Seismic reflection and transmission within an extended scatterer

Kris Innanen*
k.innanen@ucalgary.ca

FWI, RWI and nonlinear kernels

Standard FWI kernels are able to build long-wavelength model updates only in shallow depth, long-offset domains. In order to use reflections earlier in the iteration, and to construct kernels such that they illuminate deep parts of the model, multiple scattering extensions have been introduced.

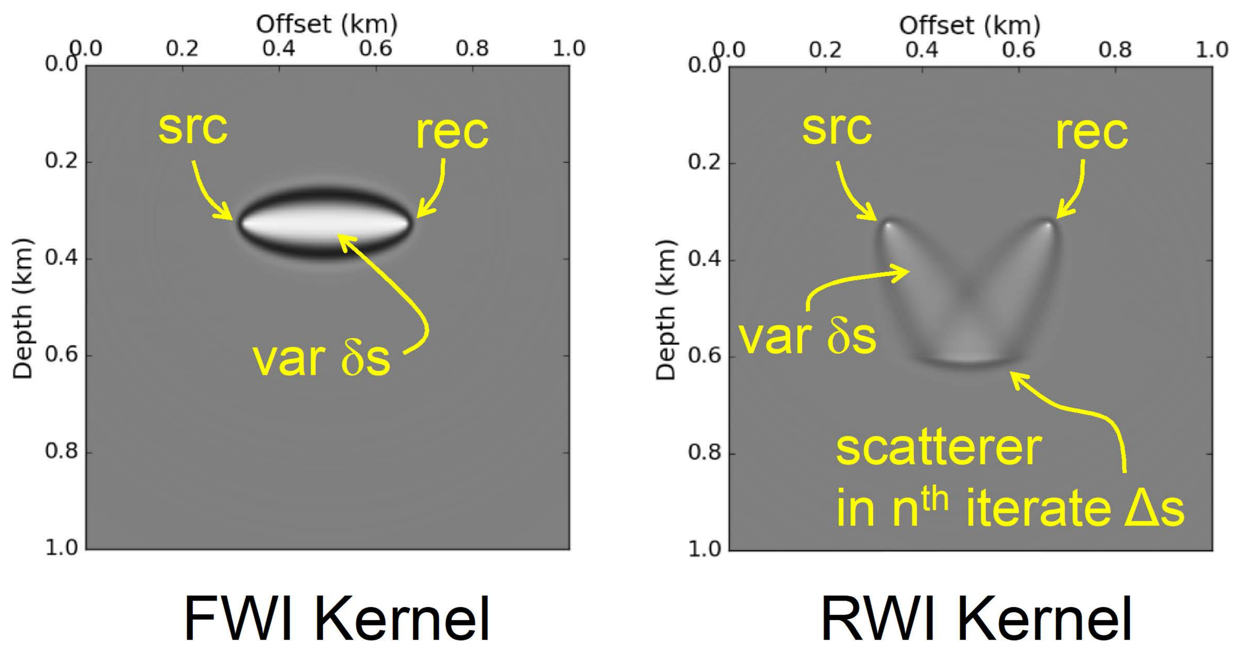

Fig. 1. Standard FWI kernels are "transmission like", building long wavelength components of the model only in shallow regions. Extensions attempt to provisionally introduce scatterers to fill in deeper regions.

...but if we start including multiple scattering in regions where the model error / perturbations are large and extended, unfamiliar behaviour ensues. Let's summarize and quantify some of this.

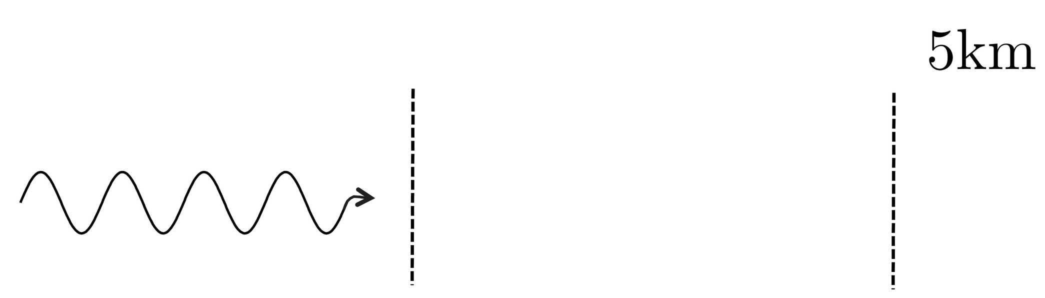

Fig. 2. We can see a lot by making 1D fields from extended perturbations (z_1 to z_2). An nth order scattering approximation of the field at z_g due to source at z_s is computed numerically.

The field P is given as a sum of terms Pn:

$$P(z, z_s, \omega) = \sum_{n=0}^{\infty} P_n(z, z_s, \omega)$$

with terms given by

$$P_n(z, z_s, \omega) = -\frac{ik}{2} \int_{\alpha(z,z_s)}^{\beta(z,z_s)} dz' \exp\left(ik|z - z'|\right) a(z') P_{n-1}(z', z_s, \omega)$$

The perturbation a contains most of the medium structure; the limits α and β can be adjusted to obtain a range of approximations.

WKBJ series: 0th order events need Nth order terms

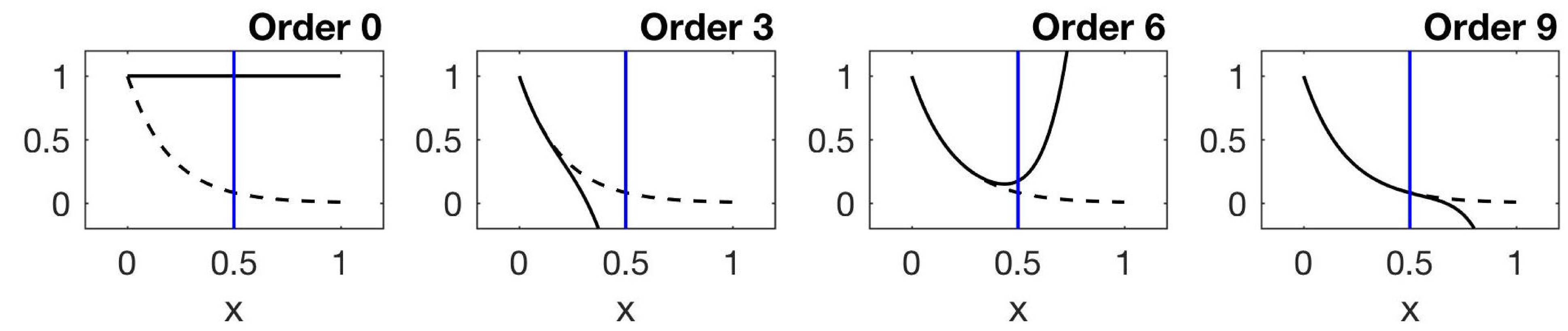

Fig. 3. A good way to understand N^{th} order approximations is to consider the Taylor's series approximation of e^{-x}. At order N, the series converges over a range $(0,x_N)$. As N increases, the range $(0,x_N)$ grows. This is roughly followed in multiple scattering.

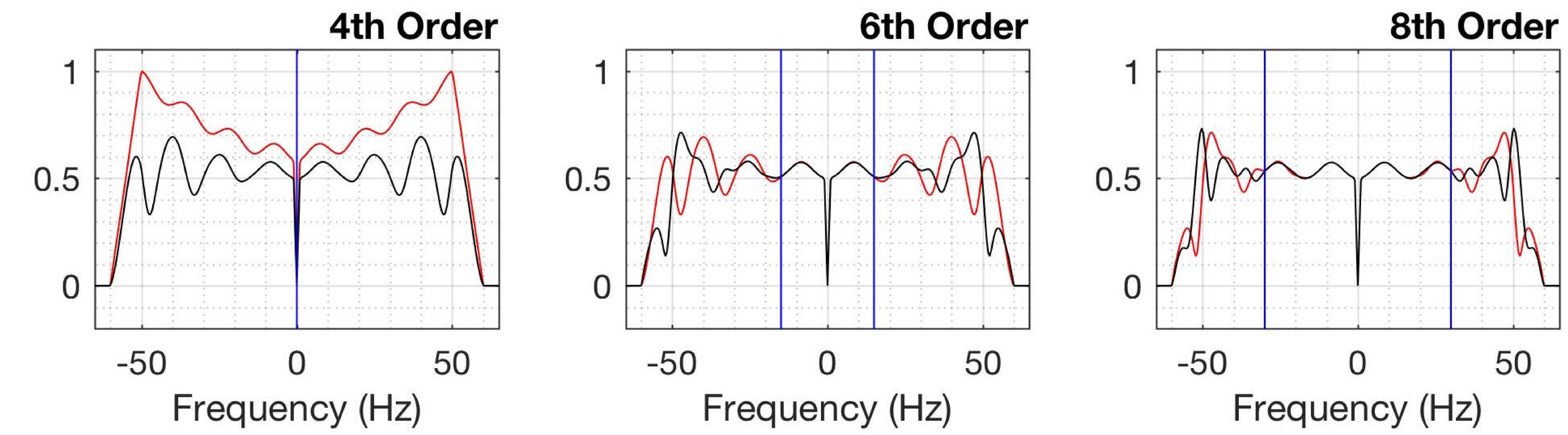

Fig. 4. This is true of $\Sigma_n^N P_n$ also, as we can see by examining increasing orders N and N-1, and deciding based on their differences what frequency band has converged.

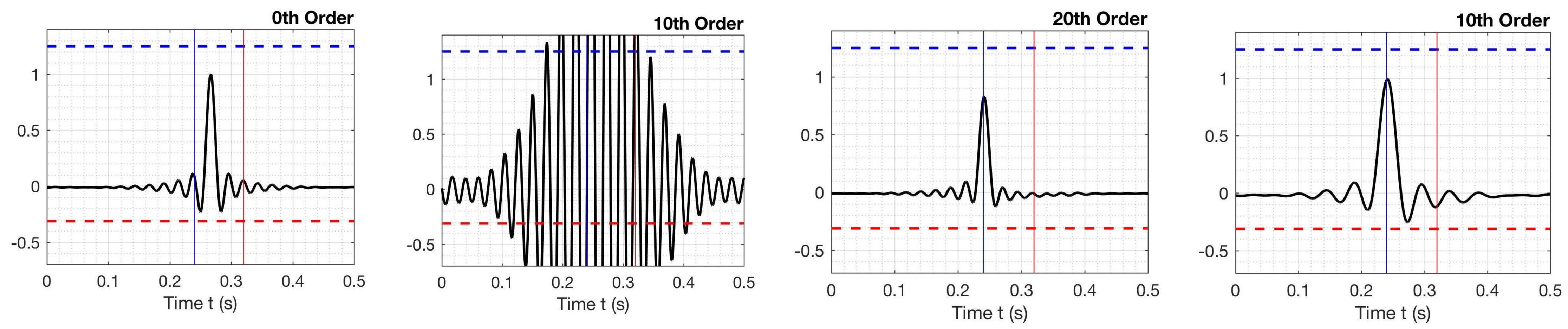

Fig. 5. In the time domain, this has real consequences for qualitative conclusions about convergence. The 0^{th} order (right) needs to be corrected towards the blue lines in amplitude and phase. For f_{max}=45Hz, the 10^{th} order approximation has not "converged" (2^{nd} panel). It takes 20 orders to converge (3^{rd} panel). But, f_{max}=10Hz, the 10^{th} order approximation *does* converge.

Care is needed when evaluating multiple scattering

See the report for more examples and uses of the Matlab code. If we wind up as a community making complex nonlinear kernels for FWI using multiple scattering, in order to overcome the low-wavenumber problem, we will likely encounter non-intuitive features, well beyond the standard "Nth order scattering = Nth order multiples" paradigm. This numerical tool may be helpful in design and appraisal in this sense.