# Towards seismic moment tensor inversion for source mechanism

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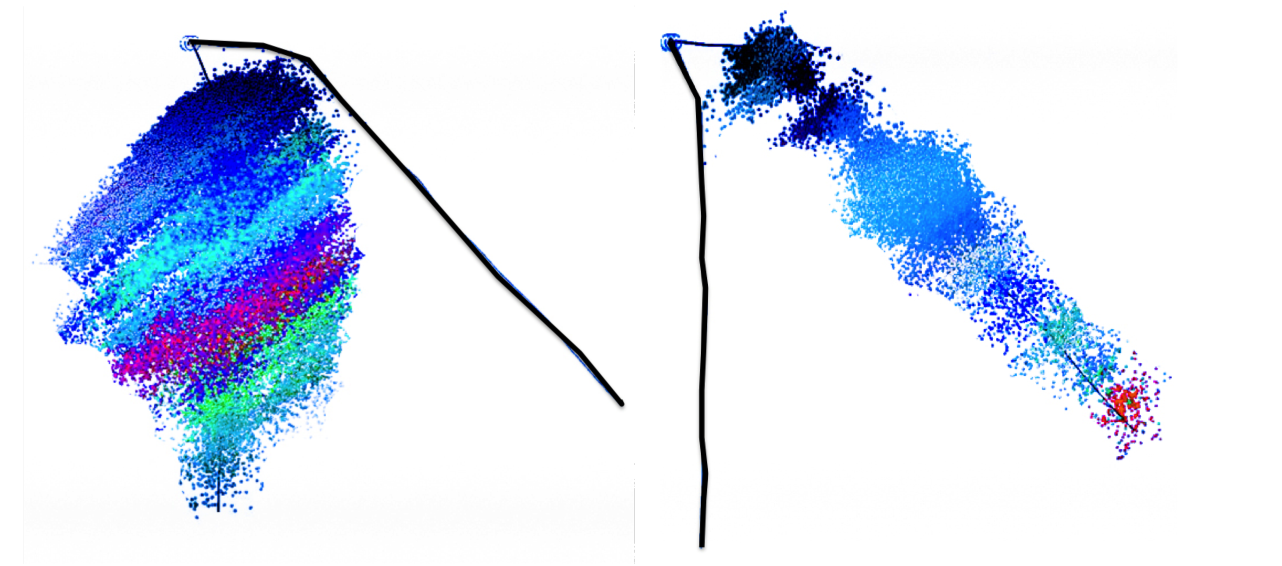
## Summary


- To obtain seismic moment tensor, $M_{pq}$, from amplitude inversion of multi-component microseismic data.
- The determination of the proper observation geometry for which the full moment tensor is resolvable: two vertical or surface receiver arrays.
- To avoid picking task, Initiated a waveform inversion for source-time function and $M_{pq}$.
- Obtained an accurate source-time function estimate.


## Motivation



- FIG.1. A hypotetical well with two side-tracks and known production rate, produced based on the first author's experience with unconventional shale reservoirs in central Alberta. **(Left)** The side-track with denser microseismic clouds and a low production rate. **(Right)** The side-track with less dense microseismic clouds and a high production rate.

- $M = M_{DC} + M_{ISO} + M_{CLVD}$
- The $M_{pq}$ ($3 \times 3$ symmetric matrix) can be decomposed into double-couple (DC), isotropic (ISO) and compensated-linear-vector-dipole (CLVD) (related to tensile fracturing) components - the tensile components might show a correlation to hydrocarbon production rate.

## Receiver geometry effect

- Isotropic homogenous medium, Far-field displacement of P- and S-wave

$$u_n(\vec{x}, t) = m_{pq}(t) * g_{np,q}(\vec{x}, t) \qquad (1)$$

- Green's functions

$$g^P_{np,q}(\vec{x}) = \frac{\gamma_n\gamma_p\gamma_q}{4\pi\rho\alpha^3 r}$$

$$g^S_{np,q}(\vec{x}) = \frac{(\delta_{np} - \gamma_n\gamma_p)\gamma_q}{4\pi\rho\beta^3 r}$$

where $\gamma_n = \frac{(\vec{x}-\vec{x}_s)_n}{r}$, $r$ is source-receiver distance

- Source-time function, $S(t)$

$$m_{pq}(t) = M_{pq}S(t)$$

- Test $M$

$$\begin{bmatrix} 1 & 6 & 0.5 \\ 6 & -2 & -1 \\ 0.5 & -1 & 4 \end{bmatrix}$$

- Linear inversion of P- and S-wave first arrival amplitudes in the time domain.

$$\begin{bmatrix} u^P_1 \\ u^P_2 \\ u^P_3 \end{bmatrix} = \begin{bmatrix} \gamma_1^3 & \gamma_1\gamma_2^2 & \gamma_1\gamma_3^2 & 2\gamma_1\gamma_2\gamma_3 & 2\gamma_1^2\gamma_3 & 2\gamma_1^2\gamma_2 \\ \gamma_2\gamma_1^2 & \gamma_2^3 & \gamma_2\gamma_3^2 & 2\gamma_2^2\gamma_3 & 2\gamma_2\gamma_1\gamma_3 & 2\gamma_2^2\gamma_1 \\ \gamma_3\gamma_1^2 & \gamma_3\gamma_2^2 & \gamma_3^3 & 2\gamma_3^2\gamma_2 & 2\gamma_3^2\gamma_1 & 2\gamma_3\gamma_1\gamma_2 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}$$

$$\begin{bmatrix} u^S_1 \\ u^S_2 \\ u^S_3 \end{bmatrix} = \begin{bmatrix} \gamma_1 - \gamma_1^3 & -\gamma_1\gamma_2^2 & -\gamma_1\gamma_3^2 & -2\gamma_1\gamma_2\gamma_3 & \gamma_3 - 2\gamma_1^2\gamma_3 & \gamma_2 - 2\gamma_1^2\gamma_2 \\ -\gamma_2\gamma_1^2 & \gamma_2 - \gamma_2^3 & -\gamma_2\gamma_3^2 & \gamma_3 - 2\gamma_2^2\gamma_3 & -2\gamma_2\gamma_1\gamma_3 & \gamma_1 - 2\gamma_2^2\gamma_1 \\ -\gamma_3\gamma_1^2 & -\gamma_3\gamma_2^2 & \gamma_3 - \gamma_3^3 & \gamma_2 - 2\gamma_3^2\gamma_2 & \gamma_1 - 2\gamma_3^2\gamma_1 & -2\gamma_3\gamma_1\gamma_2 \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{22} \\ M_{33} \\ M_{23} \\ M_{13} \\ M_{12} \end{bmatrix}$$

- System of linear equations solved with least-squares method for $M_{pq}$.
- Synthetics seismograms produced by TIGER 3D anisotropic elastic finite-difference modelling software.
- Various receiver configuration.

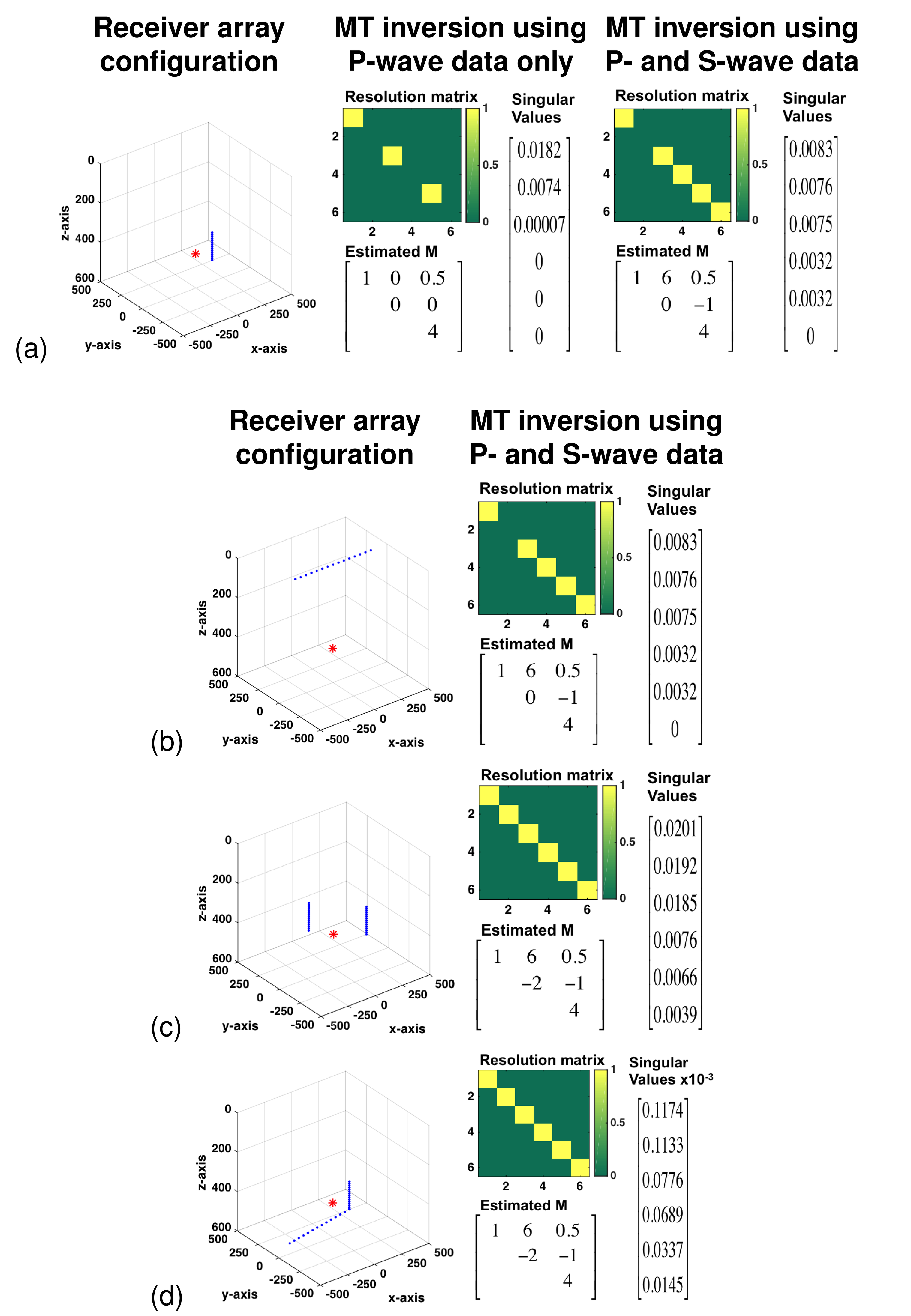



FIG.2. Moment tensor inversion for various receiver configuration. (a) A single borehole array. (b) A single surface array. (c) Two borehole receiver arrays. (d) A deviated well.

## Waveform inversion Vavryčuk and Kühn (2012)

- **Step 1:** Inversion for source-time function in the frequency domain.
- $$u_n(\vec{x}, \omega) = m_{pq}(\omega) g_{np,q}(\vec{x}, \omega)$$
- Reducing the indexes, define (for $k = 1:6$)

$$G_{k1} = g_{k1,1}, \quad G_{k2} = g_{k2,2}, \quad G_{k3} = g_{k3,3},$$
$$G_{k4} = g_{k2,3} + g_{k3,2}, \; G_{k5} = g_{k1,3} + g_{k3,1}, \; G_{k6} = g_{k1,2} + g_{k2,1}$$

- For each frequency, the linear inversion: ($N$ number of receivers)

$$\begin{bmatrix} u_1^{(1)}(w) \\ u_2^{(1)}(w) \\ u_3^{(1)}(w) \\ \vdots \\ u_1^{(N)}(w) \\ u_2^{(N)}(w) \\ u_3^{(N)}(w) \end{bmatrix} = \begin{bmatrix} G_{11}^{(1)}(\omega) & G_{12}^{(1)}(\omega) & G_{13}^{(1)}(\omega) & G_{14}^{(1)}(\omega) & G_{15}^{(1)}(\omega) & G_{16}^{(1)}(\omega) \\ G_{21}^{(1)}(\omega) & G_{22}^{(1)}(\omega) & G_{23}^{(1)}(\omega) & G_{24}^{(1)}(\omega) & G_{25}^{(1)}(\omega) & G_{26}^{(1)}(\omega) \\ G_{31}^{(1)}(\omega) & G_{32}^{(1)}(\omega) & G_{33}^{(1)}(\omega) & G_{34}^{(1)}(\omega) & G_{35}^{(1)}(\omega) & G_{36}^{(1)}(\omega) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{11}^{(N)}(\omega) & G_{12}^{(N)}(\omega) & G_{13}^{(N)}(\omega) & G_{14}^{(N)}(\omega) & G_{15}^{(N)}(\omega) & G_{16}^{(N)}(\omega) \\ G_{21}^{(N)}(\omega) & G_{22}^{(N)}(\omega) & G_{23}^{(N)}(\omega) & G_{24}^{(N)}(\omega) & G_{25}^{(N)}(\omega) & G_{26}^{(N)}(\omega) \\ G_{31}^{(N)}(\omega) & G_{32}^{(N)}(\omega) & G_{33}^{(N)}(\omega) & G_{34}^{(N)}(\omega) & G_{35}^{(N)}(\omega) & G_{36}^{(N)}(\omega) \end{bmatrix} \begin{bmatrix} m_{11}(w) \\ m_{22}(w) \\ m_{33}(w) \\ m_{44}(w) \\ m_{55}(w) \\ m_{66}(w) \end{bmatrix}$$

- Taking the inverse Fourier transform, results in $m(t)$ matrix with six time-independent moment tensor vector.
- Vasco (1989): took SVD of matrix $m(t)$, showed the eigenvector associated with the largest singular value is the source-time function.

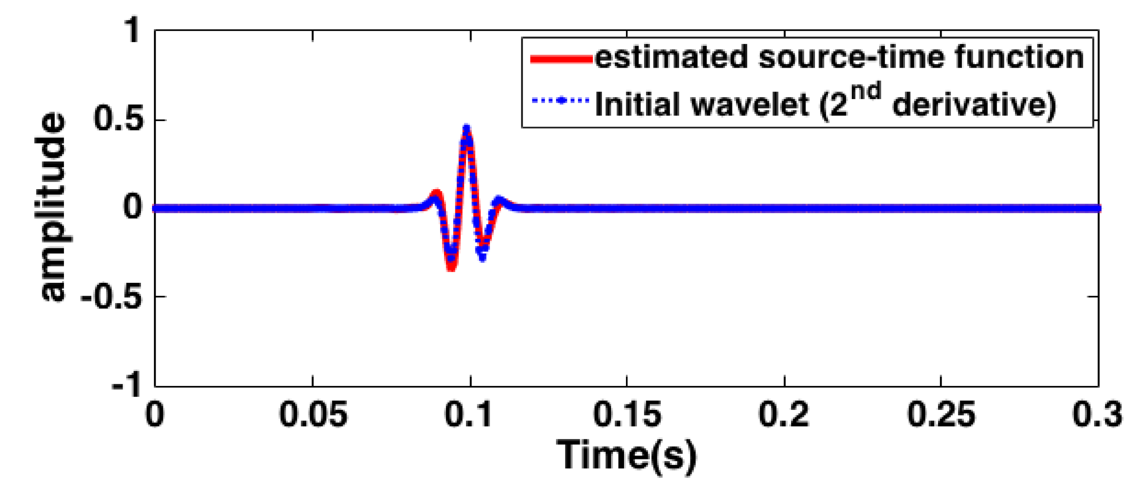



FIG.3. Comparison of the estimated source-time function with initial wavelet with the dominant frequency of 150HZ.

- **Step 2:** Time domain waveform inversion for moment tensor.
- Define elementary seismograms

$$E_{np}(\vec{x}, t) = S(t) * g_{np}(\vec{x}, t)$$

- Substituting the elementary functions into equation 1, the displacement components become

$$u_n(\vec{x}, t) = M_{pq}E_{np,q}(\vec{x}, t).$$

- Using the source-time function from Step 1, calculate the elementary seismograms from the Green's functions, and finally a waveform inversion in the time domain to obtain $M_{pq}$, **Ongoing research**.

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