# A 3D pseudo-spectral method for qP- and qSV- wave simulation in heterogeneous VTI media

Junxiao Li*, Huaizhen Chen and Kris Innanen
li.junxiao@ucalgary.ca

## Abstract

During reverse time migration in anisotropic media, P- and SV-waves are coupled and the elastic wave equation should be used. However, the crosstalks caused by the interference between different wave modes are detected. Even if an acoustic anisotropic wave equation is used instead, an undesired SV-wave energy could be generated during modeling and reverse time migration. To avoid this unwanted energy, we proposed an approximation of decoupled P-and SV- wave equation system for vertical transversely isotropic (VTI) media. The qP- and qSV- phase velocities for the approximated equations are plotted and compared with the exact and other approximations, which proves its accuracy with different Thomsen parameter sets. The H-PML in second order wavenumber domain is also proposed to eliminate the artificial boundary reflections, comparisons of different absorbing boundary layers are also illustrated to validate the wave number domain H-PML.

### DECOUPLED WAVE EQUATIONS IN VTI MEDIA

Exact dispersion relation for P and SV waves in VTI media

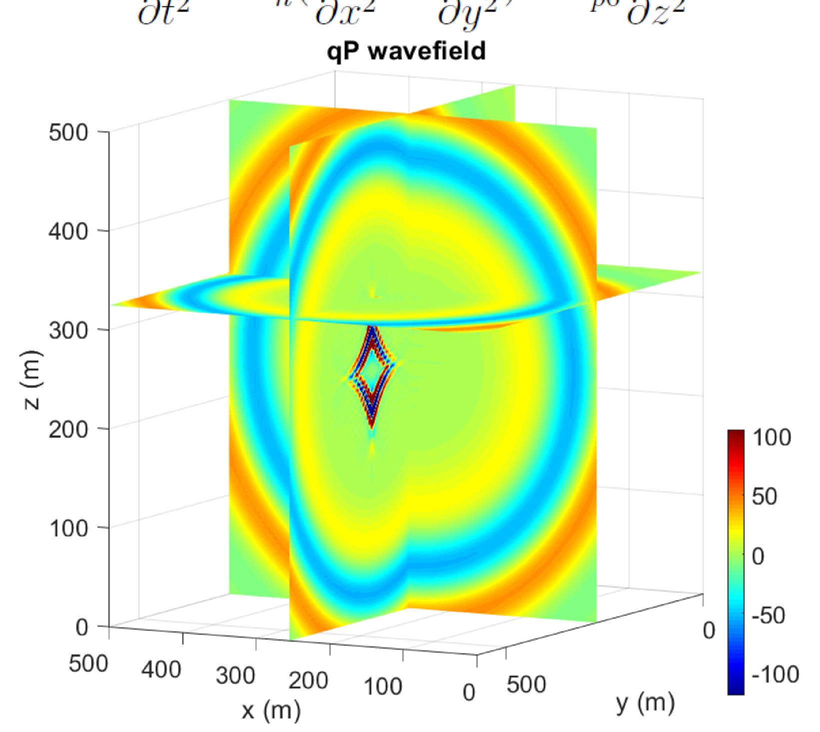
$$\frac{v^2(\theta)}{v_{p0}^2} = 1+\varepsilon\sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\left[1+\frac{2\varepsilon\sin^2\theta}{f}\right]\left[1-\frac{2(\varepsilon-\delta)\sin^2 2\theta}{f(1+\frac{2\varepsilon\sin^2\theta}{f})^2}\right]^{1/2}$$

For weak anisotropic VTI media

$$\frac{V_p^2(\theta)}{v_{p0}^2} = 1+2\delta sin^2(\theta)cos^2(\theta)+2\varepsilon sin^4(\theta),$$
$$\frac{V_s^2(\theta)}{v_{p0}^2} = 1-f+2(\varepsilon-\delta)sin^2(\theta)cos^2(\theta).$$

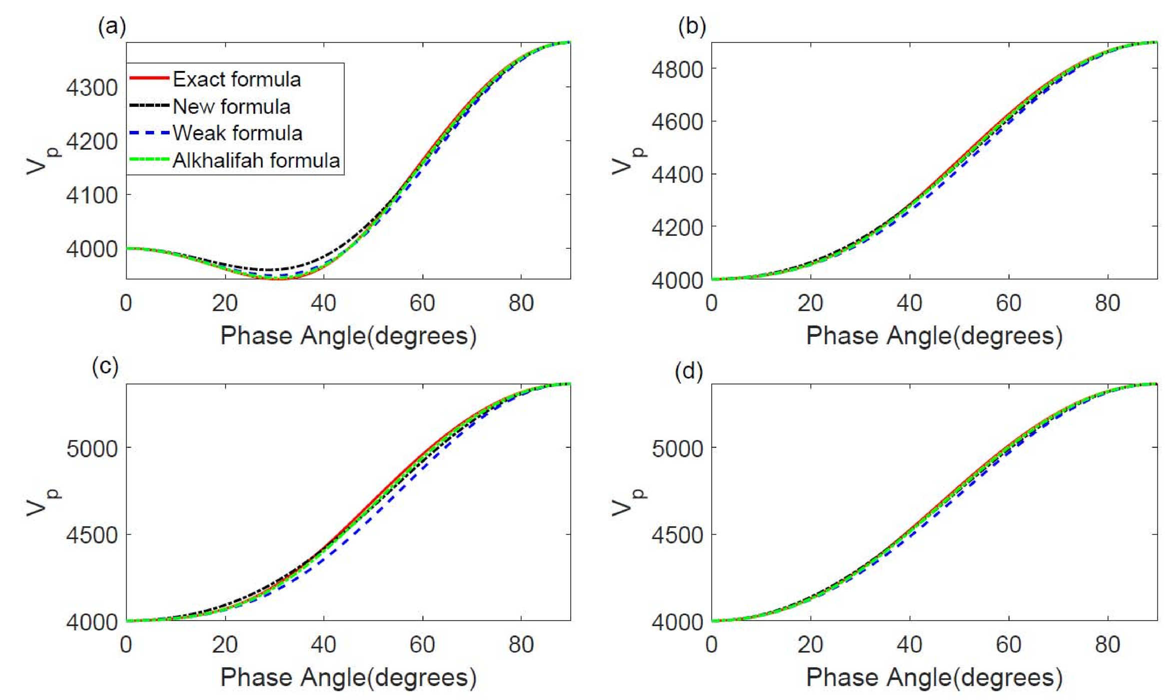
Du (Du et al.,2008) also derived the pseudo-acoustic wave equations

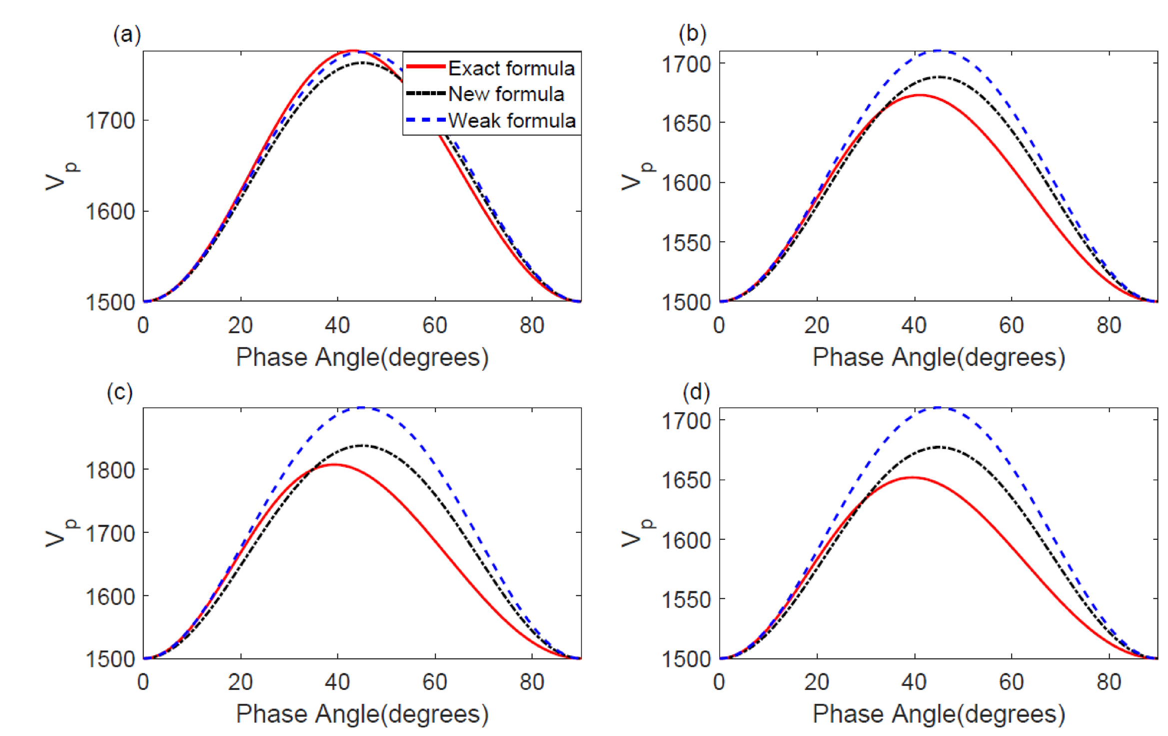
$$\frac{\partial^2 p}{\partial t^2} = v_h^2\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+v_{p0}^2\frac{\partial^2 q}{\partial z^2},$$
$$\frac{\partial^2 q}{\partial t^2} = v_n^2\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+v_{p0}^2\frac{\partial^2 q}{\partial z^2}.$$




We propose a new approximation:

$$\frac{\partial^2 p}{\partial t^2} = \left[v_h^2(k_x^2+k_y^2)+v_{p0}^2k_z^2\right]p - \frac{(v_h^2-v_n^2)}{1+\varepsilon}\left[\frac{(k_x^2+k_y^2)}{k_x^2+k_y^2+k_z^2}\right]k_z^2 p,$$
$$\frac{\partial^2 sv}{\partial t^2} = \left[v_{p0}^2(1-f)(k_x^2+k_y^2+k_z^2)\right]sv - 2\frac{(v_h^2-v_n^2)}{2+\varepsilon}\left[\frac{(k_x^2+k_y^2)}{k_x^2+k_y^2+k_z^2}\right]k_z^2 sv$$



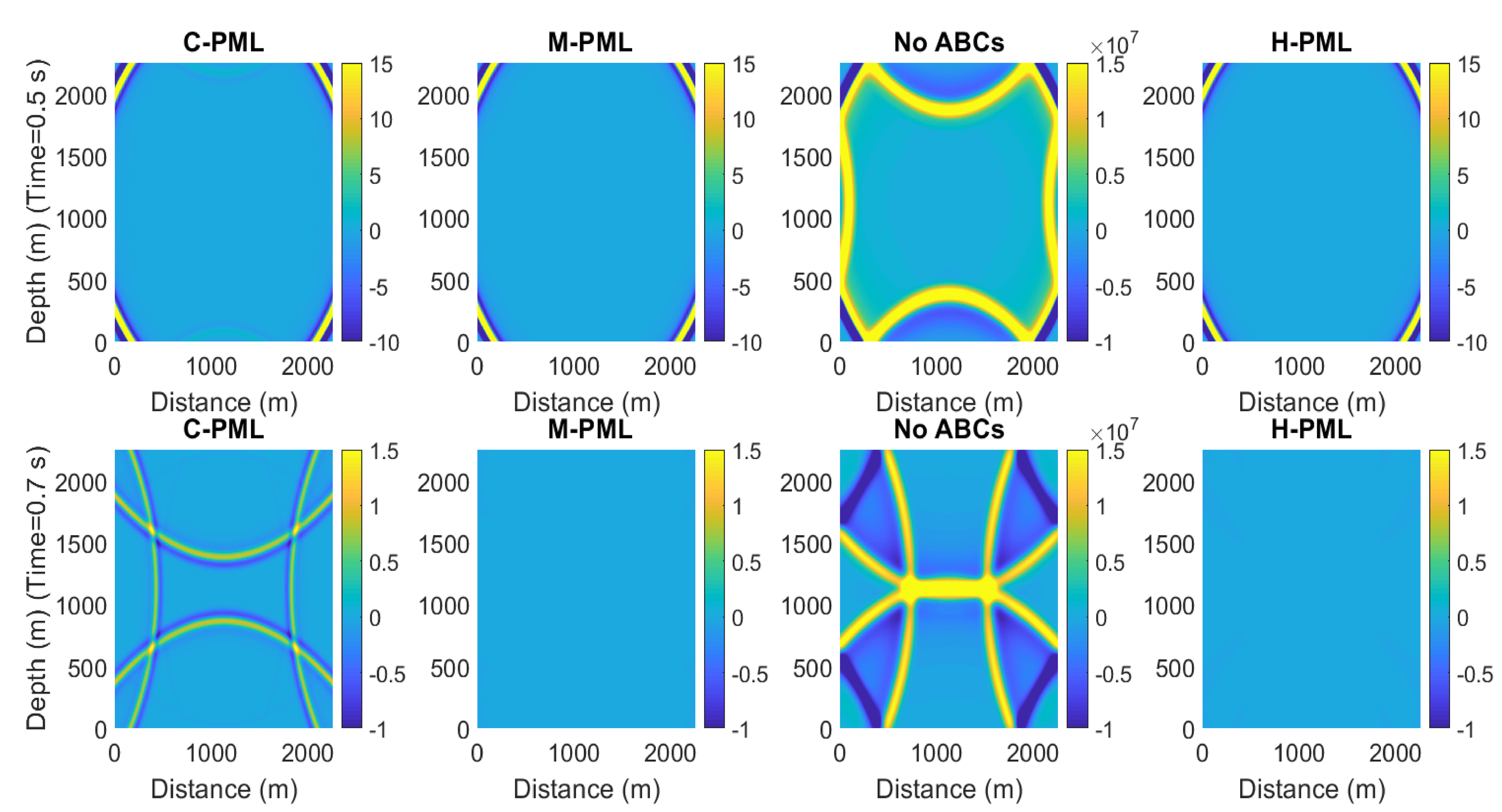

### SECOND ORDER HPML IN WAVENUMBER DOMAIN:

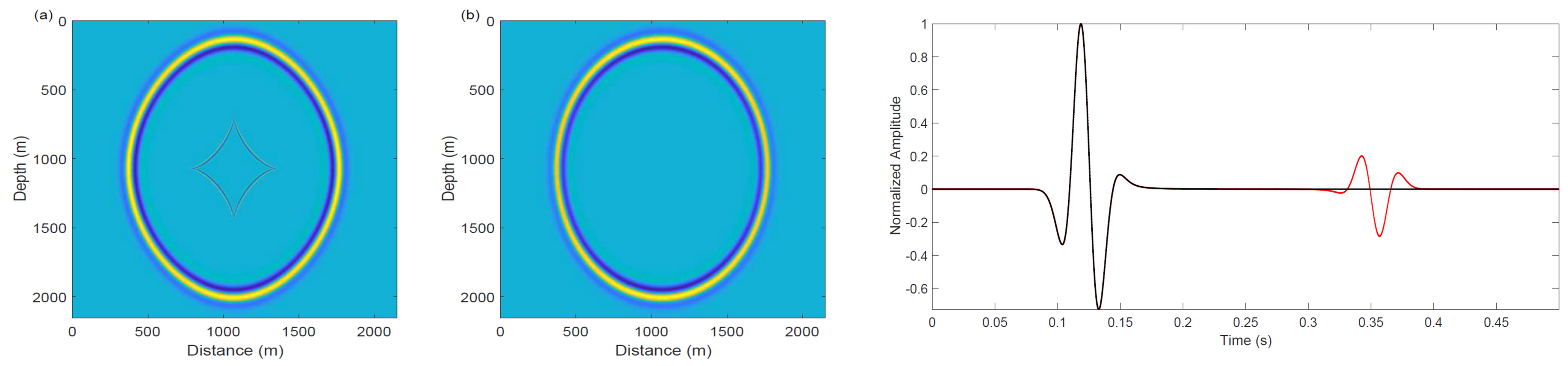
The second order spatial derivatives in wavenumber domain

$$\mathrm{DFT}^{-1}\left[(-jk_{\tilde{x}}).^2\mathrm{DFT}\right] = \frac{1}{\kappa_x}\mathrm{DFT}^{-1}\left[-jk_x\mathrm{DFT}\left(\frac{1}{\kappa_x}\mathrm{DFT}^{-1}(-jk_x\mathrm{DFT})+\psi_x\right)\right]+\phi_x$$

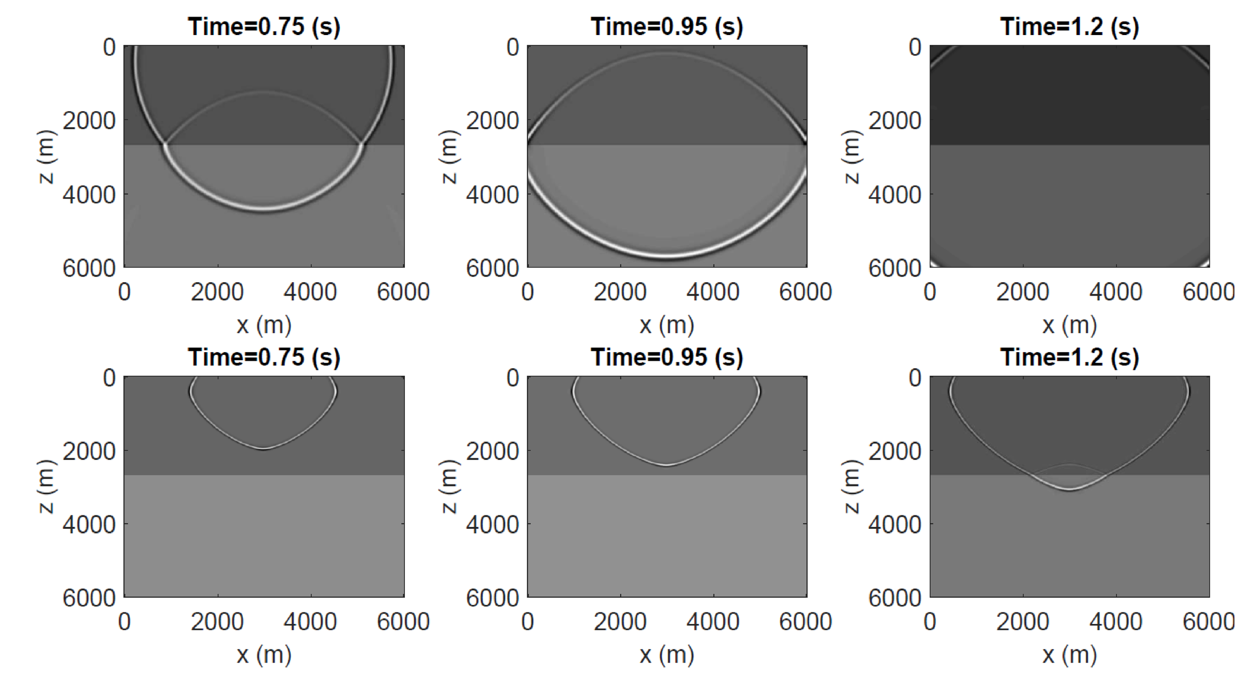
where

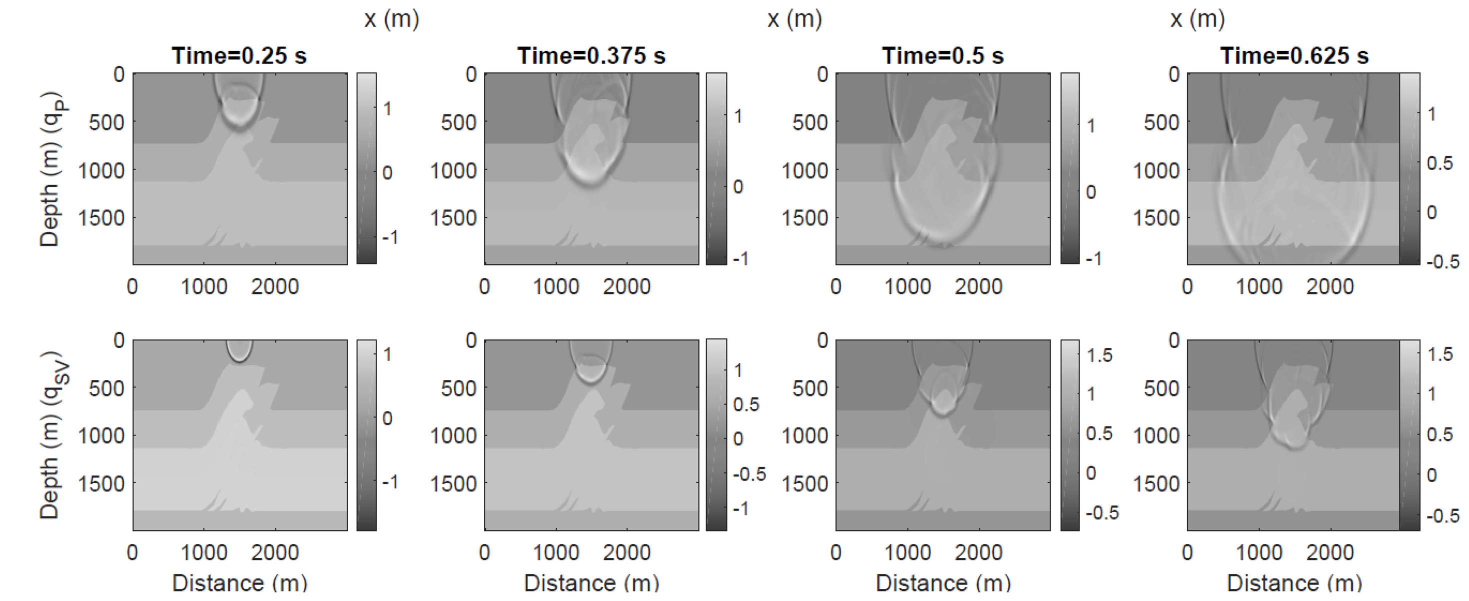
$$\phi_x = b_x\phi_x^{n-1}+c_x\left(\frac{1}{\kappa_x}\mathrm{DFT}^{-1}\left[(-jk_x)^2\mathrm{DFT}\right]^{(n-\frac{1}{2})}+\frac{\partial\psi_x^{(n-\frac{1}{2})}}{\partial x}\right)$$




### NUMERICAL EXPERIMENTS:




### Conclusions

A new approximation of decoupled qP- and qSV-wave equation set has been proposed, which appears to separate the qP-wave completely from the qSV-wave. The approximated qP- and qSV-phase velocities with different phase angle are illustrated and compared with some other methods. Compared with equations with higher accuracy, the new equation set doesn't have to deal with the high order wavenumber, which increases the computational cost and the complexity of the wave equations. In order to eliminate the wrap-around effect and boundary reflections, the H-PML is modified to be applicable for the new decoupled wave equations, which are basically composed of second-order wave number parameters. Numerical comparisons between H-PML, C-PML and M-PML in the second-order wavenumber domain are illustrated and verifies the effectiveness of the H-PML for the new approximation.

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