Fermat’s principle and ray tracing in anisotropic layered media

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Introduction
I consider the path followed by a seismic signal travelling through velocity models consisting of horizontal layers with VTI and TTI velocity anisotropy. I describe a ray-bending method based on Fermat’s principle, which states that the raypath between the source and receiver must be the one with the least travel time.

An accurate approximation for quasi-P group velocities in VTI and TTI anisotropy is given by Byun et al. (1989) and Kumar et al. (2004):

\[
V_g^2(\phi) = a_1 + a_2 \cos^2(\phi) + a_3 \cos(\phi \cdot \varphi) + a_4 \cos^2(\phi \cdot \varphi)
\]

\[
a_1 = V_0^2,
\]

\[
a_2 = 4V_{31} - 3V_0^2 - 2V_0^2,
\]

\[
a_3 = 2V_{56} - 3V_0^2 - 2V_0^2.
\]

The dip angle \(\phi\) is measured from the VTI axis of symmetry. The tilt angle \(\varphi\) between the VTI axis of symmetry and the vertical results in TTI anisotropy.

Fermat’s principle and ray bending

Figure 3 shows a bent raypath between a source S and a receiver R intersecting two boundaries. Bending at the intersection points occurs in such a way that the total path must conform to Fermat’s principle. Two-point ray tracing reduces to determining the coordinates \(x_1\) and \(x_2\) of the intersection points that minimize the total travel time \(t\).

\[
l_1 = \sqrt{(x_1-x_2)^2 + (z_1-z_2)^2},
\]

\[
l_2 = \sqrt{(x_1-x_3)^2 + (z_1-z_3)^2},
\]

\[
l_3 = \sqrt{(x_1-x_4)^2 + (z_1-z_4)^2},
\]

\[
t = u_1l_1 + u_2l_2 + u_3l_3.
\]

These equations can be extended to include more layers and more boundary intersection points.

Direct search minimization

A direct-search optimization routine used to find the points \(x_1\) and \(x_2\). For this problem, direct search is faster than a gradient based inversion scheme as there is no need to calculate and invert Jacobian matrices.

Summary

- Two-point ray tracing based on Fermat’s principle together with direct search optimization allows us to quickly produce the raypaths and arrival times for sources and receivers embedded in layered media.
- By including the Byun/Kumar formulation for qP group velocities in VTI and TTI media, the method gives an efficient and flexible means for analyzing VSP, crosswell, and microseismic monitoring travel times in anisotropic layered media.
- Sloping straight boundaries and possibly gently curving boundaries can be accommodated.

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References
