Multi-parameter acoustic full-waveform inversion: a comparison of different parameterizations and optimization methods

Wenyong Pan*, Kris Innanen and Yu Geng

wpan@ucalgary.ca

Summary

Full-waveform inversion methods allow to provide high-resolution estimates of subsurface properties but suffers from parameter crosstalk artifacts arising from the inherent ambiguities among different physical parameters, which significantly increase the non-linearity of the inverse problems. For multi-parameter acoustic FWI, density is difficult to construct, which maybe caused by the strong parameter trade-off from velocity. An appropriate parameterization in multi-parameter FWI can help avoid parameter crosstalk. We study the resolving abilities of different parameterizations for acoustic FWI. It has been proved that the second-order derivative (namely Hessian operator) is capable to suppress the parameter crosstalk artifacts. In this research, we will give example to show that the Hessian operator can be used to reduce the parameter crosstalk artifacts. We will also illustrate the performances of different optimization methods.

Optimization Methods for FWI

Newton-type optimization methods (e.g., full Newton (FN) and Gauss-Newton (GN) methods) use the quadratic search direction and exhibit fast convergence given a limited number of unknown parameters. In multi-parameter FWI, It has been proved that the Hessian operator can mitigate the parameter crosstalk. Though Newton-type methods benefit from fast convergence rate, the computation, storage and inversion of Hessian at each iteration are prohibitively expensive, which limits their applications for large-scale inverse problems in exploration geophysics.

Gradient-based methods (e.g., steepest-descent (SD) and non-linear conjugate-gradient (NCG) methods) approximate the Hessian matrix as an identity matrix and they are computationally more attractive than the Newton-type ones when inverting a large number of unknown model parameters. The SD method simply determines the search direction to be the negative of the gradient. In NCG method, the search direction is just a linear combination of current gradient and previous search direction. The gradient-based methods are known to converge globally, but possibly very slowly.

Quasi-Newton methods provide an attractive alternative to Newton-type and gradient-based methods by approximating the inversion Hessian iteratively instead of constructing the Hessian matrix.

BFGS method is one popular quasi-Newton strategy to approximate the inverse Hessian iteratively using the changes of the model and gradient. In the BFGS updating formula, we are given a symmetric and positive definite matrix that approximates the inverse of the Hessian, and a pair of vectors that indicates the model and gradient changes. Using these vectors, we compute the inverse Hessian approximation by the following formula:

$$\mathcal{H}_{k+1} = \mathbf{v}_k^\dagger \mathcal{H}_k \mathbf{v}_k + \mathbf{w}_k \mathbf{s}_k \mathbf{s}_k^\dagger,$$

. A limited-memory BFGS (L-BFGS) method was developed by storing the model and gradient changes from a limited number L of previous iterations (typically $L<10$)

In Hessian-free optimization method, the search direction is computed by approximately solving the Newton equations through a matrix-free fashion of the conjugate-gradient (CG) algorithm:

$$\mathbf{H}_k \Delta \mathbf{m}_k = -\mathbf{g}_k$$

We also develop an L-BFGS preconditioning scheme for the HF optimization method, namely the L-BFGS-GN method. Furthermore, the L-BFGS preconditioner is constructed with the diagonal Hessian approximations as initial guess.

Numerical Results

In the numerical section, we will first illustrate the performances of different optimization methods for mono-parameter FWI and show the quadratic convergence of the Hessian-free Gauss-Newton method. We also give numerical example to show the effectiveness of Hessian in mitigating the parameter crosstalk artifacts and compare the performances of different optimization methods.

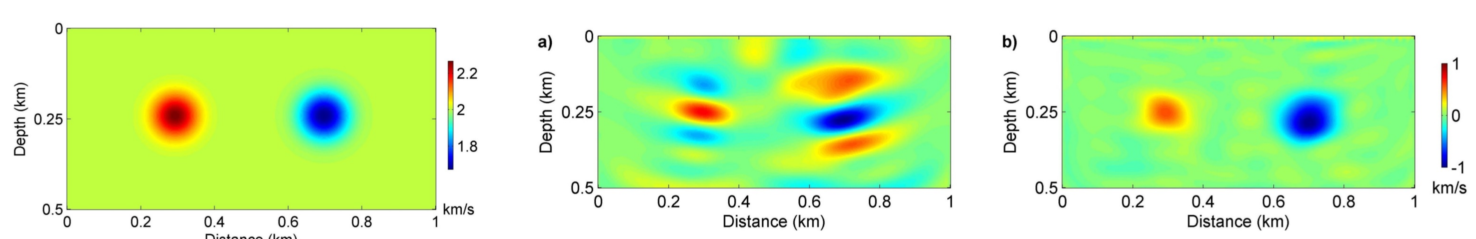

Figure 1. The left figure show the true P-wave velocity model. Initial model is homogeneous. The middle and right figures show the gradient update and Gauss-Newton update.

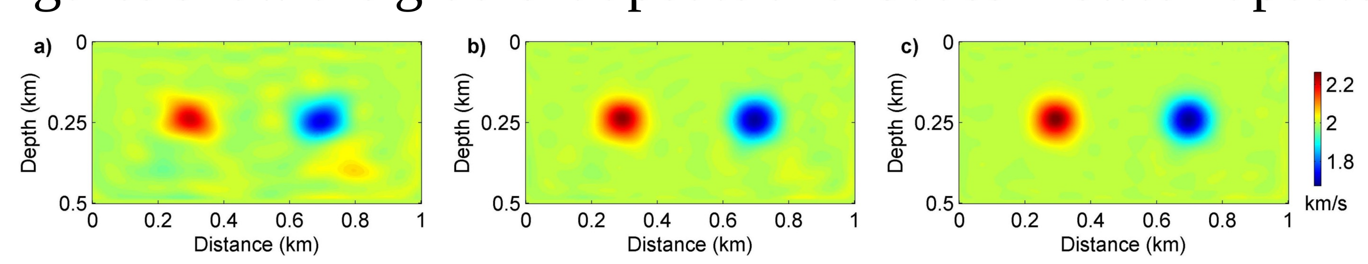

Figure 2. (a)-(c) show the inverted models using SD method, L-BFGS and Hessian-free Gauss-Newton methods respectively.

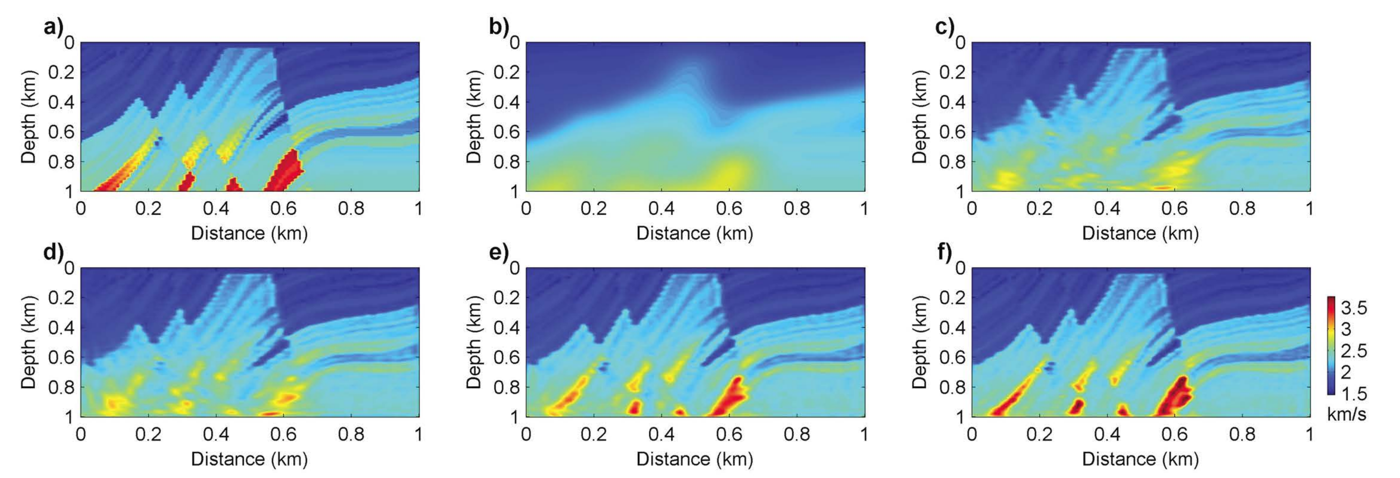

Figure 3. (a) and (b) show the true and initial models. (c), (d), (e) and (f) show the inverted models using SD, L-BFGS, non-preconditioned GN and L-BFGS preconditioned GN methods respectively.

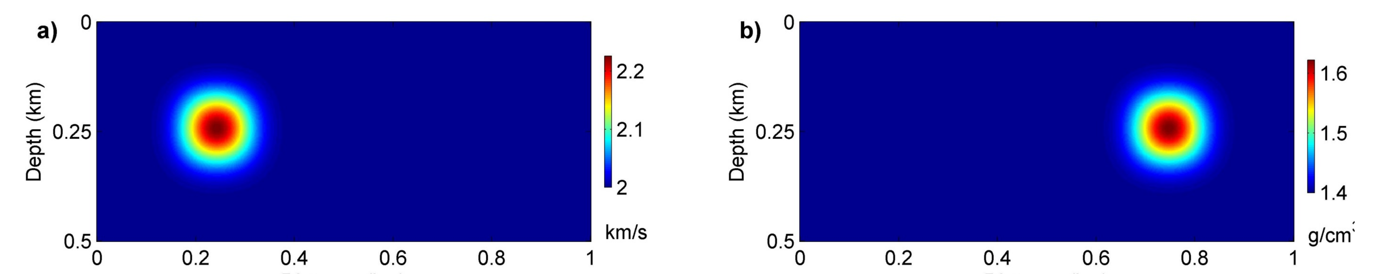

Figure 4. (a) and (b) show true P-wave and density models.

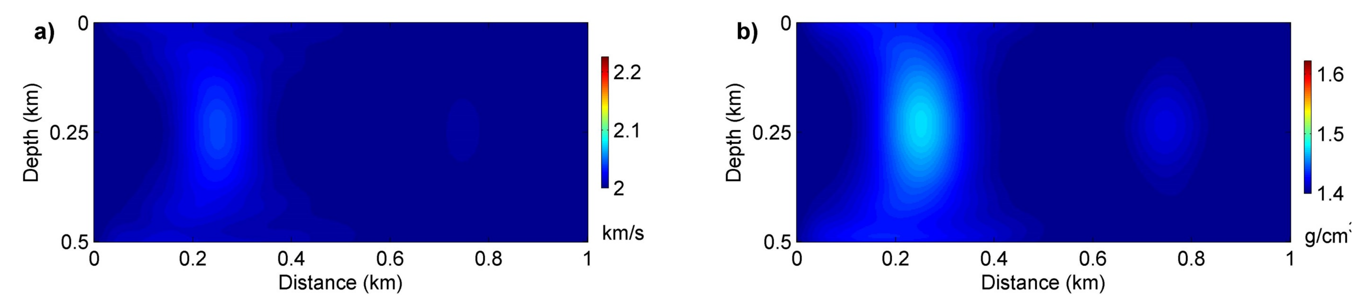

Figure 5. Inverted models with SD method.

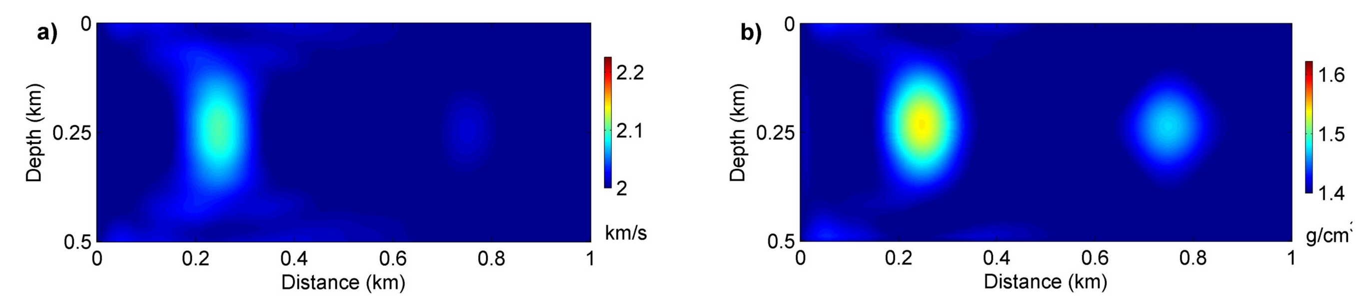

Figure 6. Inverted models with L-BFGS method.

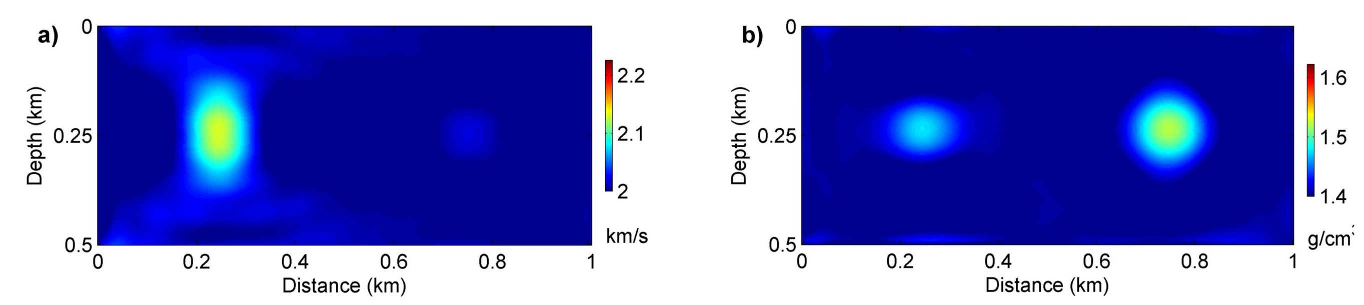

Figure 7. Inverted models with HFGN method.

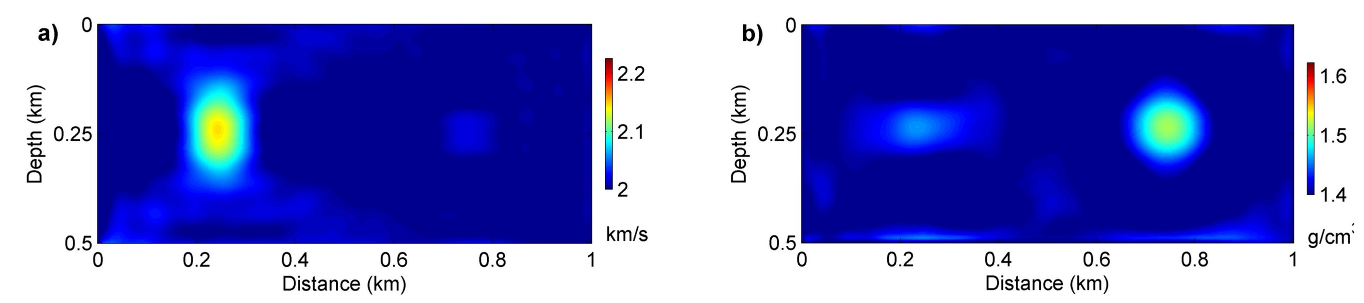

Figure 8. Inverted models by preconditioned HFGN method.

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