Borehole reflection extraction using K-L transform and azimuth ambiguity elimination by 3D borehole RTM
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Abstract
Sonic reflection logging, as one of the recently developed borehole geophysical schemes, is capable of providing with a clear view of the structures up to 40 m away from well site theoretically. Under acoustic well logging conditions, it is critical to effectively extract the reflection signals from the acoustic full waveforms in acoustic reflection well logging data processing. The Karhunen-Loeve transformations combined a band limit filter is used to extract reflections of interest out of dominant head waves. The azimuth ambiguity is another issue due to the intrinsic defect of the 2D data processing that treats recorded real data as a 2D data set, which inevitably leads us to take for granted that the data (which actually may be from every possible direction of underneath formations) is only from one direction. Therefore, the 3D reverse time migration in the borehole environment is proposed and applied after the reflections have been extracted.

Karhunen-Loeve (KL) transformation
The typical sonic log can be considered as a N-dimensional vector $x = (x_1, x_2, \ldots, x_N)$. The mean value of this N-dimensional vector can be described as $\mu_x = \frac{1}{N} \sum x_i$. Its covariance matrix thus can be written as,

$$C_x = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu_x)(x_i - \mu_x)^T$$

Assume $\lambda_i$ ($i = 1, 2, \ldots, N$) being the eigenvalue of the covariance matrix with its corresponding eigenvector $e_i$ ($i = 1, 2, \ldots, N$). Thus, we have an orthogonal matrix of $A$. Let $Y = A(X - \mu_x)$, we get $X = A^T Y + \mu_x$.

Let us consider the first $k$ largest eigenvectors $A_k$, we have an approximate matrix of $X$, $\tilde{X} = A_k Y + \mu_x$. There, the mean square error can be written as

$$e(x) = \frac{1}{N} \sum_{i=1}^{N} (x_i - \tilde{x}_i)^2$$

Conclusions
The KL transformation is applied in this paper to separate reflections away from direct signals in acoustic reflection well logging data. Based on energy difference of each signal component, the direct waves can be efficiently removed. Comparisons with MSTC method both from synthetic data and laboratory data show KL transformation is capable of providing much more precise reflection signals. Imaging results of the two horizontal models using 2D borehole RTM illustrate the generation of azimuthal ambiguity. In order to eliminate the azimuth ambiguity due to the mechanism of borehole radiation and reception response in 2D environment, the 3D borehole RTM is proposed to solve the azimuth ambiguity problem, taking the advantage of 8 omnidirectional hydrophones evenly spaced around the borehole to receive reflections from all directions. The imaging results of the 3D synthetic model show the azimuth ambiguity problem can be fixed even with a monopole source.

3D borehole RTM scheme
The basic theory of RTM is simple, which mainly is composed of simulation of source wavefield, backward simulation of the received waveforms and the application of imaging condition, where, the forward and backward wavefield simulation are realized by the staggered-grid finite difference (FD) method in this paper.

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