Azimuthally-dependent scattering potentials and full waveform inversion sensitivities in low-loss viscoelastic orthorhombic media

Shahpoor Moradi* and Kris Innanen

*moradis@ucalgary.ca

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Introduction

The problem of seismic wave scattering from anisotropic and attenuative inclusions is analyzed within the mathematical framework of the Born approximation. Specifically, a Born scattering model is used to extract scattering potentials, which generalize linearized reflection coefficients and sensitivity kernels, and which in the latter form are a basis for multi-parameter seismic full waveform inversion (FWI) updates. To derive the scattering potentials, a point scatterer comprising a perturbation in each medium property is inserted in a homogeneous isotropic background. The amplitudes, or scattering radiation patterns, associated with incoming and outgoing wave vector pairs provide the weights used to simultaneously invert for viscoelastic and anisotropic medium properties. Analysis of the angle-dependence of the scattering patterns provide qualitative and quantitative insight into inter-parameter trade-offs and cross-talk. We explicitly derive scattering potentials for elastic and viscoelastic P-to-P, P-to-SV and P-to-SH waves in a weak anisotropic, low-loss viscoelastic orthorhombic media. We assume the background or reference medium to be either isotropic-elastic or isotropic-viscoelastic. The results generalize reflection coefficient expressions derived from linearization of exact solutions of the Zoeppritz equation for transversely isotropic viscoelastic media with both vertical (VTI) and horizontal (HTI) axes of symmetry.

Viscoelastic orthorhombic media

Stiffness tensor components are defined in terms of anisotropic parameters as

$$
\begin{aligned}
C_{22} &= C_{33}\left(1+2\varepsilon^{(1)}\right),\\
C_{11} &= C_{33}\left(1+2\varepsilon^{(2)}\right),\\
C_{66} &= \frac{1}{2}(C_{55}+C_{44})+\gamma^{(1)}C_{55}+\gamma^{(2)}C_{44},\\
C_{23} &\approx C_{33}\left(1+\delta^{(1)}\right)-2C_{44},\\
C_{13} &\approx C_{33}\left(1+\delta^{(2)}\right)-2C_{55},\\
C_{12} &\approx C_{11}\left(1+\delta^{(3)}\right)-2C_{66}.
\end{aligned}
\tag{1}
$$

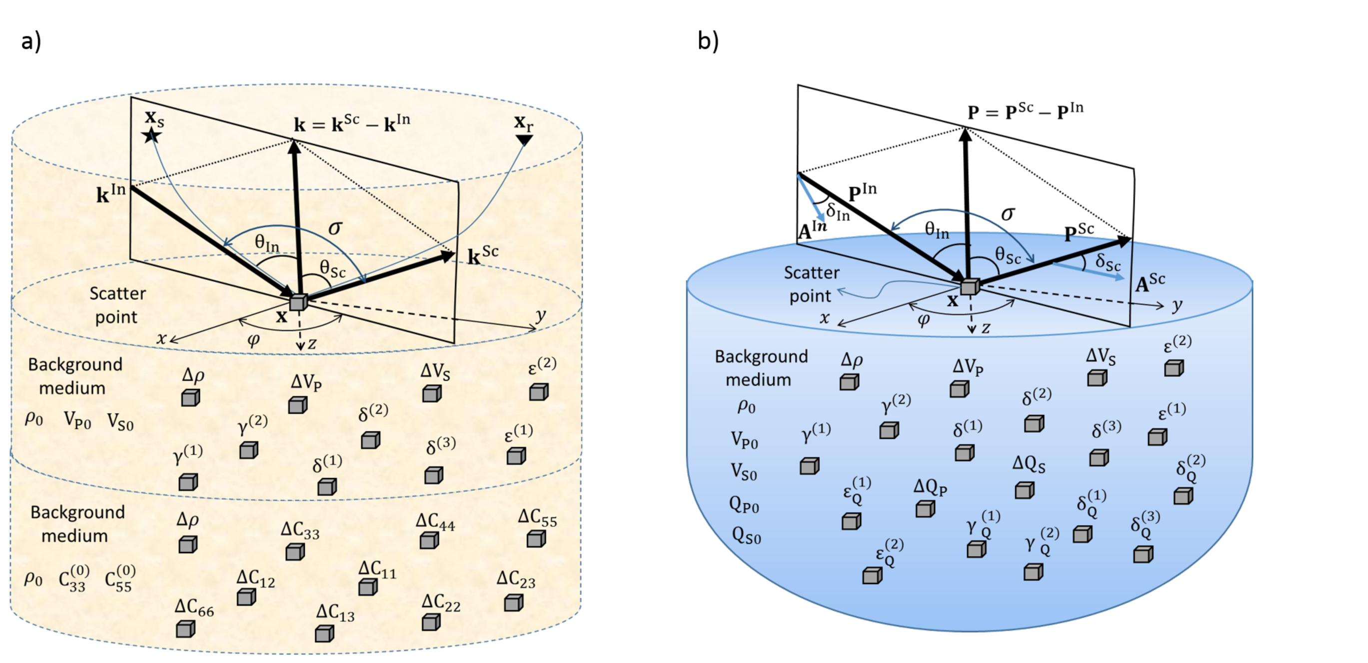

Figure 1: a) Schematic description of Born scattering. $\mathbf{k}^{\mathrm{In}}$ is the slowness vector for incident ray and $\mathbf{k}^{\mathrm{Sc}}$ slowness vector for scattered ray; $\mathbf{k}$, is the difference of incident and scattered slowness vectors and these three vectors define the scattering plane; θ^{In}, the angle between $\mathbf{k}^{\mathrm{In}}$ and $\mathbf{k}$ is incident angle; θ^{Sc}, the angle between $\mathbf{k}^{\mathrm{Sc}}$ and $\mathbf{k}$ is the scattered angle; angle between $\mathbf{k}^{\mathrm{Sc}}$ and $\mathbf{k}$ is scattered angle, σ, the angle between $\mathbf{k}^{\mathrm{In}}$ and $\mathbf{k}^{\mathrm{Sc}}$ is the opening angle. The bottom of Figure a is the schematic illustration of breakdown of the orthorhombic media into isotropic background medium and differences in medium properties for Thomsen (top) and stiffness tensor model parametrization (bottom). b) Viscoelastic orthorhombic volume scattering model. $\mathbf{P}_{\mathrm{In}}$ is the incident propagation vector; $\mathbf{P}_{\mathrm{Sc}}$ is the reflected(scattered) propagation vector; $\mathbf{A}_{\mathrm{In}}$ is the incident attenuation vector; $\mathbf{A}_{\mathrm{Sc}}$ is the scattered attenuation vector; δ_{In} is the incident attenuation angle and δ_{Sc} is the scattered attenuation angle.

Scattering potentials

The main assumption behind the Born approximation states that the actual medium where the wave propagates in it, slightly differs from a homogeneous background medium. The difference between the actual and background media represents the small perturbations of the medium. An elastic homogeneous model of background medium is characterized by its density ρ^0 and its stiffness tensor $C^{(0)}_{ijkl}$, such that the actual medium properties can be written as

$$\rho = \rho_0 + \Delta\rho, \tag{2}$$

$$C_{ijkl} = C^{(0)}_{ijkl} + \Delta C_{ijkl}. \tag{3}$$

Scattering potential is given by

$$
\begin{aligned}
\mathrm{S} =& [\rho]\frac{\Delta\rho}{\rho_0} - [C_{33}]\frac{\Delta C_{33}}{\rho_0} - [C_{44}]\frac{\Delta C_{44}}{\rho_0} - [C_{55}]\frac{\Delta C_{55}}{\rho_0}\\
&- [\gamma^{(1)}]\gamma^{(1)} - [\gamma^{(2)}]\gamma^{(2)} - [\varepsilon^{(1)}]\varepsilon^{(1)} - [\varepsilon^{(2)}]\varepsilon^{(2)}\\
&- [\delta^{(1)}]\delta^{(1)} - [\delta^{(2)}]\delta^{(2)} - [\delta^{(3)}]\delta^{(3)}.
\end{aligned}
\tag{4}
$$

Where the straight bracket [...] denotes the sensitivity of scattering potential to each parameter. For incident inhomogeneous wave, scattering potential is given by $\mathrm{S}_{\mathrm{PP}} = \mathrm{S}^{\mathrm{E}}_{\mathrm{PP}} + i\mathrm{S}^{\mathrm{H}}_{\mathrm{PP}} + i\mathrm{S}^{\mathrm{IH}}_{\mathrm{PP}}$, where contribution of the inhomogeneity of the wave is $\mathrm{S}^{\mathrm{IH}}_{\mathrm{PP}}$.

Radiation patterns: P-to-P wave

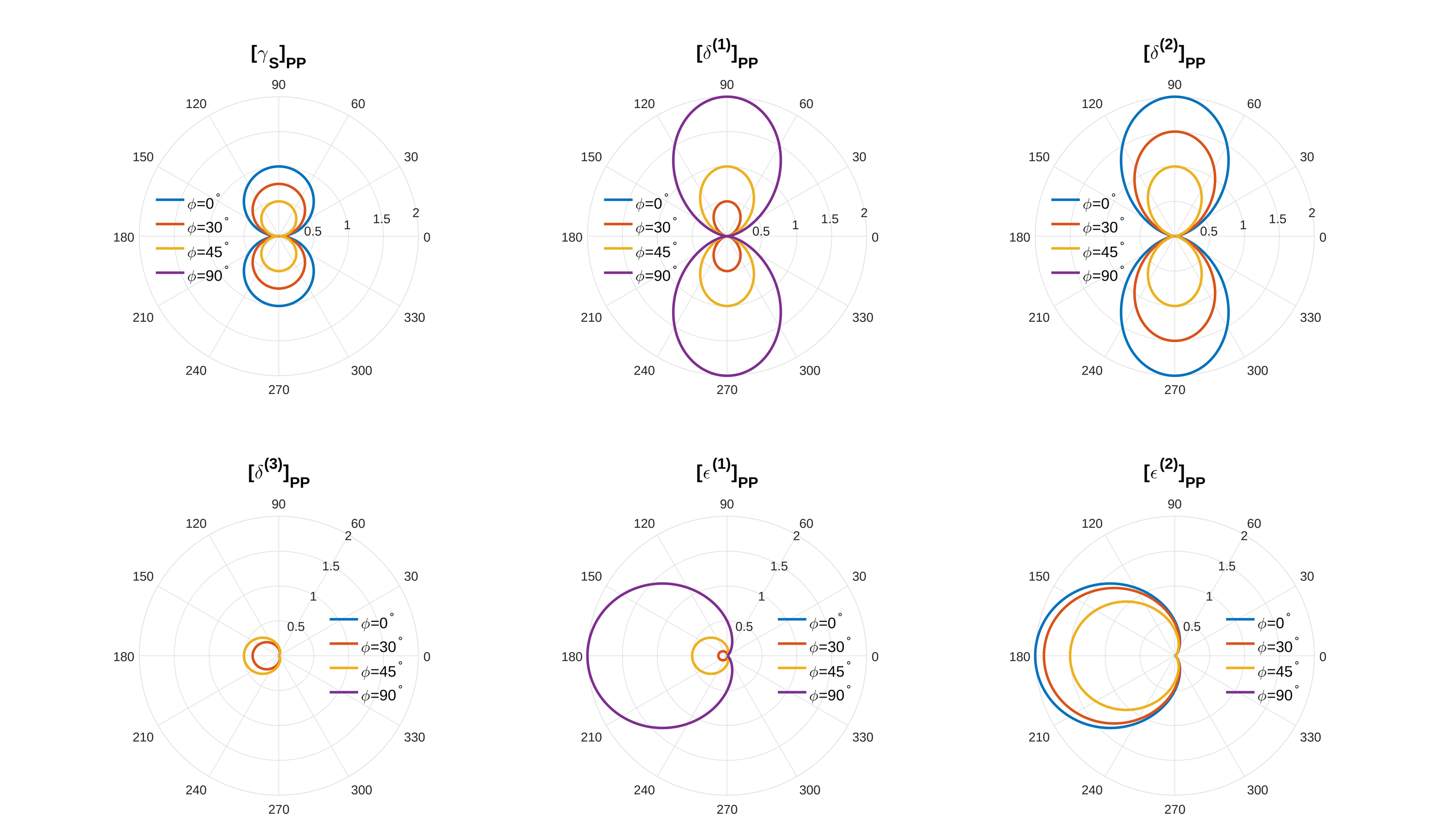

Figure 2: P-to-P radiation patterns induced by anisotropic parameters versus opening angle $\sigma_{\mathrm{PP}} = 2\theta_{\mathrm{P}}$. The six anisotropic Thomsen parameters are placed into the homogeneous isotropic background with $\mathrm{V_{SP}} = \mathrm{V_{S0}}/\mathrm{V_{P0}} = 1/2$.

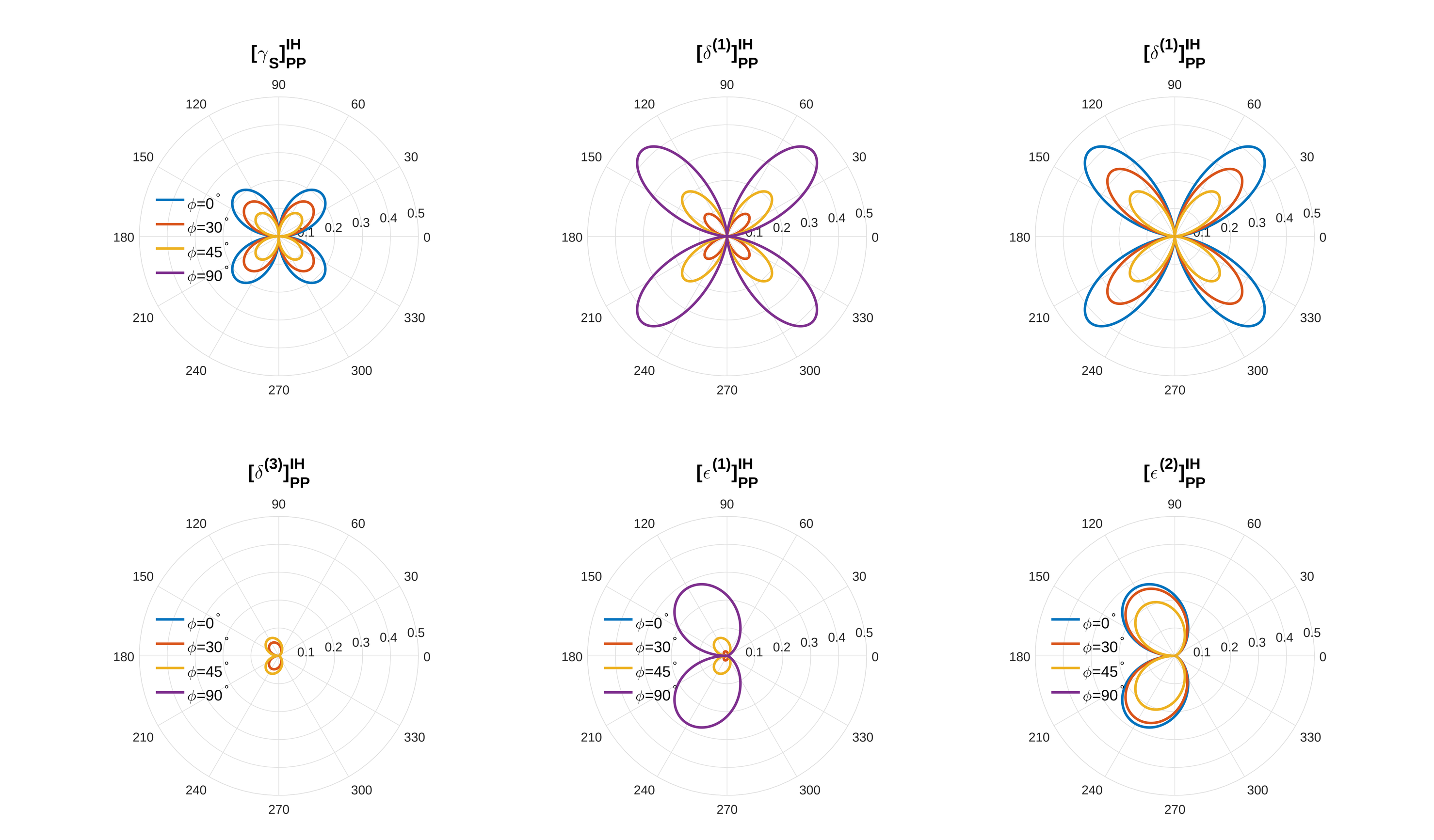

Figure 3: Inhomogeneous part of the P-to-P scattering potential. The six anisotropic Thomsen parameters are placed into the homogeneous isotropic background medium with $\mathrm{Q_{P0}} = 10$ and $\mathrm{Q_{S0}} = 8$.

Radiation patterns: P-to-S wave

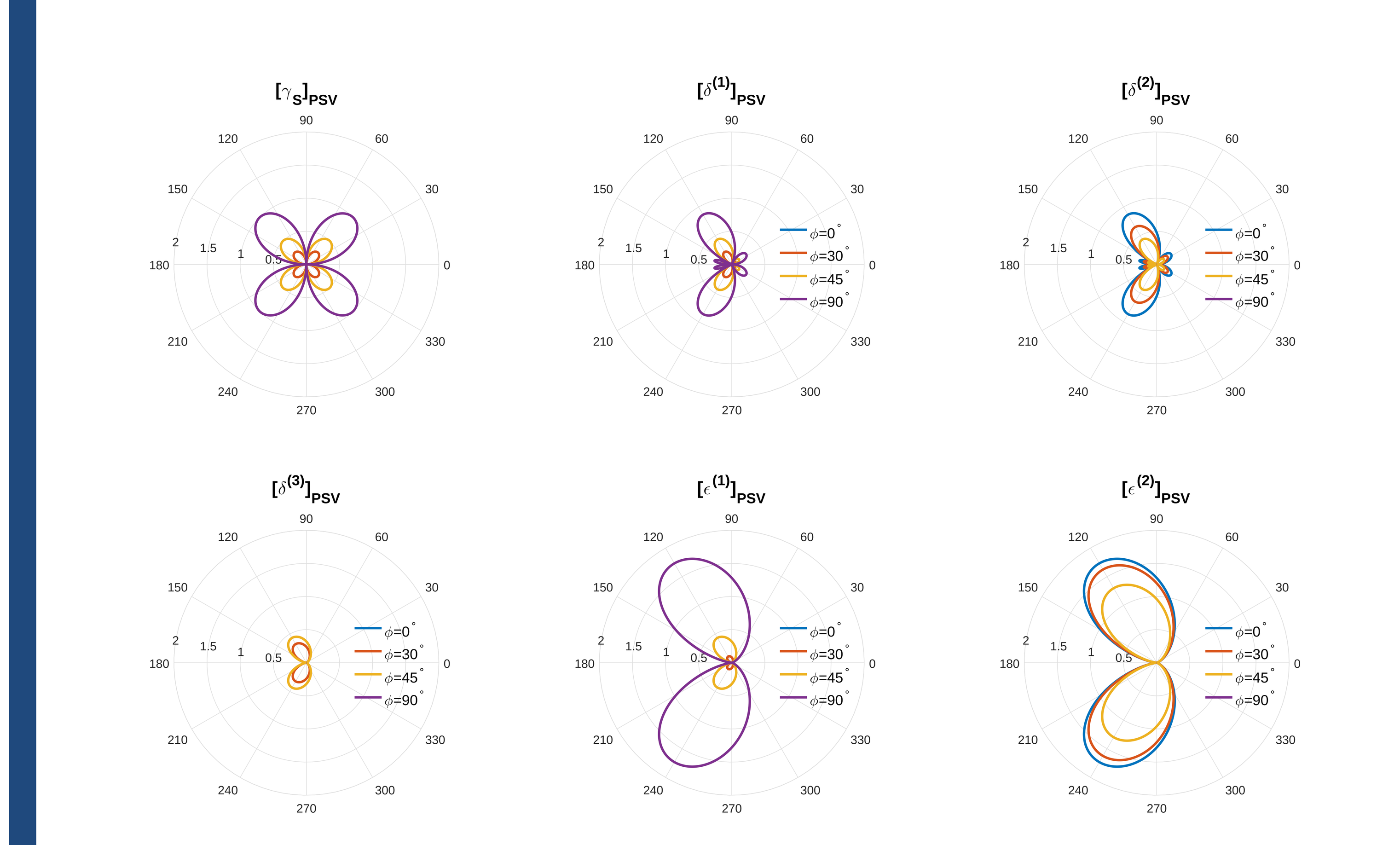

Figure 4: P-to-SV radiation patterns induced by anisotropic parameters versus opening angle $\sigma_{\mathrm{PS}} = (\theta_{\mathrm{P}} + \theta_{\mathrm{S}})$ for different values of azimuth angles $\varphi = 0°, 30°, 45°$ and $90°$. The six anisotropic Thomsen parameters are placed into the homogeneous isotropic background with $\mathrm{V_{SP}} = \mathrm{V_{S0}}/\mathrm{V_{P0}} = 1/2$. All plots in this figure are plotted at the same scale.

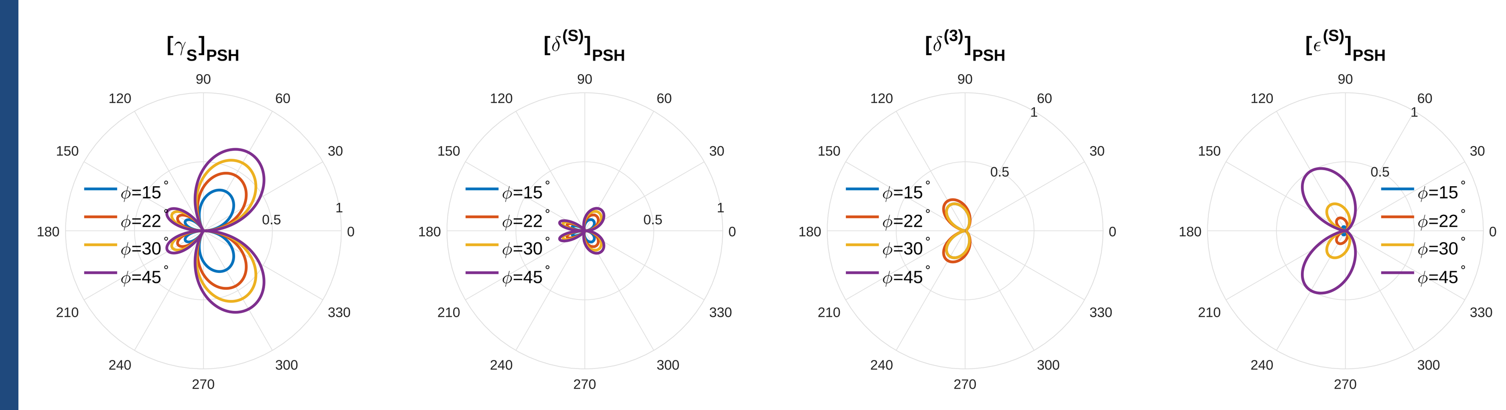

Figure 5: P-to-SH radiation patterns induced by anisotropic parameters.

Conclusions

Scattering potentials for attenuative anisotropic media provides a simple tool to evaluate the Fréchet kernels, and this is relevant to FWI applications where Fréchet kernels are regarded as a sensitivity kernels. Moreover, the study of scattering potentials highlights the dependency of linearized reflection coefficients to anisotropy and attenuation. Attenuation and anisotropy are essential in amplitude variation with offset (AVO) trends as they changes the amplitude and phase of the scattered wave field from geological interfaces. In this research, we derived the analytic forms of the components of the scattering potentials for scattering of the homogeneous and inhomogeneous waves in attenuative orthrohmobic media. These expressions for scattering potentials which are the sensitivity kernels are involved in building the framework for FWI.

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Bibliography

Please see the reports for references.