FWI without tears: a forward modeling free gradient

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Abstract

Full waveform inversion (FWI), a machine learning algorithm with, has goal to find the Earths model parameters that minimize the difference of acquired and synthetic shots. This work presents some new and promising approximations for the gradient. Initially, the gradient is computed on the classic and a band limited impedance inversion is applied on each trace. The second approximation is to understand the gradient as a residual impedance inversion between iterations of the pre stack depth migrated shots, so no forward modeling is needed. The last one is to use a post stack depth migration and compute the impedance inversion after. All approximations are able to add features to the initial model, each one with a different final resolution. However, the computer specification require to run each method differs abruptly, while the first one requires several nodes for parallel processing, the others two methods could be done on a personal laptop.

Introduction and Theory

Full waveform inversion is a least squares minimization based algorithm. Its goal is to estimate the best model in which the computed synthetic data is equals to the acquired data. In other words, it minimizes the objective function:

$$C(\mathbf{m}) = ||\mathbf{d}_0 - \mathbf{d}(\mathbf{m})||^2 = ||\Delta \mathbf{d}(\mathbf{m})||^2 \quad (1)$$

The model is estimated iteratively and the gradient method is the one chosen for this work task:

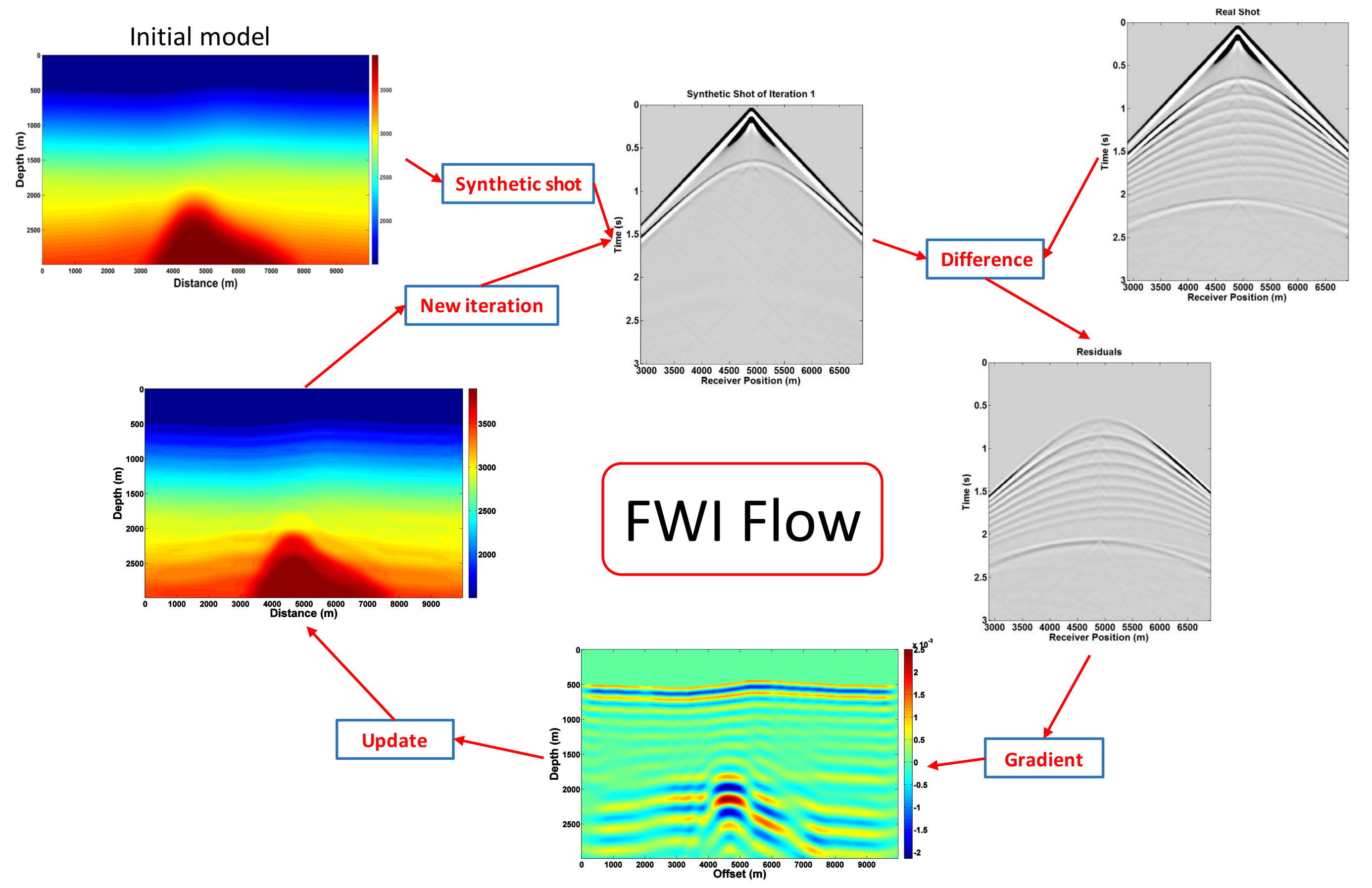
$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n \mathbf{g}_n \quad (2)$$

where g is the gradient, α is the step length and n is the iteration number. This method requires a forward modeling on each shot position to compute the residuals (difference between synthetic and real shots), migrate each one of them, stack and apply an impedance inversion to estimate the gradient plus a forward modeling to compute the step length . Each one of these steps are very well known seismic processing tools. With that in mind, equation 2 can be written as a seismic processing work flow, and the update is intuitive:

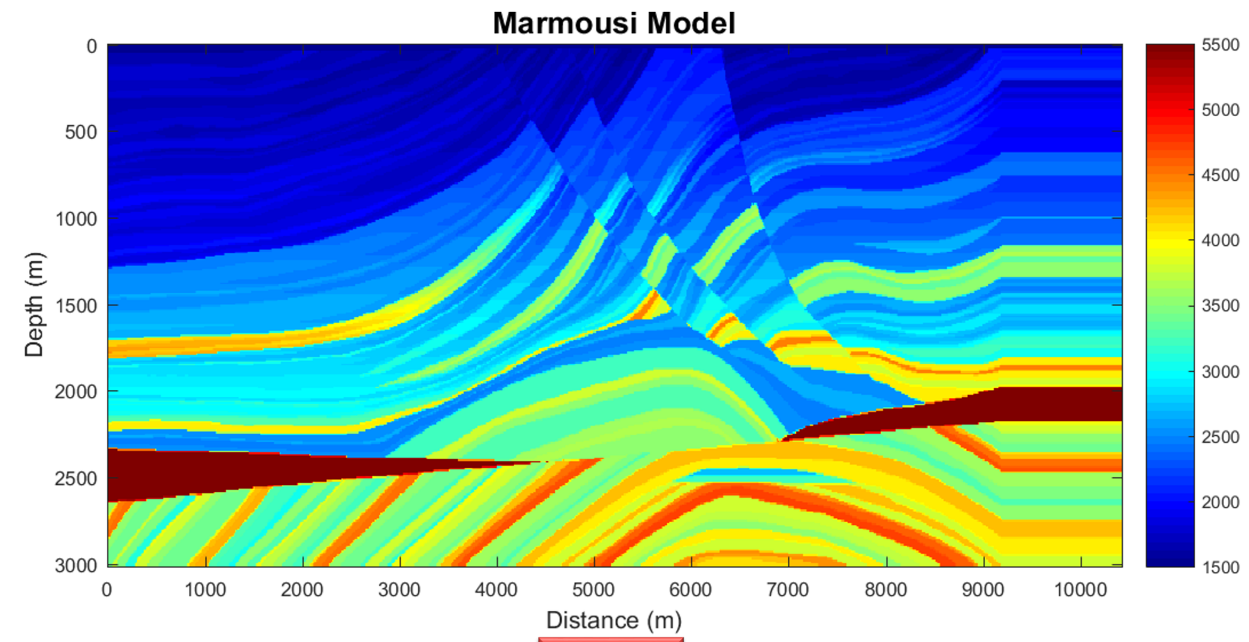
$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n I\left\{S\left[M\left(\mathbf{d}_0 - \mathbf{d}_n\right)\right]\right\} \quad (3)$$

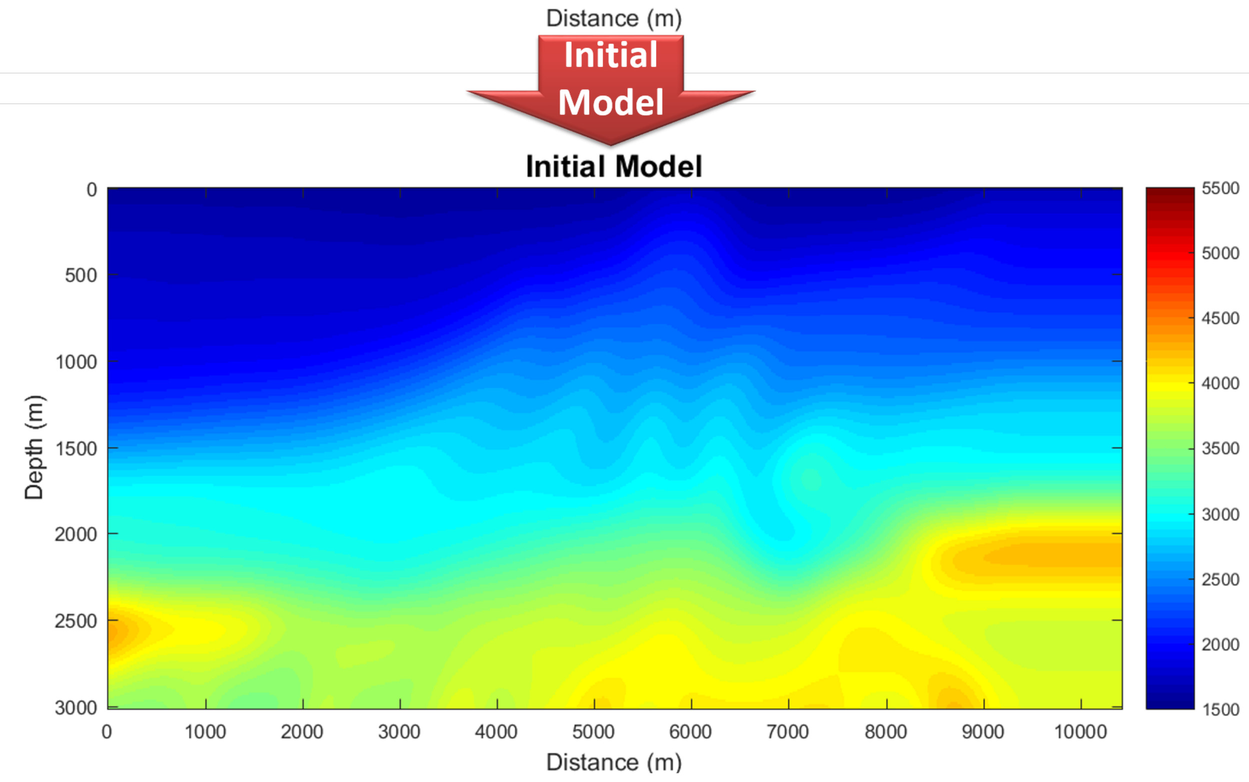
On equation 3 the gradient is opened in terms of the migration operator M (the PSPI is used), the stacking operator S and the impedance inversion operator I (it is done by a band limited impedance inversion, or BLIMP, algorithm using the initial model to fill the low frequency content).

The algorithm runs iteratively and follows the schematic flow below:



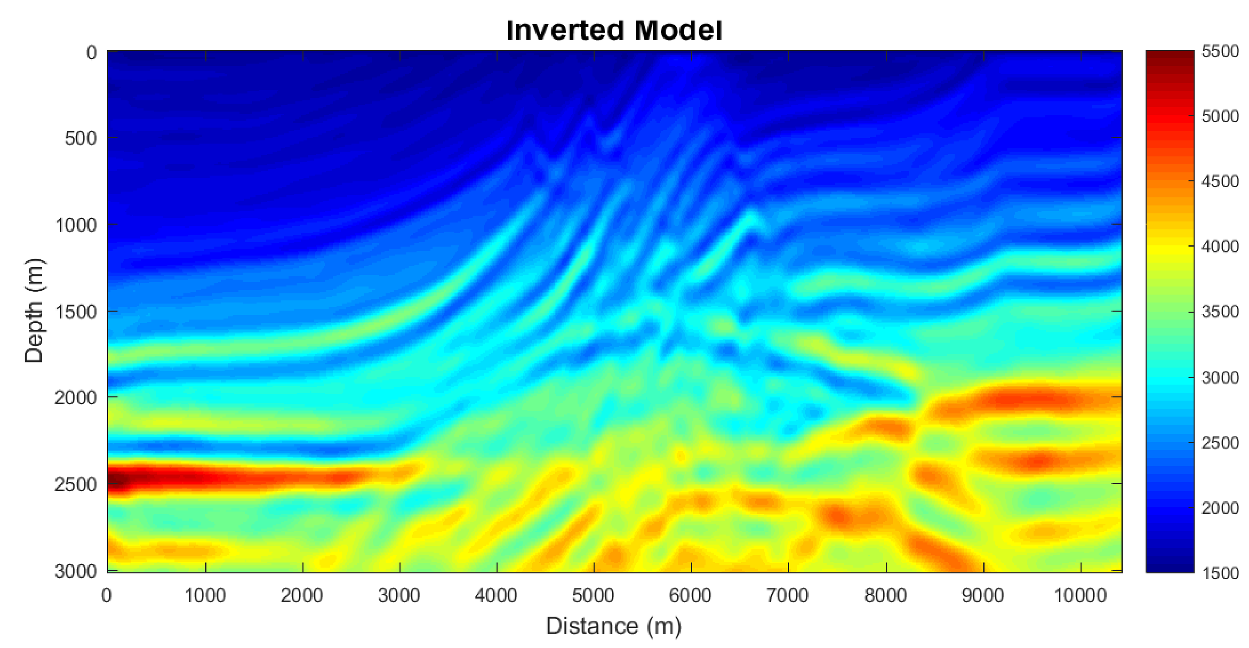
Synthetic Survey





Synthetic survey is done on the Marmousi model and its smoothed version is used as initial guess.

Results: Classic FWI



Inversion based on equation 2 and a BLIMP inversion of the gradient.

New Approximation

The new approximation starts from the assumption that all the seismic processing tools on equation 3 are linear and it can be written as:

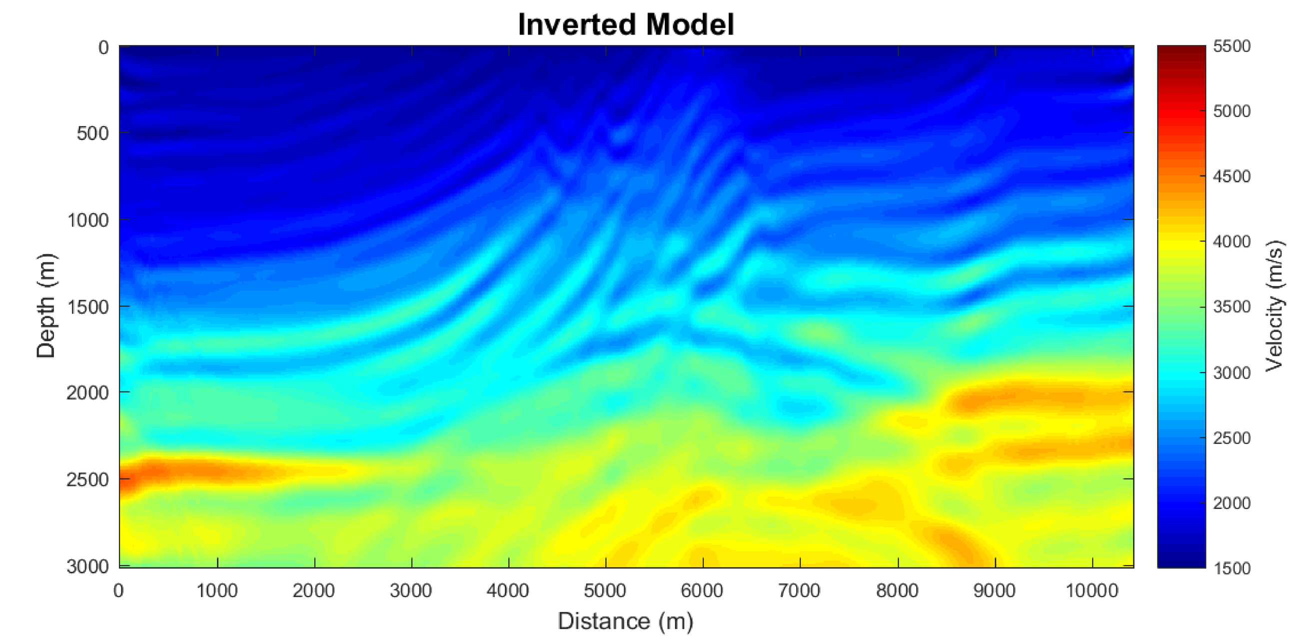
$$\begin{aligned}\mathbf{m}_{n+1} &= \mathbf{m}_n - \alpha_n(I\left\{S\left[M\left(\mathbf{d}_0\right)\right]\right\} - \underbrace{I\left\{S\left[M\left(\mathbf{d}_n\right)\right]\right\}}_{\text{Current model}}) \\ &= \mathbf{m}_n + \alpha_n(I\left\{S\left[M\left(\mathbf{d}_0\right)\right]\right\} - \mathbf{m}_n) \quad (4)\end{aligned}$$

Equation 4 tells us that the gradient can be understood as a residual impedance between the impedance inversion of the migrated and stacked acquired data and the current model. No forward modeling is required during the gradient estimation. Two synthetic data are only needed to calculate the step length, and this independent to the number of shots in the project. Going even furtherer, the order of the migration and stacking operators can be changed. Equation 4 becomes:

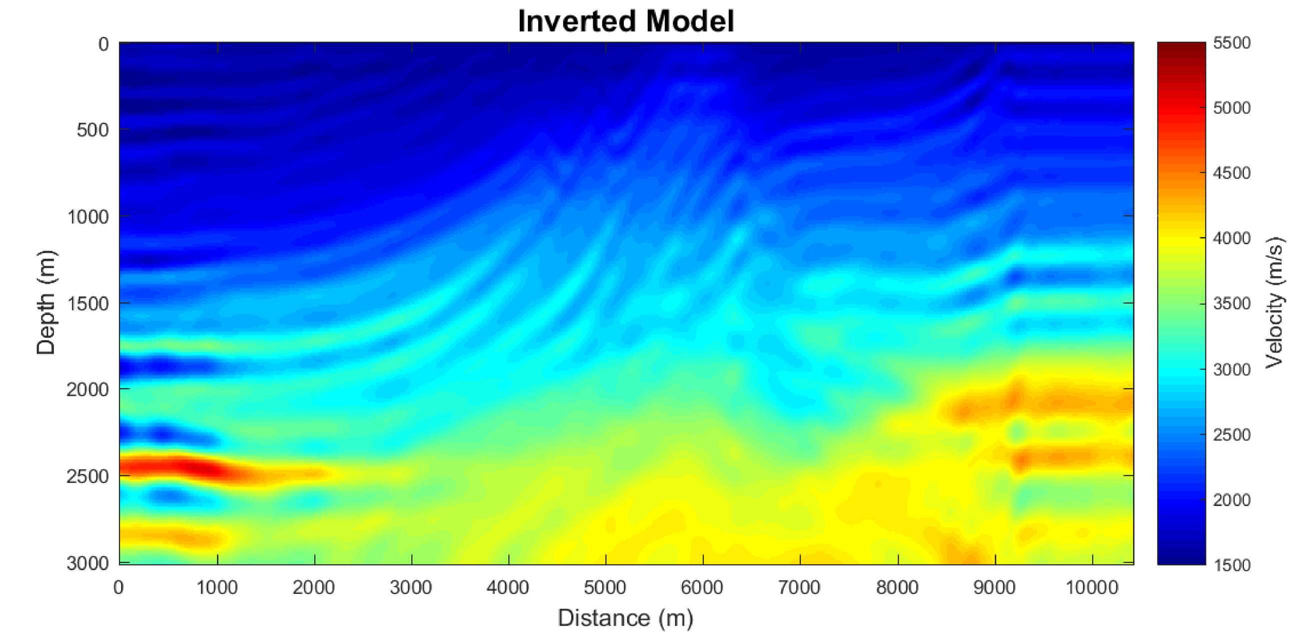
$$\mathbf{m}_{n+1} = \mathbf{m}_n + \alpha_n(I\left\{M\left[S\left(\mathbf{d}_0\right)\right]\right\} - \mathbf{m}_n) \quad (5)$$

Estimating the gradient is reduced to a post-stack depth migration and impedance inversion of the acquired data.

Results: Forward Modeling Free Gradient

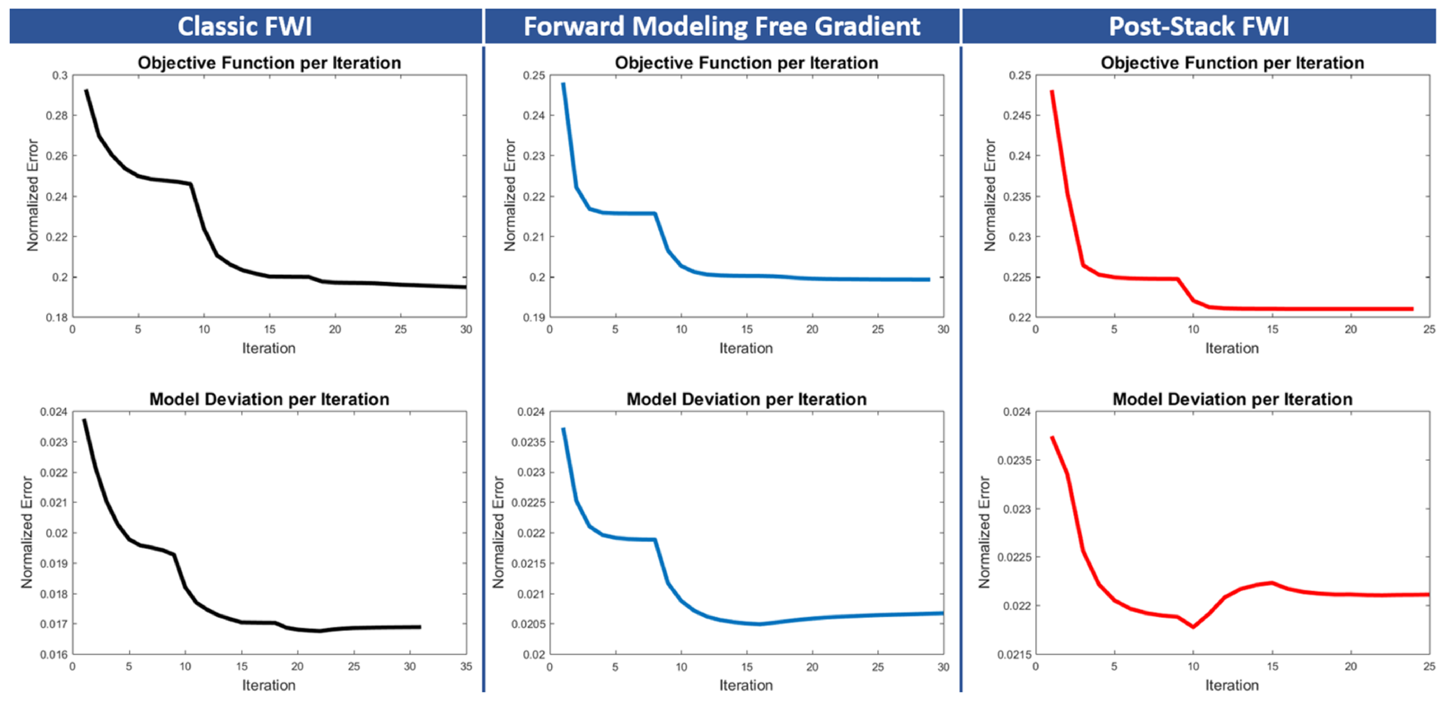


Forward modeling free gradient FWI inversion (equation 4).



Post-stack method (equation 5) resulted on the model above.

Results: Errors Plots



Shot errors (objective function) and model deviation plots of all the tests done.

Conclusions

We presented new ways to estimate the gradient of a FWI algorithm. Initially, a BLIMP converted the migrated residuals from reflection coefficients to velocity using the initial model as pilot, resulting on a high resolution inversion. However, it requires synthetic data to be computed on every shot position and iteration. We came with the understanding of the gradient as the residual difference of the impedance inversion of the migrated and stacked acquired data and the current model, reducing the need of forward modelings only to estimate the step length and the inverted model is comparable with the classic FWI. In the end, we changed the order of the processing tools (migration and stack) for a post-stack forward modeling free gradient FWI with promising results.

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