Comparison between least-squares reverse time migration and full-waveform inversion

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Abstract

The inverse problem in exploration geophysics usually consists of two parts: seismic imaging and velocity model constructing. We compare the algorithms for least-squares reverse time migration (LSRTM) and full-waveform inversion (FWI) and use numerical examples to understand the differences. LSRTM uses Born approximation as the modelling method because it requires the adjoint of migration (linear inversion), while FWI uses finite-difference modelling because it does not require an adjoint-pair operator (non-linear inversion). Linearized Born modelling can update model perturbations by a linear conjugate gradient method, but may have severe inaccuracies and inversion noise if the initial model is poor. Both, FWI and LSRTM depend on the initial model largely, but FWI has a mechanism to improve the velocities and LSRTM does not. Conversely, FWI suffers from cycle skipping while LSRTM does not. For LSRTM, the long wavelength components of the gradient are considered to be noise, while for FWI they are considered to be signal.

Least-squares reverse time migration

The conventional imaging condition of RTM can be described as

$$I(x) = \int dx_s dx_r dt d\tau G(x_s, x, \tau)\, G(x, x_r, t-\tau) d(x_s, x_r, t).$$

$G(x_s, x, \tau)$ and $G(x, x_r, t-\tau)$ are the source and receiver Green's function respectively and $d(x_s, x_r, t)$ is the seismic reflection data in common RTM imaging.

The objective function of LSRTM is

$$J(\Delta m) = \left|\left|\Delta d - A\Delta m\right|\right|^2.$$

To get the m that produces the best predictions, the objective function should be minimum, which happens when:

$$\frac{\partial J(\Delta m)}{\partial \Delta m} = 0.$$

Therefore, we may get the least-squares solution:

$$\Delta m = \left(A^{\dagger}A\right)^{-1}A^{\dagger}\Delta d,$$

where † is the conjugate transpose.

Full-waveform Inversion

Full waveform inversion is a nonlinear inversion method. Similar to LSRTM, it starts from a least-squares problem:

$$J(m) = \left|\left|d_{obs} - d_{syn}\right|\right|^2,$$

when the minimum value of the misfit function is reached.

Ignoring the high-order terms, we have

$$\frac{\partial J(m+\Delta m)}{\partial m} \approx \frac{\partial J(m)}{\partial m} + \frac{\partial^2 J(m)}{\partial m^2}\Delta m = 0,$$

and

$$\Delta m = -\left(\frac{\partial^2 J(m)}{\partial m^2}\right)^{-1}\frac{\partial J(m)}{\partial m}.$$

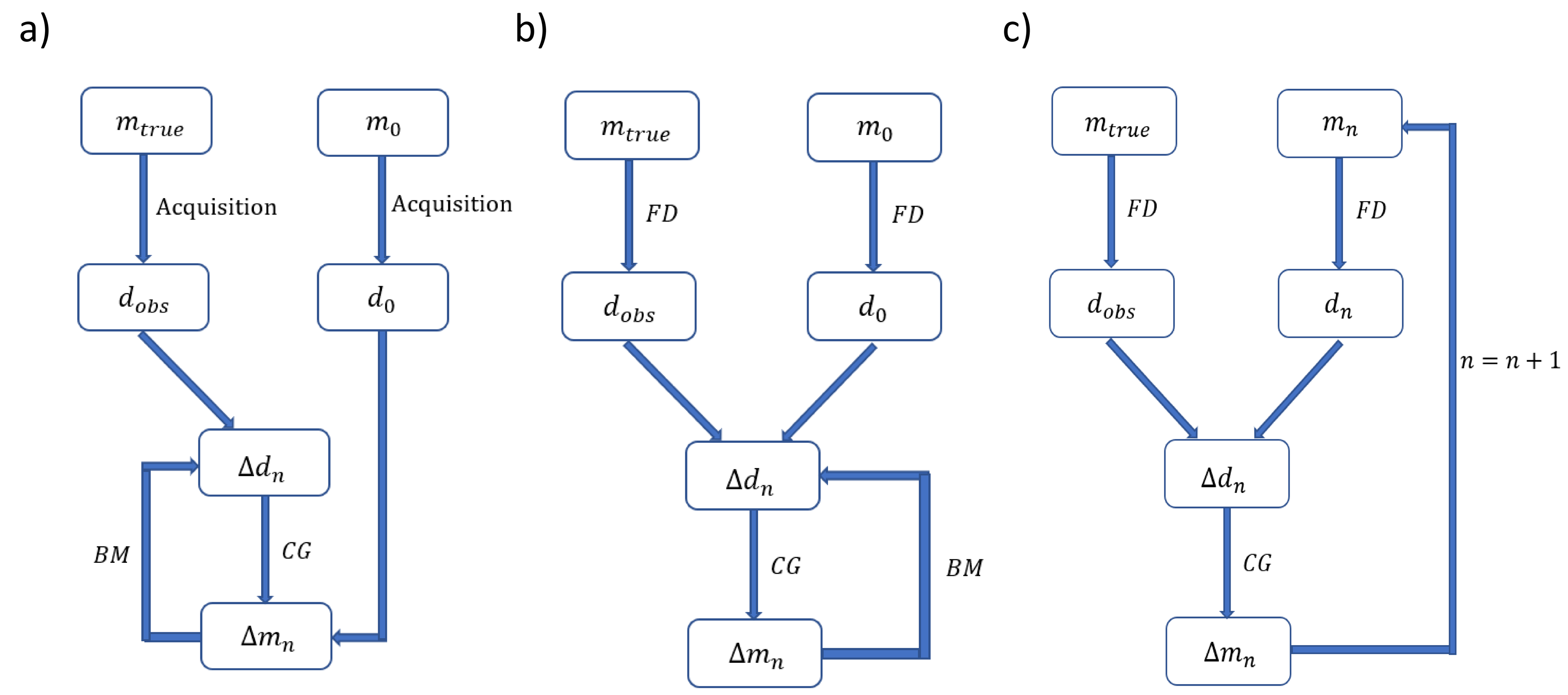

Fig 1. The workflows for a) classical LSRTM, b) Proposed LSRTM and c) classical FWI

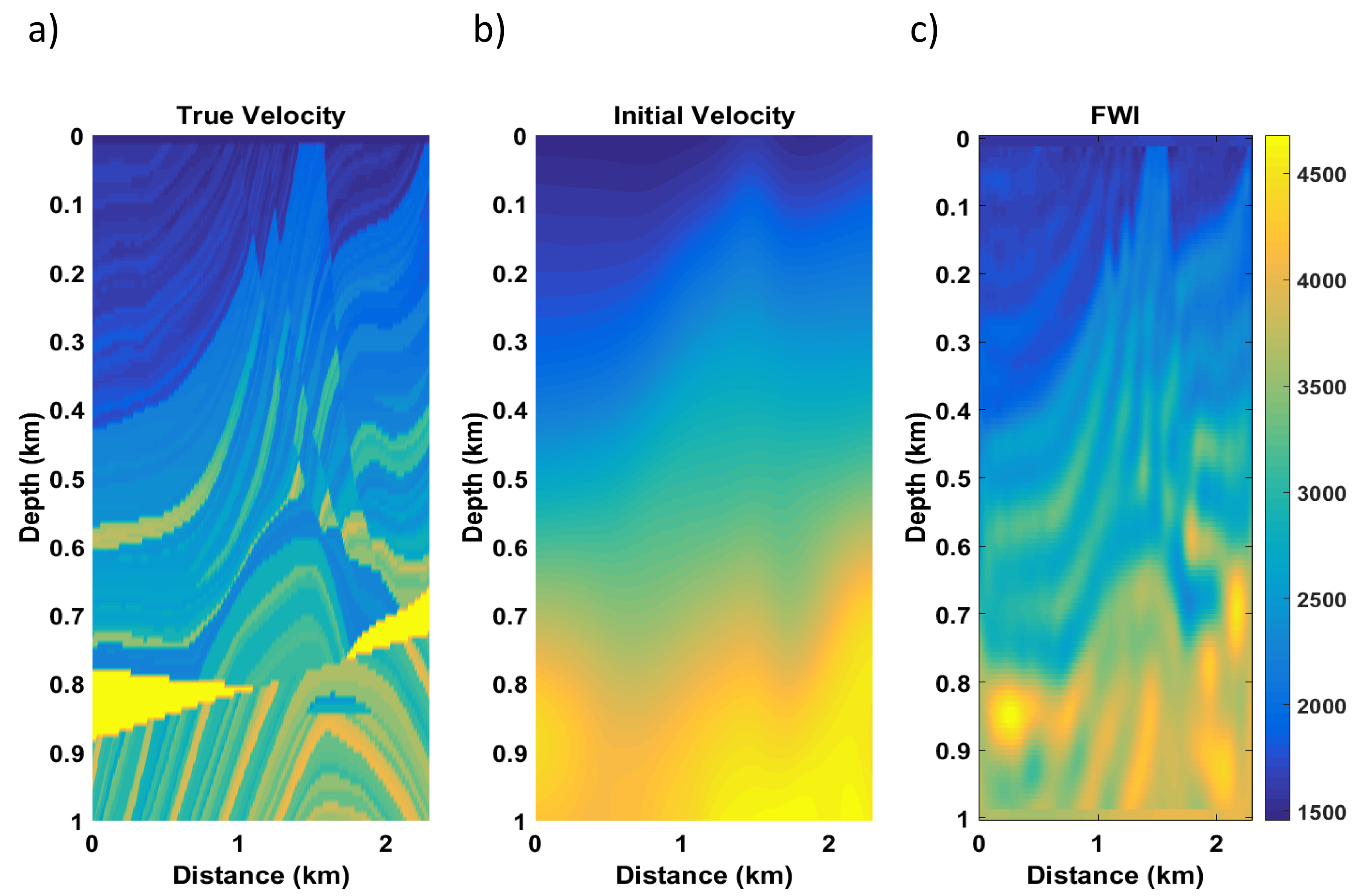

Fig 2. a) The true velocity model of Marmousi b) The initial model applied Gaussian smoother and c) FWI result from the smoothed model

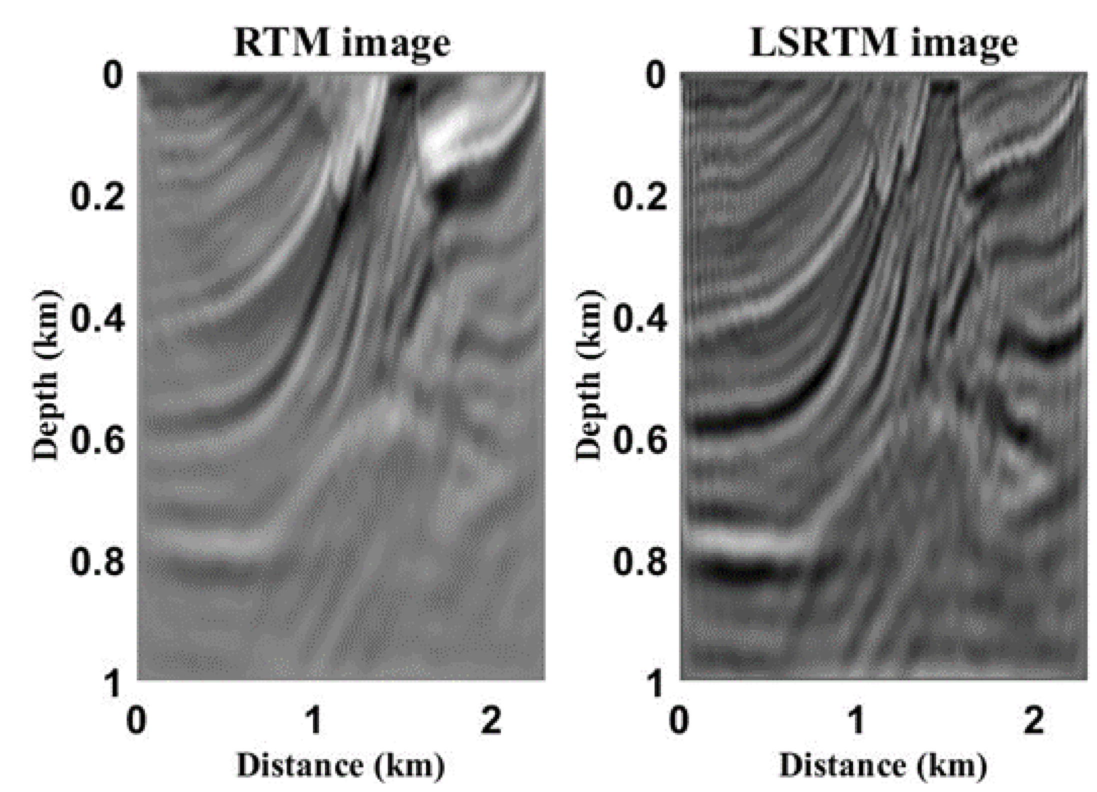

Fig 3. RTM image and LSRTM image of the initial model using the workflow in Fig 1b (proposed LSRTM).

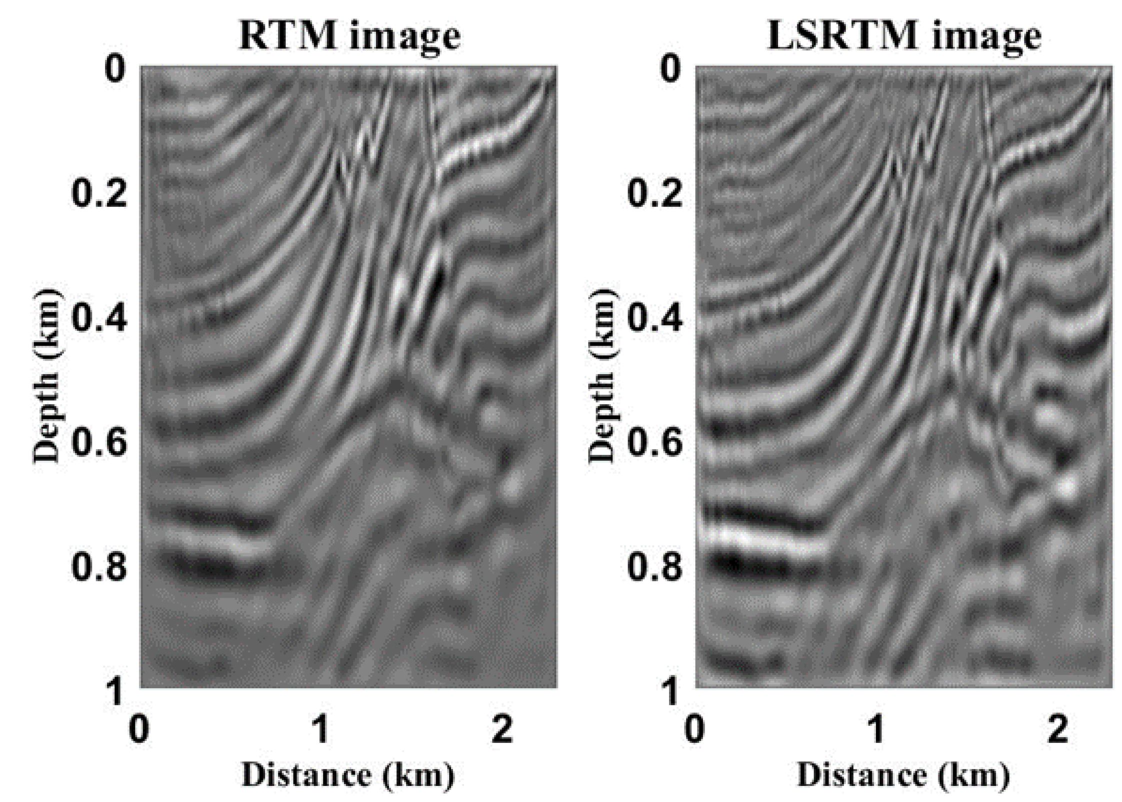

Fig 4. RTM and LSRTM image of FWI result using proposed LSRTM

Summary

LSRTM uses a linearized wave equation based on the Born approximation, which allows one to use a linear inversion method with parameterized step size calculation. FWI uses the finite-difference method which is more precise than Born modeling because it produces first and multiple scattering waves, but it requires to use a non-linear inversion algorithm to correct for the model updates at each iteration. In LSRTM the high frequency components are emphasized to produce a model of reflectivities. FWI focuses on the low frequency components to correct for the background velocity model. In LSRTM the inversion result will be correct only if the background velocity model is accurate, but not otherwise.

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