Comparison of travltime computation and ray tracing methods

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Motivation

Travel times and ray paths of the propagation of seismic body wave in heterogenous media are used in seismic tomography, imaging and inversion processes. In this study, we review the seismic ray theory, basic principles of the fast marching, wavefront construction and paraxial method. We analyze their differences and similarities to investigate the effectiveness of these methods in refraction tomography and seismic imaging.

Seismic ray theory

High frequency approximation of the solution of elastodynamic equation leads to solutions in different forms. For kinematic ray tracing, the solution leads to the eikonal equation and the ray equations.

- Elastodynamic equation:

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_I \qquad (1)$$

- Eikonal equation:

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{c^2} \qquad (2)$$

- Ray equations:

$$\frac{d\vec{x}}{ds} = c\vec{q} \qquad (3)$$

$$\frac{d\vec{q}}{ds} = \vec{\nabla}[\frac{1}{c}] \qquad (4)$$

Grid based methods

Finite difference solution to eikonal equation (Vidale 1988):

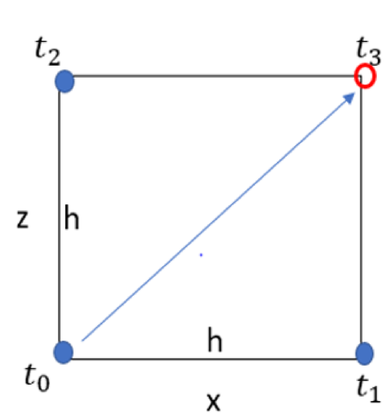
Eikonal equation:

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = s(x,z)^2 \qquad (5)$$

Average finite difference approximation of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial z}$:

$$\frac{\partial T}{\partial x} = \frac{1}{2}(t_0 + t_2 - t_1 - t_3) \qquad (6)$$

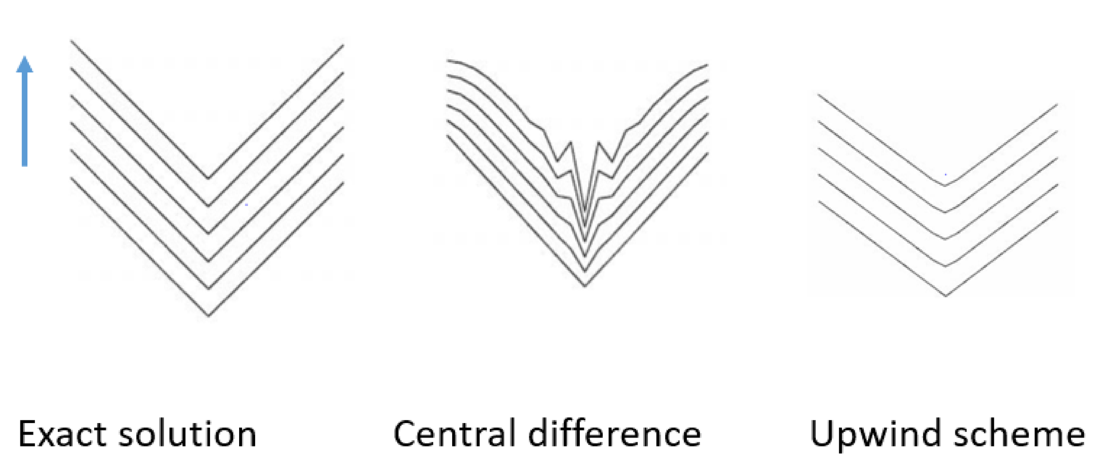
$$\frac{\partial T}{\partial z} = \frac{1}{2}(t_0 + t_1 - t_2 - t_3) \qquad (7)$$



Substitude (6) and (7) into (5):

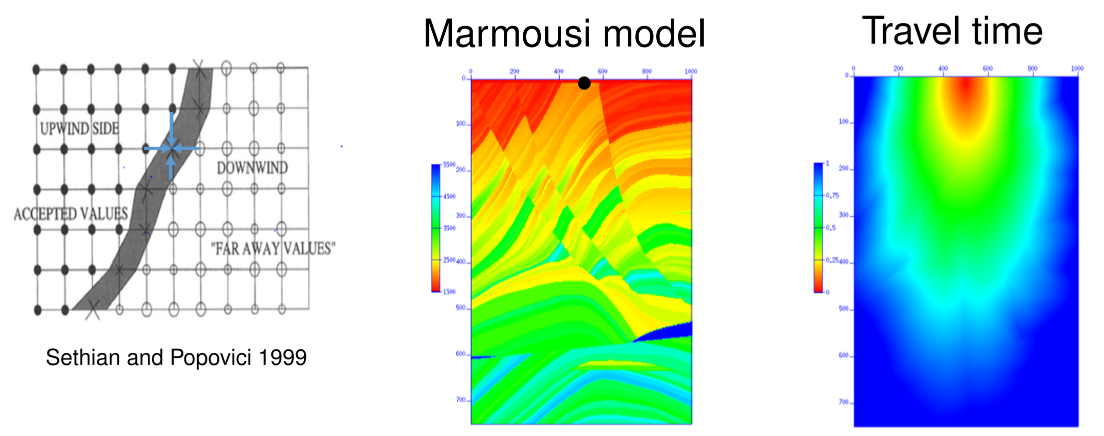
$$t_3 = t_0 + \sqrt{2(hs)^2 - (t_2 - t_1)^2} \qquad (8)$$

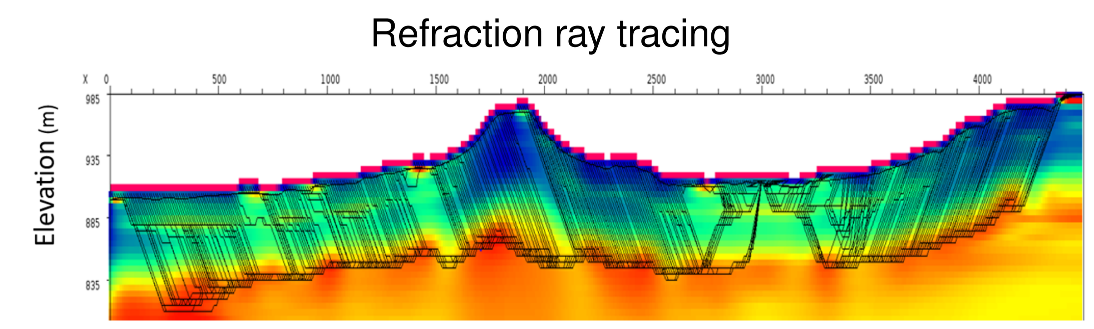
Fast marching method and upwind scheme (Sethian and Popovici 1999):



Grid based method

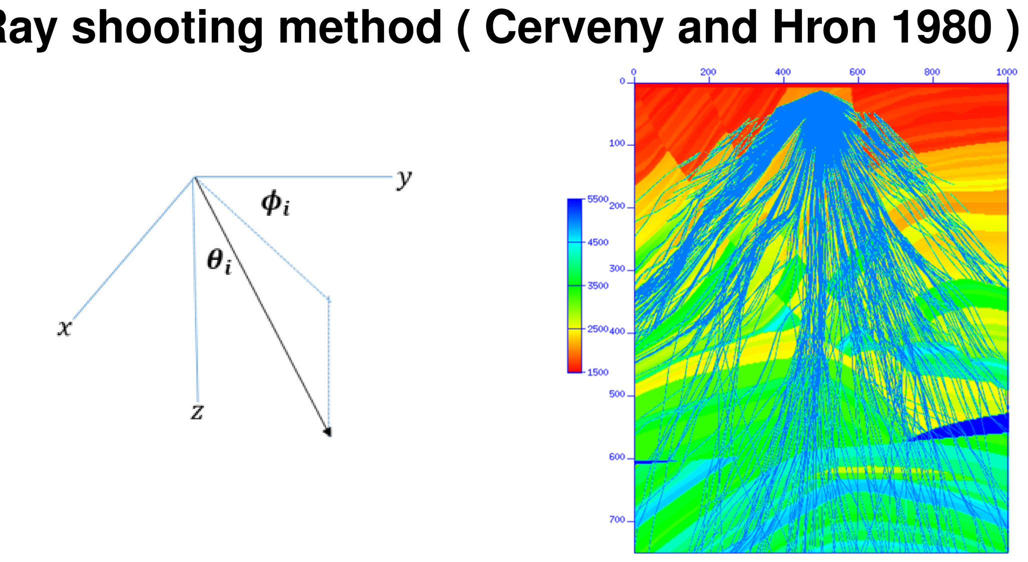
- **Fast marching method (Sethian and Popovici 1999)**





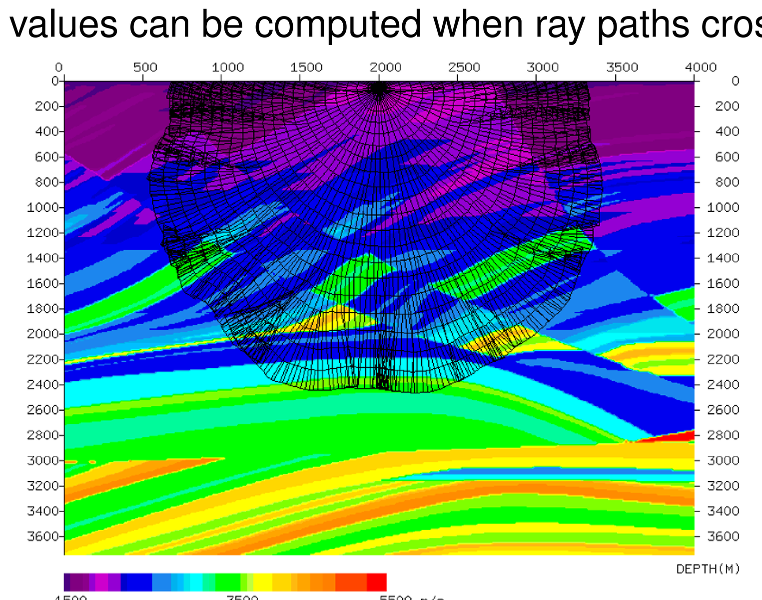
Ray based method

- **Ray shooting method (Cerveny and Hron 1980)**



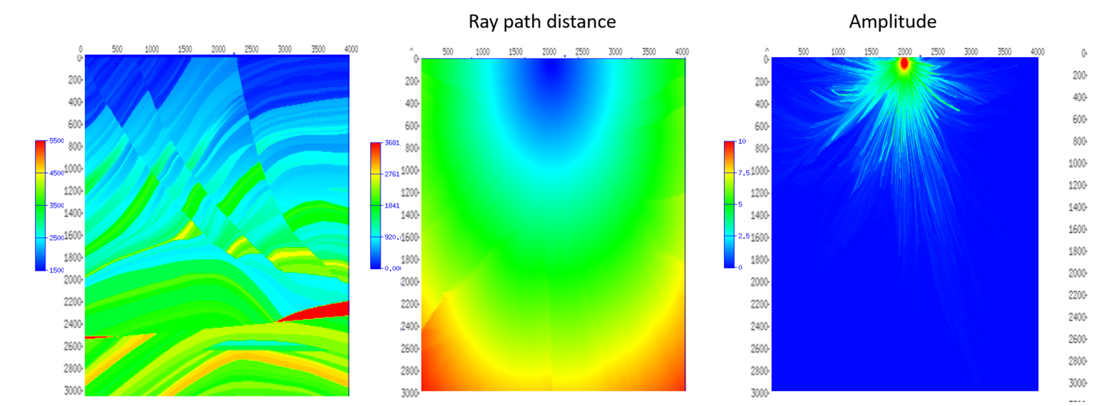
- Loop on take off angles θ_i, ϕ_i
  Initial values at first depth step
  $\frac{d\vec{x}}{ds} = (sin\theta_i cos\phi_i, sin_i sin\phi_i, cos\theta_i)$
  $\vec{q} = \frac{1}{c(X_s, Y_s, Z_s} \frac{d\vec{x}}{ds}$
- Loop on steps until end is reached
  1: Solve ODE (2) for next $\vec{x}$
  2: Solve ODE (3) for next $\vec{q}$
- Next step
- Next take off angle

- **Wavefront construction method (Vinje 1993)**
  - Shoot out rays of equal time step. End points of these rays form the new wavefront
  - Uses new wavefront to propagate new rays for another time step
  - Geometric spreading can be computed from ratio of cress-sectional areaa
  - Multiple values can be computed when ray paths cross

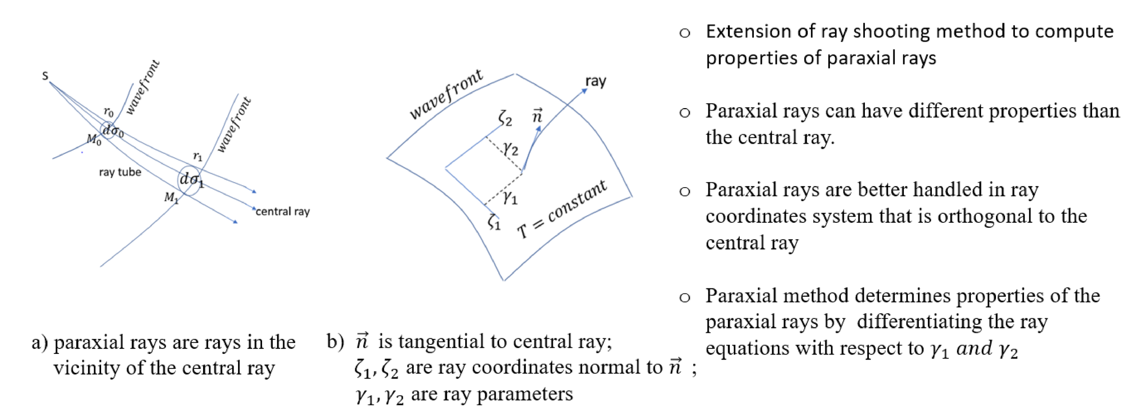


Ray based method

- **Wavefront construction method**



- **Paraxial method (Beydoun and Keho 1987)**



- **Dynamic ray tracing equations**

$$Q_i = \frac{\partial x_i}{\partial \gamma}; P_i = \frac{\partial p_i}{\partial \gamma} \qquad (9)$$

$$\frac{dQ_i}{ds} = C(,k)Q_kP_i + cP_i \qquad (10)$$

$$\frac{dP_i}{ds} = \frac{\partial^2}{\partial x_i \partial x_k}(\frac{1}{c})Q_k \qquad (11)$$

- **Paraxial ray tracing equations**

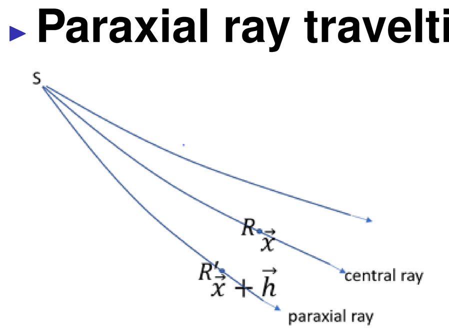
$$\frac{d\delta x_i}{ds} = c_{,k}\delta x_k p_i + c\delta p_i \qquad (12)$$

$$\frac{d\delta p_i}{ds} = \frac{\partial^2}{\partial x_i \partial x_k}(\frac{1}{c})x_k \qquad (13)$$

- **Geometrical spreading**

$$d\sigma = Q_1 Q_2 d\gamma_1 d\gamma_2 \qquad (14)$$

- **Paraxial ray traveltimes**



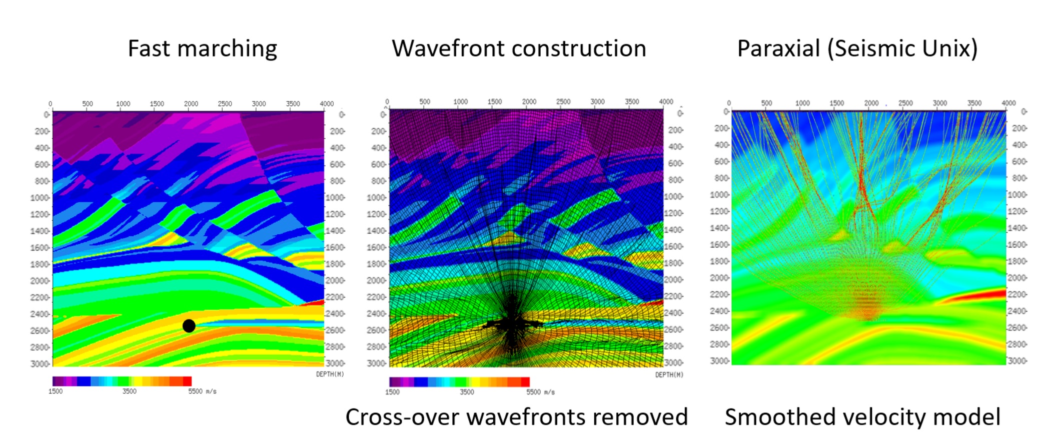
$$T(\vec{x}+\vec{h}) = T(\vec{x}) + T_{,j}(\vec{x})h_j + \frac{1}{2}T_{,jk}(\vec{x})h_jh_k \qquad (15)$$

$$T_{,j} = P_j \qquad (16)$$

$$T_{,jk} = P_{jn}Q^{-}_{nk}1 \qquad (17)$$

Comparison of ray tracing methods

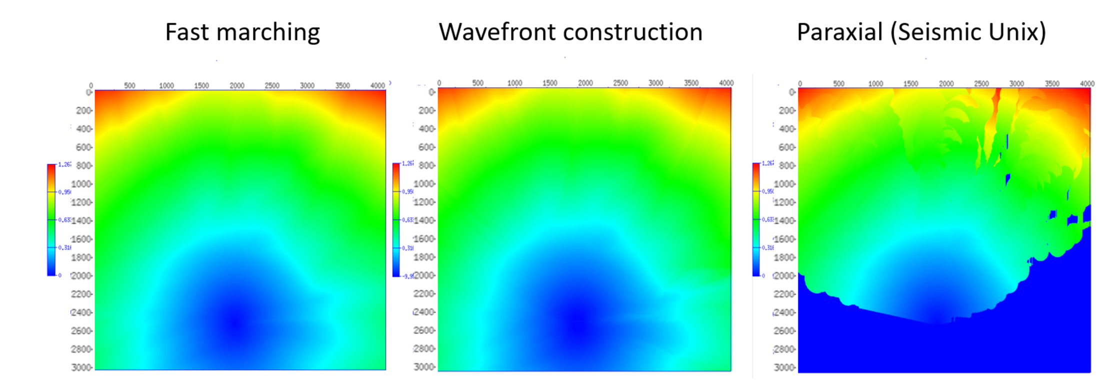
- Source is placed at the depth of 2500m
- Receivers are placed at the surface



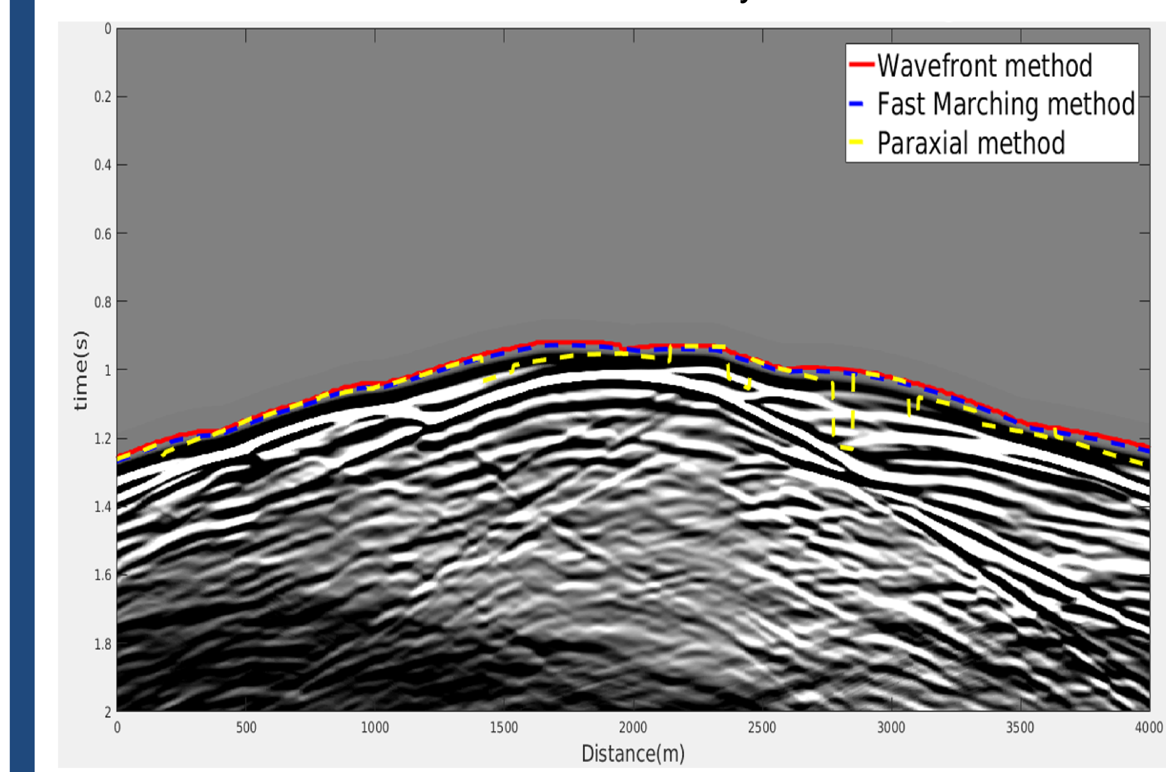
Rays from WFC and paraxial methods can cross-over in area with complex velocity structure and result in multi-arrivals at the same grid point. Multi-arrivals can be used for different branches of traveltime including most energetic arrival.

Comparision of ray tracing methods

Caustics are removed in WFC to output minimum travel times



Traveltimes overlaid on finite difference synthetic shot record



- Travel times from WFC and fast marching are almost identical
- Travel times from paraxial method agree with the other methods at most locations except at locations where rays diverge

Summary

- Fast marching method
  - **Advantages:** Unconditional stable. Can handle turning rays. Does not have shadow zones problem.
  - **Disadvantages:** Does not compute ray paths and amplitude. Cannot compute multi-arrivals.
  - **Application:** refraction tomography
- Wavefront construction method
  - **Advantages:** Stable if appropriate velocity smoothing is applied; however accuracy can decrease with increasing smoothing. Can handle turning rays. Does not have shadow zones problem. Can compute multi-arrivals and amplitude. Faster than fast marching method, if large time step size is used.
  - **Application:** refraction tomography and depth imaging
- Paraxial method
  - **Advantages:** More accurate traveltime interpolation in the vicinity of central ray than classical ray shooting method. Compute multi-arrivals and amplitude.
  - **Disadvantages:** Cannot handle turning ray.
  - **Application:** depth imaging,gaussian beam migration

Acknowledgements

We thank the sponsors of CREWES and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13 for supporting this project.