A framework for linear and nonlinear S-wave and C-wave time-lapse difference AVO

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Introduction

The elastic properties of a rock change when the pressure and fluid flow is altered in a reservoir due to production (Greaves 1987). This raise the necessity of multicomponent 4D time-lapse analysis in a reservoir (Stewart et al. 2002 and 2003). A framework has been formulated to model linear and nonlinear elastic time-lapse difference AVO for P-P sections (Jabbari and Innenan 2013). The study described here focuses on applying the perturbation theory in a time-lapse amplitude variation with offset (Time-lapse AVO) method to model a framework for describing the difference data for the converted and shear waves (Figure 1).

Figure 1: Displacement amplitude of an incident P-wave with related reflected and transmitted P and S waves.

Theory

The P-wave and S-wave velocities and the density change from the time of the baseline survey relative to the monitoring survey. Let \( V_{p0}, V_{s0}, \rho_0 \) be the rock properties of the cap rock and reservoir at the time of the baseline survey. Amplitudes of reflected and transmitted waves impinging on the boundary of a planar interface between these two elastic media are calculated through setting the boundary conditions in the Zoeppritz equations in a matrix form (Aki and Richards 2002). Reflection coefficients are determined for the baseline and monitor surveys using the same method. Rock properties for the cap rock are the same, but the reservoir properties change to \( V_{p0}, V_{s0}, \rho_0 \) for the monitor survey. The difference data reflection coefficients between the baseline and monitor survey is calculated as:

\[
\begin{align*}
\Delta R_{PnP}(t) &= R_{PnP}(t) - R_{PnP}(t) \\
\Delta R_{PnS}(t) &= R_{PnS}(t) - R_{PnS}(t) \\
\Delta R_{SnS}(t) &= R_{SnS}(t) - R_{SnS}(t)
\end{align*}
\]

In our time lapse study we have considered two groups of perturbation parameters (Innenan 2013 and Stoll 2012). The first group expresses the perturbation caused by propagating the wavefield from the first medium to the second medium in the baseline survey:

\[
b_{p0} = 1 - \frac{V_{p0}}{V_{s0}}, \quad b_{s0} = 1 - \frac{V_{s0}}{V_{p0}}, \quad b_{p0} = 1 - \frac{\rho_0}{\rho_{p0}}
\]

The second group is to account for the change from the baseline survey relative to the monitor survey, the time lapse perturbation, we define:

\[
a_{p0} = 1 - \frac{V_{p0}}{V_{s0}}, \quad a_{s0} = 1 - \frac{V_{s0}}{V_{p0}}, \quad a_{p0} = 1 - \frac{\rho_0}{\rho_{m}}
\]

Elastic parameters in Zoeppritz matrixes may be re-defined in terms of perturbations in the P- and S-waves velocities and the densities, and reflection coefficients for the shear and converted wave can be re-computed in terms of these perturbation parameters. Therefore, Equations (1) can be expanded in orders of all six perturbations as:

\[
\begin{align*}
\Delta R_{PnP}(t) &= \Delta R_{PnP}^{(1)}(t) + \Delta R_{PnP}^{(2)}(t) + \Delta R_{PnP}^{(3)}(t) + \ldots \\
\Delta R_{PnS}(t) &= \Delta R_{PnS}^{(1)}(t) + \Delta R_{PnS}^{(2)}(t) + \Delta R_{PnS}^{(3)}(t) + \ldots \\
\Delta R_{SnS}(t) &= \Delta R_{SnS}^{(1)}(t) + \Delta R_{SnS}^{(2)}(t) + \Delta R_{SnS}^{(3)}(t) + \ldots \\
\end{align*}
\]

Numerical example

In this section, we examine the derived linear and non linear difference time lapse AVO terms qualitatively with a numerical example for the PS and SP waves. The exact difference data are compared with our derived linear and higher order approximations in Figure 2. The second and third approximations are in better agreement with the exact difference data, especially for angles below the critical angle, which correspond to the range of study in this project.

Figure 2: \( \Delta R_{PS} \) and \( \Delta R_{SP} \) for the exact, linear, second, and third order approximation. Elastic incidence parameters: \( \theta_{inc} = 2000^\circ \text{m/s}, \phi_{inc} = 1500^\circ \text{m/s}, \text{and } \varphi \in [2.0^\circ \text{c}] \). Baseline parameters: \( V_{p0} = 3000^\circ \text{m/s}, V_{s0} = 1700^\circ \text{m/s}, \text{and } \varphi \in [2.1^\circ \text{c}] \). Monitor parameters: \( V_{p0} = 4000^\circ \text{m/s}, V_{s0} = 1900^\circ \text{m/s}, \text{and } \varphi \in [2.3^\circ \text{c}] \).

Changes in the fluid saturation and pressure will have an impact in elastic parameters of subsurface, such as, P, and S wave velocities and density, which can be approximated by applying time-lapse AVO analysis methods. Jabbari and Innenan (2013) have already investigated P-wave time-lapse AVO and showed that adding the higher order terms in \( \Delta R_{PnP} \) to the linear approximation for difference time-lapse data increases the accuracy of the \( \Delta R_{PnP} \) and corrects the error due to linearizing \( \Delta R_{PnP} \). In the current research, we extended this work by formulating a framework for the difference reflection data in \( \Delta R_{PS}, \Delta R_{SP}, \text{and } \Delta R_{SP} \). The results showed that, including higher order terms in \( \Delta S \) for shear and converted waves improves the accuracy of approximating time-lapse difference reflection data, particularly for large contrast cases. Comparing linear, second, and third order terms for \( \Delta R_{PnP} \) and \( \Delta R_{SP} \) indicates that, as we are moving toward higher order approximations; \( \Delta R_{PS} \) and \( \Delta R_{SP} \) are different. This confirms the difference between exact \( \Delta R_{PS} \) and \( \Delta R_{SP} \) which does not show up in the linear approximation case.

Conclusions

References


Bibliography

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