Introduction

Multicomponent surveying has developed rapidly in both land and marine acquisition with many applications in seismology including reservoir monitoring. This raises the necessity of multicomponent 4D time-lapse analysis in a reservoir (Stewart et al., 2003). A framework has been formulated to model linear and nonlinear elastic time-lapse difference for P-P sections (Jabbari et al., 2015). The study described here focuses on applying linear and nonlinear time-lapse amplitude variation with offset methods to model the difference data for converted wave and to investigate the difference between SP and PS wave in nonlinearity.

Theory

Consider two seismic experiments involved in a time-lapse survey, a baseline survey followed by a monitoring survey. Let \( V_{Pb}, V_{Sb}, \rho_b \) and \( V_{Ps}, V_{Ss}, \rho_s \) be the rock properties of the cap rock and reservoir. Now let consider a P wave and an S wave which are impinging on the boundary of a planar interface between the two elastic media; cap rock overlying the reservoir. Amplitudes of reflected and transmitted P and S waves are calculated through setting the boundary conditions in the Kirchhoff equations which can be rearranged in matrix form e.g. (Keys, 1989):

\[
\begin{bmatrix}
R_{Pb} \\
R_{Ps} \\
T_{Pb} \\
T_{Ps}
\end{bmatrix} = \begin{bmatrix}
P_S \\
R_S \\
T_S \\
T_P
\end{bmatrix} \begin{bmatrix}
\rho_S \\
V_{Pb} \\
V_{Ps} \\
\rho_P \\
V_{Pb} \\
V_{Ps} \\
\rho_P \\
V_{Pb} \\
V_{Ps} \\
\rho_P
\end{bmatrix}
\]

(1)

where \( P, S, b, p, S, p, S, \) and \( b \) are matrices and vectors which their elements are functions of P incident angle, S incident angle, and elastic properties which are defined as:

\[
A_x = \frac{\rho_{P}}{\rho_{S}}, \quad B = \frac{V_{Pb}}{V_{Ps}}, \quad C_x = B^{-1}, \quad D_x = \frac{V_{Ps}}{V_{Pb}}, \quad E = \frac{V_{Ps}}{V_{Ps}}, \quad F = \frac{V_{Ps}}{V_{Ps}}.
\]

(2)

Reflection coefficients for both converted waves are determined by forming an auxiliary matrix \( P_{SP} \) by replacing the second column of \( P \) with \( b \), and then forming another auxiliary matrix \( S_{SP} \) by replacing the second column of \( S \) with \( b_S \) to:

\[
R_{SP}(\phi) = \frac{\text{det}(P_{SP})}{\text{det}(P)} \quad R_{SP}(\phi) = \frac{\text{det}(S_{SP})}{\text{det}(S)}
\]

(3)

\( R_{SP} \) and \( R_{SP} \) for the baseline and monitor surveys are calculated using the method explained above, where rock properties for cap rock are the same, but reservoir properties change from \( V_{Pb}, V_{Ps}, \rho_b \) (replace \( x = b \) in Equation 2) at the time of the baseline survey to \( V_{Pm}, V_{Sm}, \rho_m \) (replace \( x = m \) in Equation 2) at the time of the monitor survey.

\[
\Delta R_{SP}(\phi) = R_{SP}(m)(\phi) - R_{SP}(b)(\phi)
\]

(4)

We have considered two groups of perturbation parameters (Stolt and Weglein, 2012). The first group expresses the perturbation in the baseline survey. The second group expresses the time-lapse perturbation.

Theory continued

\[
a_{SP} = 1 - \frac{V_{Sb}^2}{V_{Sb}^2} = 1 - \frac{V_{Ps}^2}{V_{Ps}^2}, \quad a_p = 1 - \frac{\rho_b}{\rho_p}
\]

(5)

Re-defining elastic parameters in terms of perturbation parameters, Equation 4 can be calculated and then expanded in first and second order for all six perturbations, \( \sin^2 \phi \) and \( \sin \phi \).

\[
\Delta R_{SP}(\phi) = \Delta R_{SP}^{(1)}(\phi) + \Delta R_{SP}^{(2)}(\phi) + \Delta R_{SP}^{(3)}(\phi) + \ldots
\]

\[
\Delta R_{SP}(\phi) = \Delta R_{SP}^{(1)}(\phi) + \Delta R_{SP}^{(2)}(\phi) + \Delta R_{SP}^{(3)}(\phi) + \ldots
\]

(6)

Results

The linear, second, and third order terms for time-lapse difference data for PS and SP converted wave are calculated and can be found in the context of the report. In this section, we examine the derived linear and nonlinear difference time-lapse AVO for PS and SP converted wave qualitatively with numerical examples. In the first example, the data used by Landro (2001) are applied. The exact data and results obtained are compared with the calculated linear and higher order approximations in Figure 2 and Figure 3. Results are also compared for the higher contrast in seismic parameters in the reservoir after the production.

Conclusions

Jabbari et al. (2015) have shown that adding the higher order terms in \( \Delta R \) to the linear approximation for difference time-lapse data increases the accuracy of \( \Delta R \). In this study we focused on the difference between \( \Delta R_{SP} \) and \( \Delta R_{SP} \) for SP and PS converted wave. The results showed that, including higher order terms in \( \Delta R \) for converted wave improves the accuracy of approximating time-lapse difference reflection data, particularly for large contrast cases. Comparing linear, second, and third order terms for \( \Delta R_{SP} \) and \( \Delta R_{SP} \) indicates as we are moving toward higher order approximations; \( \Delta R_{SP} \) and \( \Delta R_{SP} \) are different. This confirms the difference between exact \( \Delta R_{SP} \) and \( \Delta R_{SP} \) which does not show up in the linear approximation case.

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Bibliography