Frequency domain adaptive waveform inversion

Matt Eaid*, Scott Keating and Kris Innanen
mveaid@ucalgary.ca

ABSTRACT

- Classic full waveform inversion (FWI) suffers from a problem related to local optimization known as cycle skipping.
- Alternative formulations of the FWI objective function, based on extension of the model space in nonphysical parameters, have been shown to be more robust to cycle skipping.
- Adaptive waveform inversion (AWI), originally derived in the time domain, extends the model space in convolutional Weiner filter coefficients.
- We present a frequency domain alternative, and discuss special considerations for frequency domain implementation.

CONVENTIONAL FULL WAVEFORM INVERSION

Conventional FWI minimizes an objective function of the form in equation (1) to iteratively invert for a model of subsurface parameters,

$$f = \frac{1}{2}\sum \|\delta \mathbf{d}\|_2^2. \tag{1}$$

Where $\delta\mathbf{d}$ is the data residual. The gradient for this objective is given by,

$$\frac{\partial f}{\partial \mathbf{m}} = \sum \mathrm{U}\lambda \tag{2}$$

where U is a forward propagated wavefield that is correlated with the back propagated wavefield λ. The back propagated wavefield λ is formed using the data residual as a source.

ADAPTIVE WAVEFORM INVERSION

Adaptive waveform inversion computes a matching filter w that when convolved with the predicted data, provides a least squares fit to the observed data,

$$\mathbf{w} = \left(\mathbf{P}^{\mathrm{T}}\mathbf{P}\right)^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{d} \tag{3}$$

The algorithm then matches the predicted data to the observed by solving an objective function of the form in equation (4), which drives the filter towards an identity filter, in this case a zero lag delta spike

$$f = \frac{1}{2}\frac{\|\mathbf{T}\mathbf{w}\|^2}{\|\mathbf{w}\|^2} \tag{4}$$

where $\mathbf{T}$ is a weighting matrix that forces w towards a delta spike by penalizing large lags. AWI shares a similar gradient with FWI,

$$\frac{\partial f}{\partial \mathbf{m}} = \sum \mathrm{U}\phi \tag{5}$$

where, U is the same forward propagated wavefield, and ϕ is the back propagated wavefield using weighted versions of the filter coefficients as the source δs,

$$\delta \boldsymbol{s} = \alpha(t)\mathbf{w} \tag{6}$$

FREQUENCY DOMAIN ADAPTIVE WAVEFORM INVERSION

In the frequency domain the filter and source take the form,

$$\mathbf{w} = \frac{\mathbf{p}^*\mathbf{d}}{\mathbf{p}^*\mathbf{p} + \mu A_{\mathrm{MAX}}} \tag{7}$$

$$\delta \boldsymbol{s} = \alpha(\omega)\mathbf{w} \tag{8}$$

The challenge of frequency domain AWI is devising a robust form for $\mathbf{T}$. Figure 1 shows that to penalize filter coefficients at large lag, we could penalize low frequency errors in our filter.

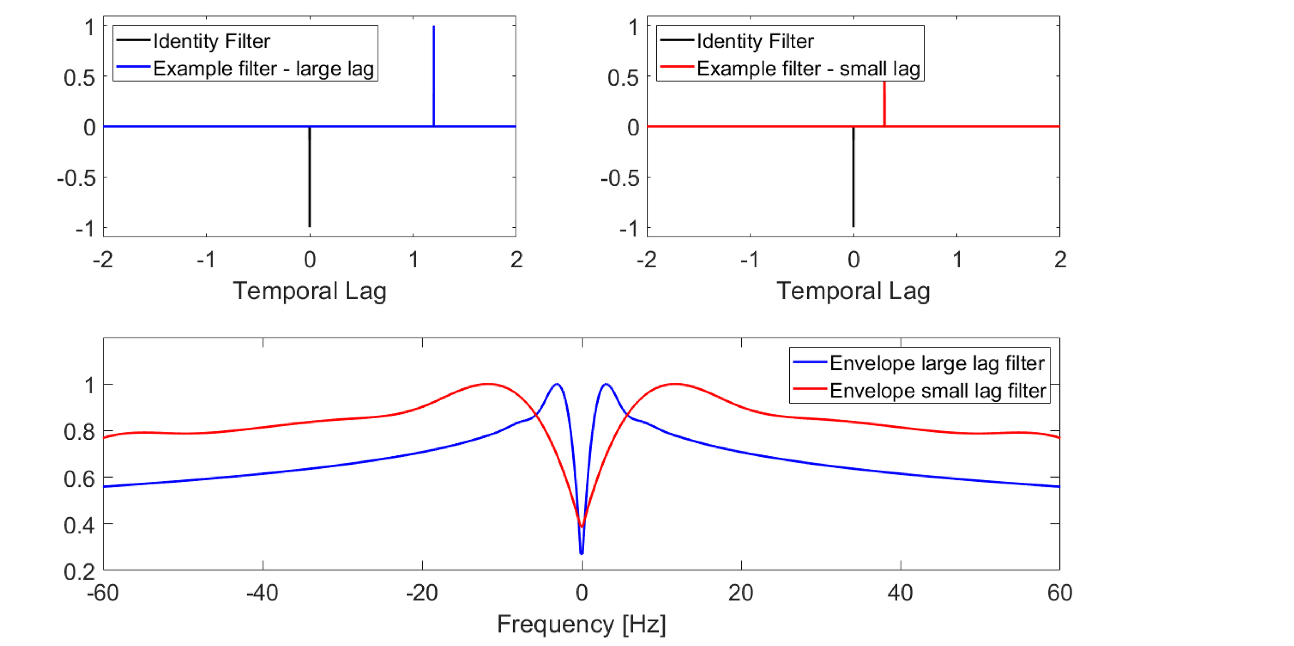

FIG. 1. (Top left) Example of large lag filter in blue and identity filter in black, (top right) example of small lag filter in red and identity filter in black, (bottom) envelope of amplitude spectrum formed by subtraction of the identity filter and the example filter for above two cases.

EXAMPLE 1: BALL MODEL

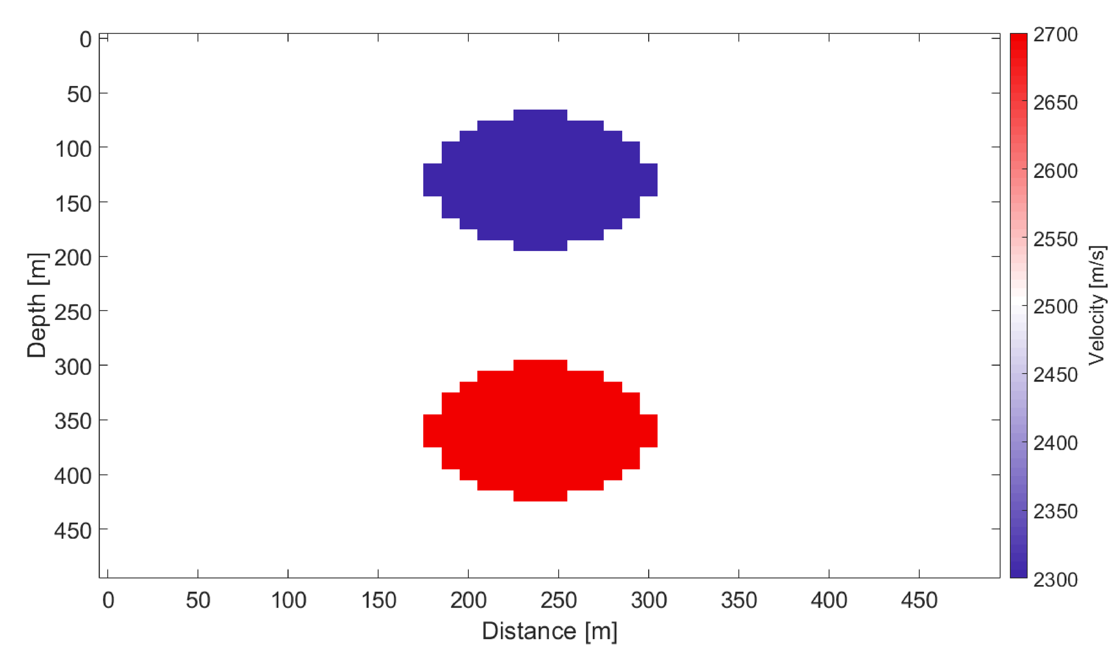

FIG. 2. True velocity model producing data that is not cycle skipped.

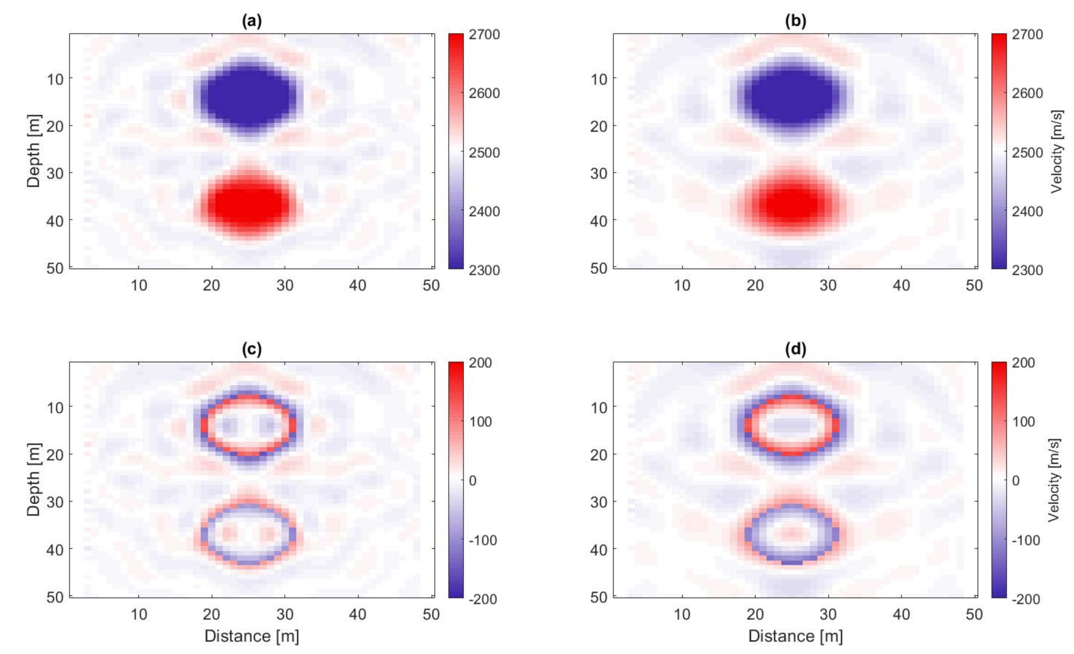

FIG. 3. Inversion results using 10 bands, 5 iterations per band of steepest descent optimization from 3-20 Hz. (a) FWI inversion results, (b) AWI inversion result, (c) difference between FWI result and true model, (d) difference between AWI result and true model.

EXAMPLE 2: GAUSSIAN ANOMALY

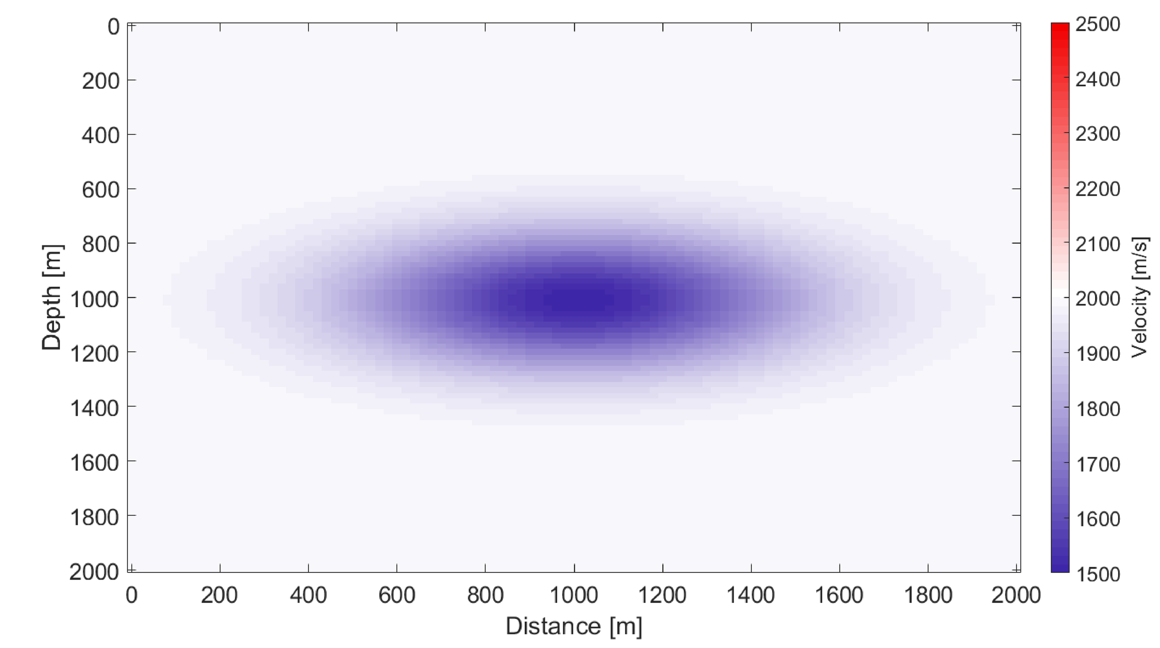

FIG. 4. True velocity model producing cycle skipped data.

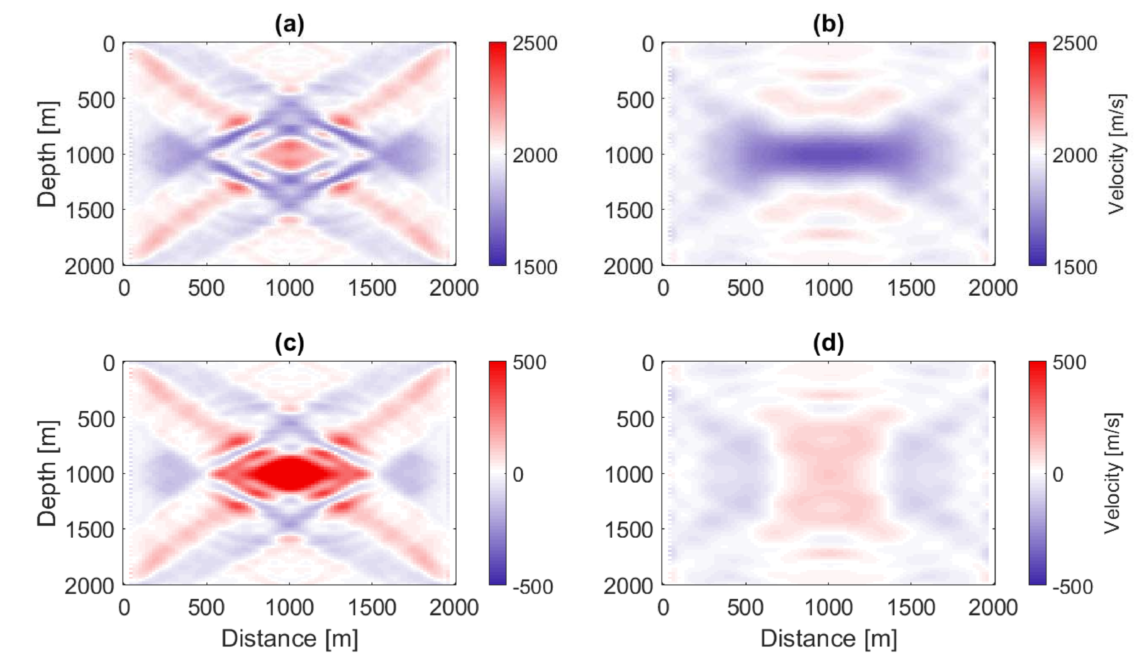

FIG. 5. Inversion results using 1 band, 25 iterations per band of steepest descent optimization from 3-20 Hz. (a) FWI inversion results, (b) AWI inversion result, (c) difference between FWI result and true model, (d) difference between AWI result and true model.

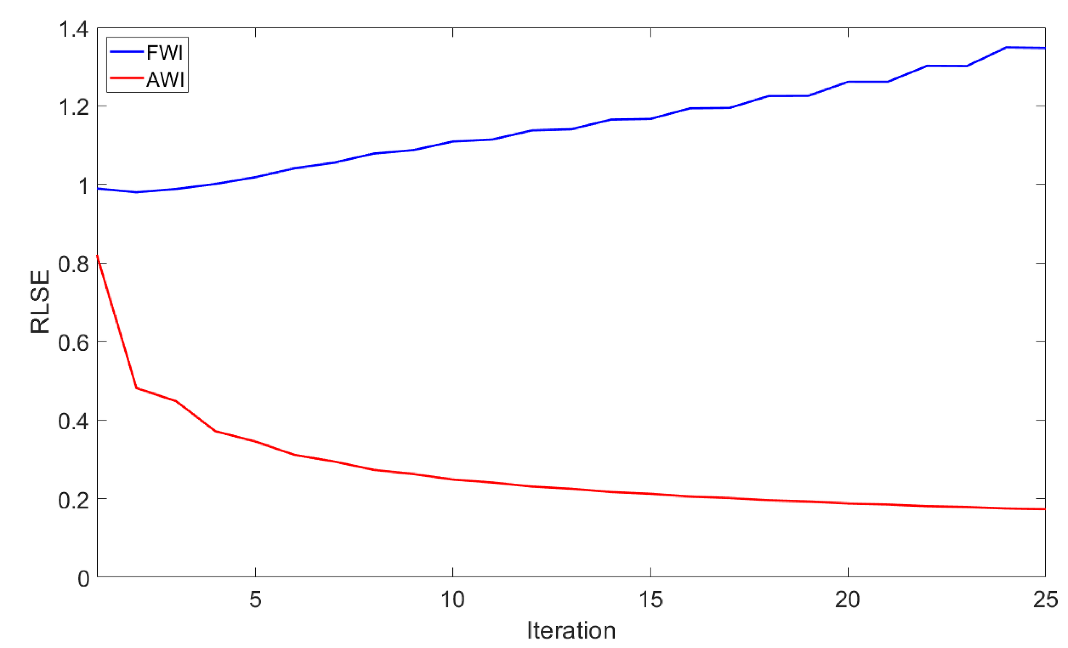

FIG. 6. Relative least-squared model error for FWI (blue), AWI (red).

CONCLUSIONS

- FWI is particularly sensitive to cycle skipped data.
- AWI derived in the frequency domain was shown to be more robust in the presence of cycle skipped data.
- The frequency domain method at this point is not as robust as the time domain method, more robust formulations of the weighting matrix $\mathbf{T}$ are under development.

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