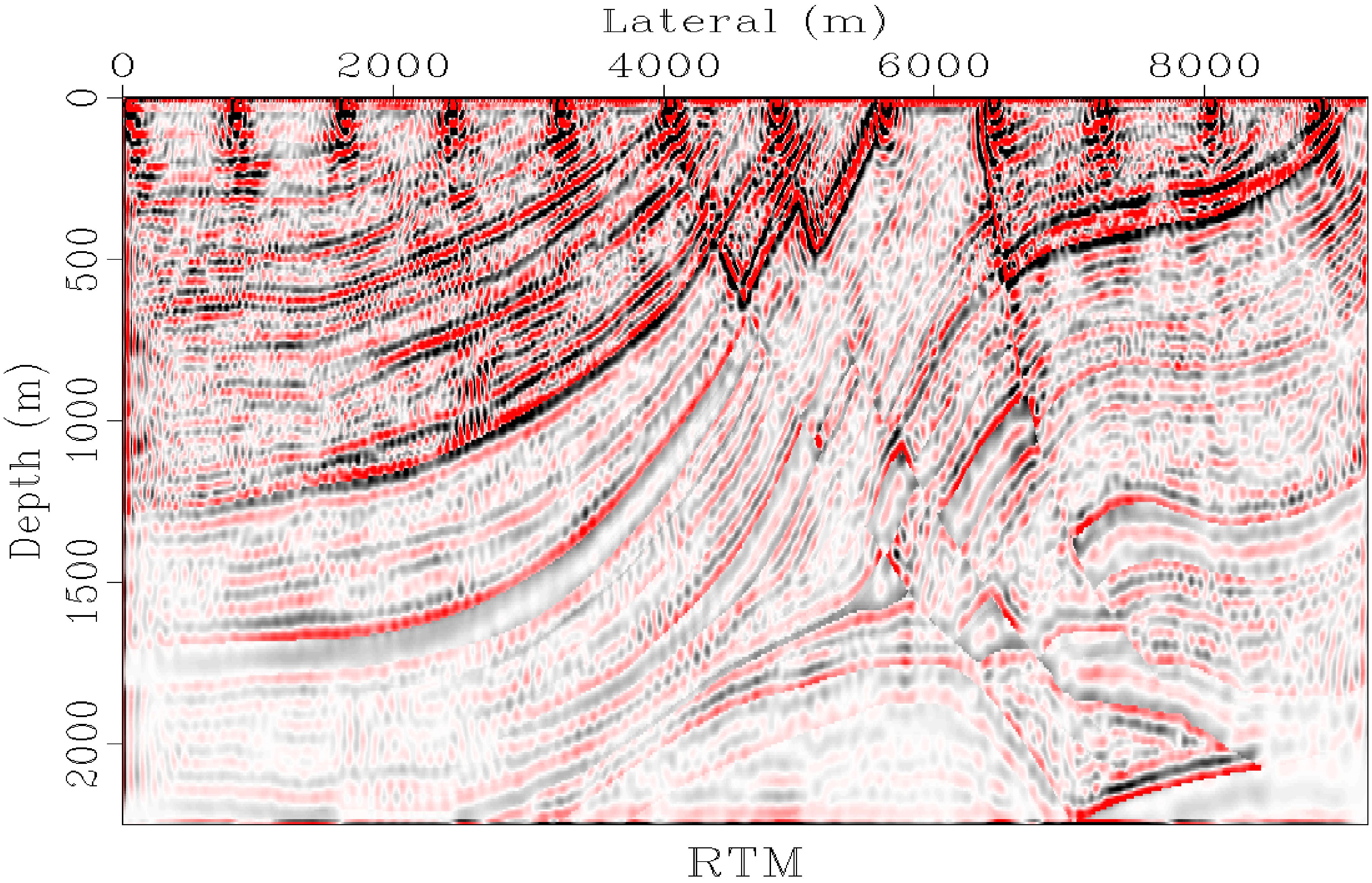


Nz: 282
Nx:1151
dx: 8m
dz: 8m
Nr: 384
Ns: 12

Examples: Marmousi RTM time domain



Examples: Marmousi RTM frequency domain



FWI

$$J(m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s}\left|\left|d_{obs} - d_{syn}\right|\right|^2$$

LSRTM

$$\Phi(m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s}||d - Lm||^2$$

Formulations

FWI

$$J(m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s}\left|\left|d_{obs} - d_{syn}\right|\right|^2$$

Model

Finite-difference

LSRTM

$$\Phi(m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s}\left|\left|d - Lm\right|\right|^2$$

Reflectivity

Born modeling

Gradient & Hessian

If we use the FWI objective function as the initial objective function

$$\delta J(m + \delta m) \approx \delta m^T g + \delta m^T H \delta m = 0$$

$$g = \frac{\partial J}{\partial m} \quad H = \frac{\partial^2 J}{\partial m^2}$$

We can have the linear relationship

$$H\delta m = -g$$

By iteratively solving this equation, we can have the image space objective function

$$\Phi(\delta m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s} ||H\delta m + g||^2$$

Gradient & Hessian

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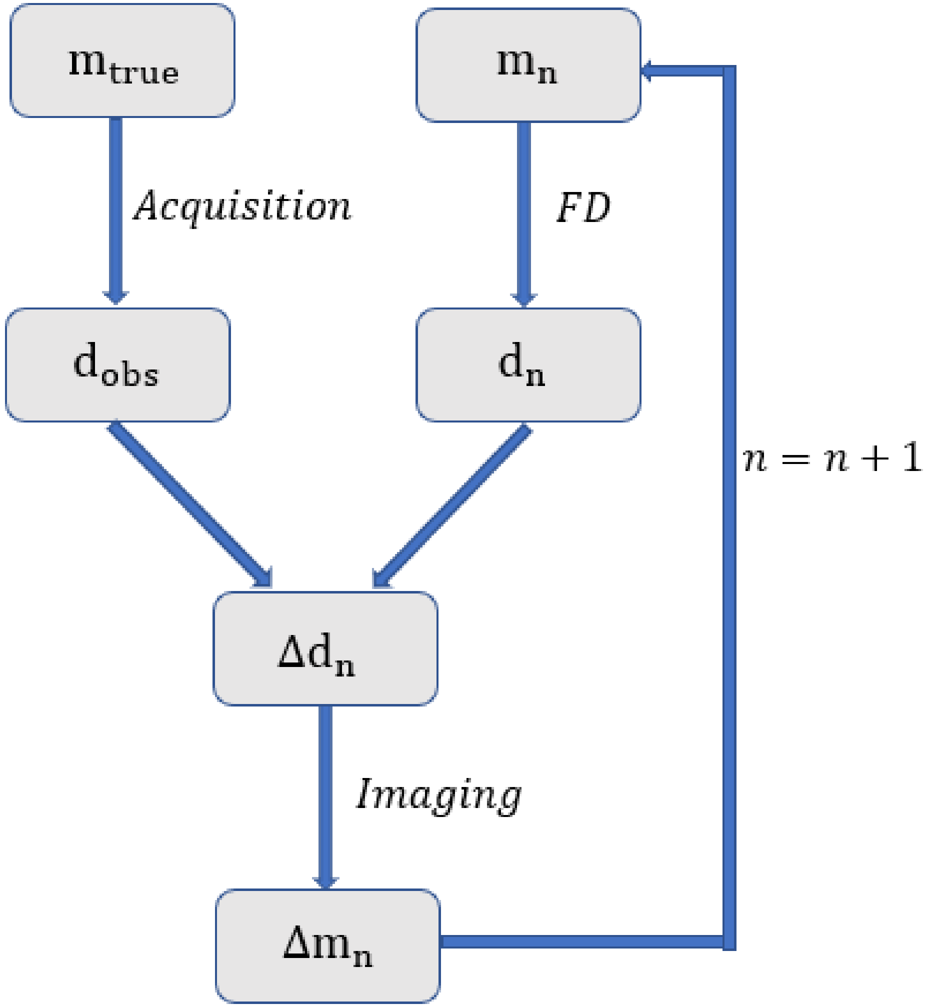
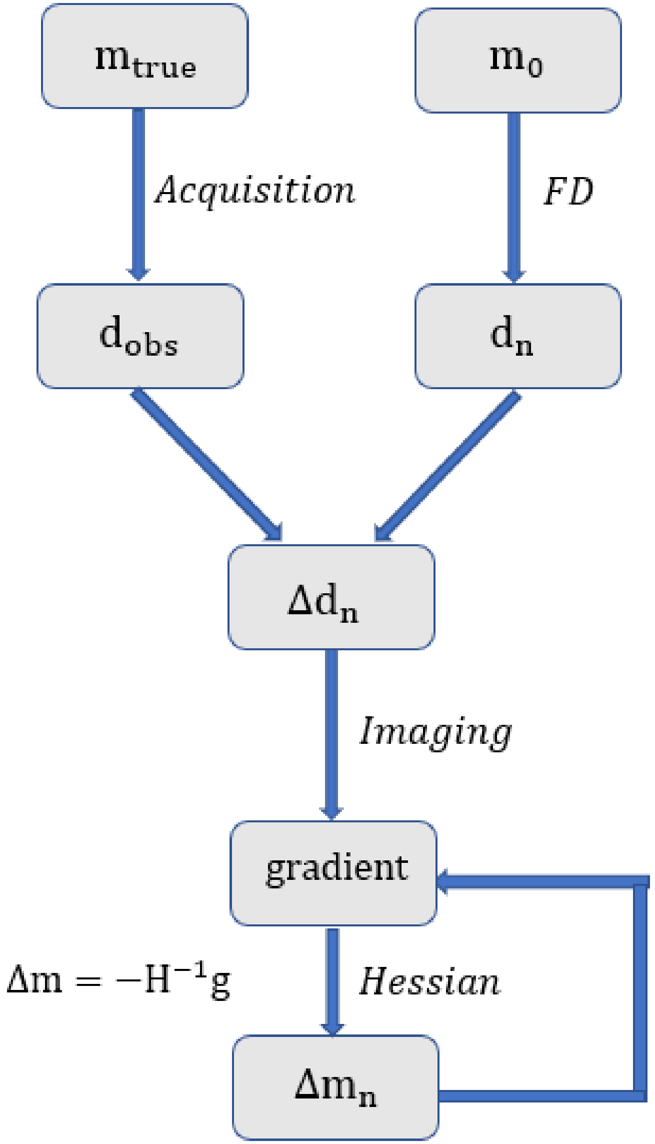
The data space objective function

$$\Phi(\delta m) = \frac{1}{2}\sum_{n_\omega}\sum_{n_s}||d - L\delta m||^2$$

Work Flow

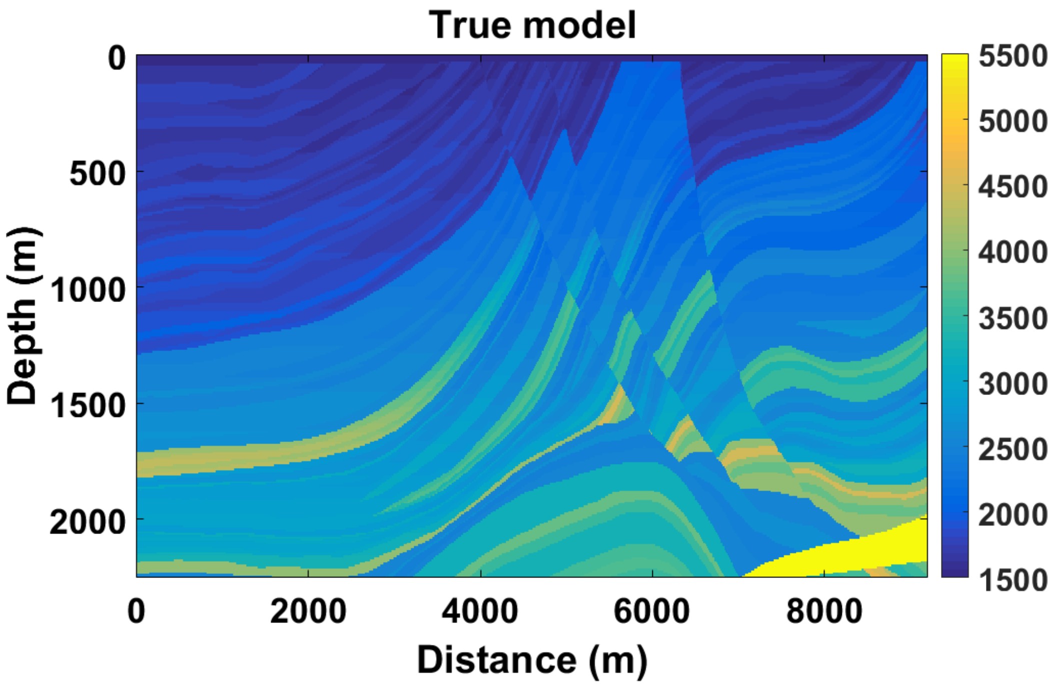


FWI



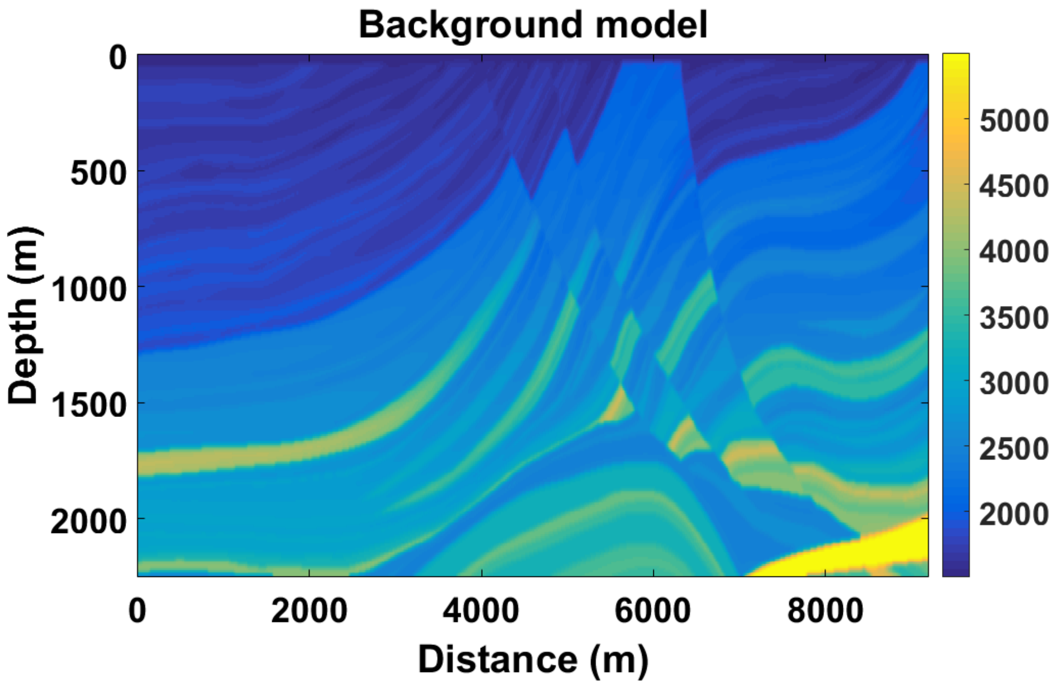
FWI-based LSRTM

Examples: Marmousi

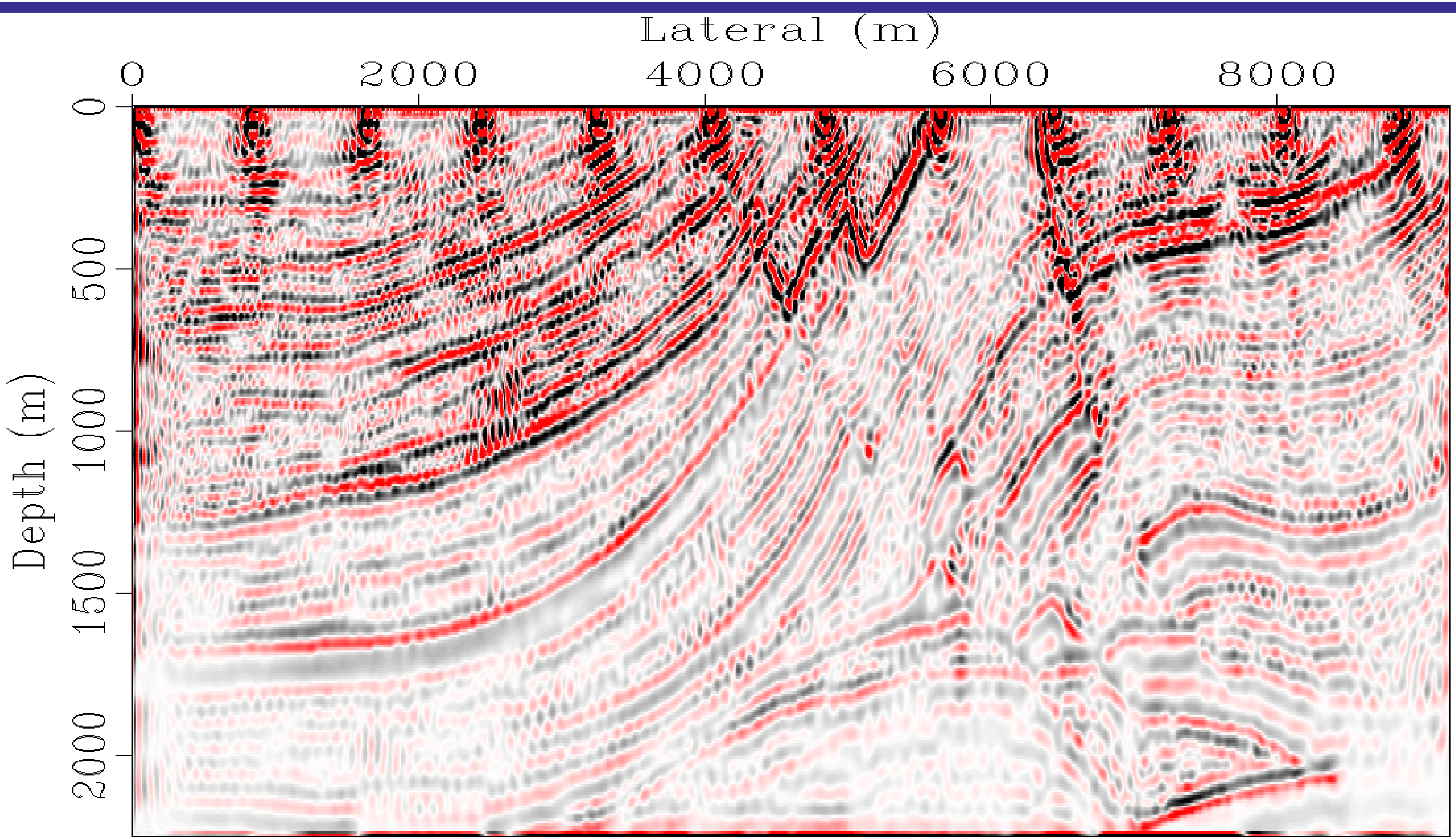


Nr: 384

Ns: 12

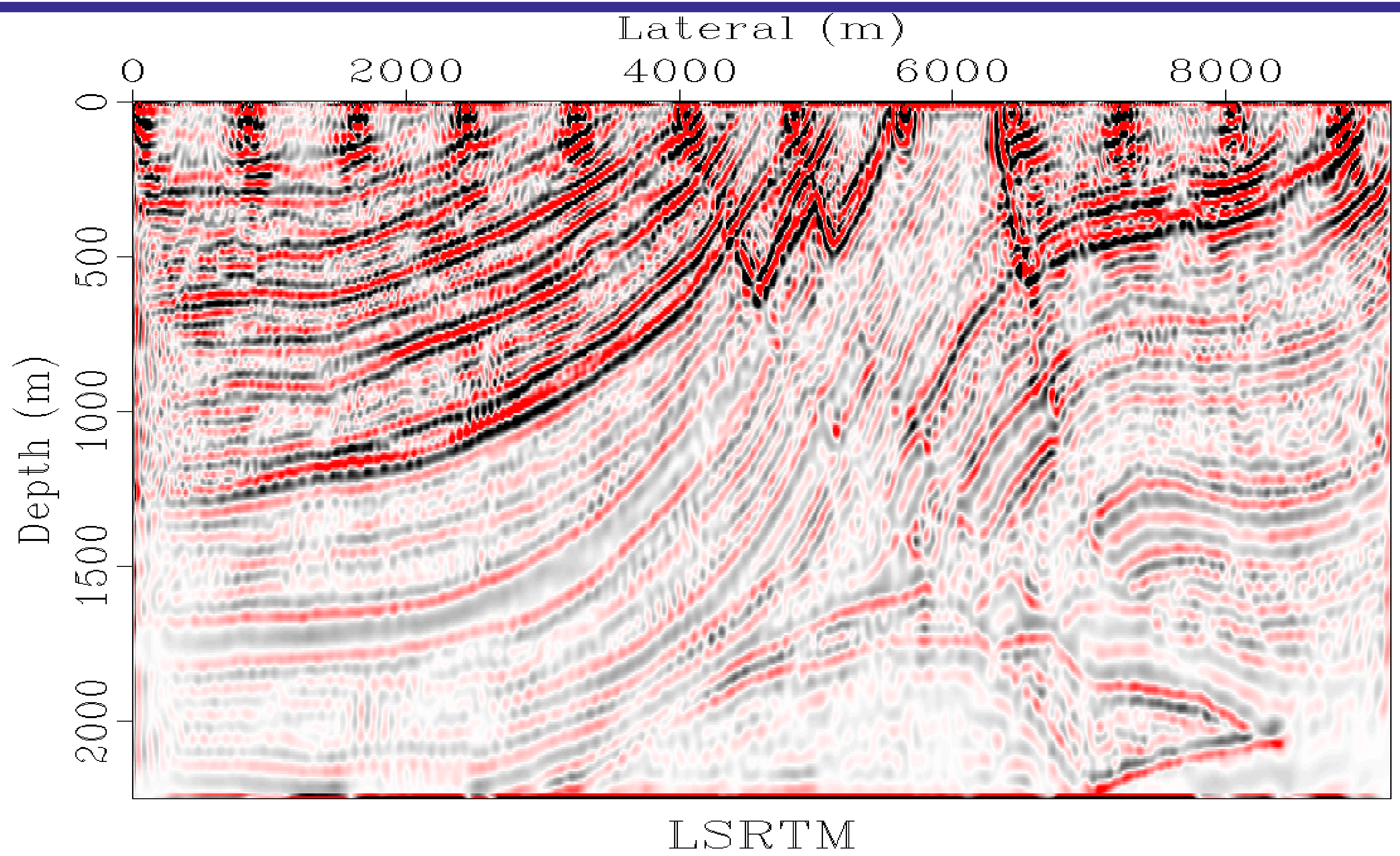


Examples: Marmousi (FWI-based LSRTM)



RTM

Examples: Marmousi (FWI-based LSRTM)



Summary & Future Study

Summary

- Time & Frequency domain LSRTM are presented
- Frequency method is able to use matrix formulation to solve wave equation, which is easier for the adjoint state method
- Using the LU decomposition, frequency method can achieve multi-source modeling
- However, the frequency method suffers from the large computation cost and requires large memory
- Also, the quality of imaging is not as good as time domain and depends on the frequency spacing

Future Study

- Implement standard LS migration in frequency domain and compare with the time domain method

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Questions?