Comparison between time and frequency domain least-squares reverse time migration

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Abstract

We compare the algorithms of least-squares reverse time migration (LSRTM) in both time and frequency domain and propose a full waveform inversion (FWI) based LSRTM method in the frequency domain. First we show the mathematical equivalence between the gradient of the FWI objective function and the reverse time migration (RTM) imaging condition. Then, we use the FWI formulation with the truncated Newton's method, to solve the linear equation which relates Hessian, model perturbation and the gradient by linear conjugate gradient method. We use simple layer models to compare the two formulations, LSRTM in time and frequency domain. Because of convergence problems that we have not solved yet, we get lower resolution images with the frequency domain FWI-LSRTM method. On the other hand, when the model is inaccurate, the reflector depth seems less affected in the frequency domain. The FWI-based LSRTM method seems to be more robust to velocity errors even if we don't correct the background model as usually done in FWI. Low frequencies seem to be less affected by the inaccurate velocities, and by model smoothness than the high frequencies, suggesting using methods from low frequencies to constraint the high frequencies can help to develop a more robust LSRTM.

Frequency domain LSRTM

The imaging condition of conventional LSRTM in frequency domain is:

$$\mathbf{m}_{mig}(\mathbf{x}) = \sum_{n_s}\sum_{n_\omega} \frac{1}{\omega^2}\mathrm{Re}(\delta \mathbf{u}(\mathbf{x},\omega)\mathbf{G}_0^\dagger(\mathbf{x_s}|\mathbf{x})\mathbf{G}_0^\dagger(\mathbf{x}|\mathbf{x}')\mathbf{f}^\dagger(\mathbf{x},\omega)).$$

where $\mathbf{G}_0^\dagger(\mathbf{x_s}|\mathbf{x})$ and $\mathbf{G}_0^\dagger(\mathbf{x}|\mathbf{x}')$ represent the conjugate transpose of the Green's functions. For the FWI-based LSRTM, the gradient of FWI objective function is

$$\mathbf{g} = -\sum_{\mathbf{n}_\omega}\sum_{\mathbf{n_s}} \mathrm{Re}\left(\frac{\partial \mathbf{d}_{\mathbf{syn}}^\dagger}{\partial \mathbf{m}}\delta \mathbf{d}\right) = \mathbf{R}^T\mathbf{R}\sum_{\mathbf{n}_\omega}\sum_{\mathbf{n_s}}\omega^2\mathrm{Re}\left(\mathbf{G}^\dagger(\mathbf{x}|\mathbf{x}')\mathbf{G}^\dagger(\mathbf{x_s}|\mathbf{x})\mathbf{f}^\dagger(\mathbf{x_s},\omega)\delta\mathbf{u}\right)$$

Comparing with these two imaging conditions, we find that the imaging condition of RTM is equal to the gradient of the FWI objective function, except for the small coefficient changes. Using the truncated Newton's method, the Hessian vector product is

$$\mathbf{H}\delta\mathbf{m} = \mathbf{u}\frac{\partial \mathbf{A}(\mathbf{m},\omega)}{\partial \mathbf{m}}\mathbf{A}^{-1}(\mathbf{m},\omega)\mathbf{R}\mathbf{R}^\dagger(\mathbf{A}^{-1}(\mathbf{m},\omega))^\dagger\left(\frac{\partial \mathbf{A}(\mathbf{m},\omega)}{\partial \mathbf{m}}\right)^\dagger\mathbf{u}^\dagger\delta\mathbf{m}$$

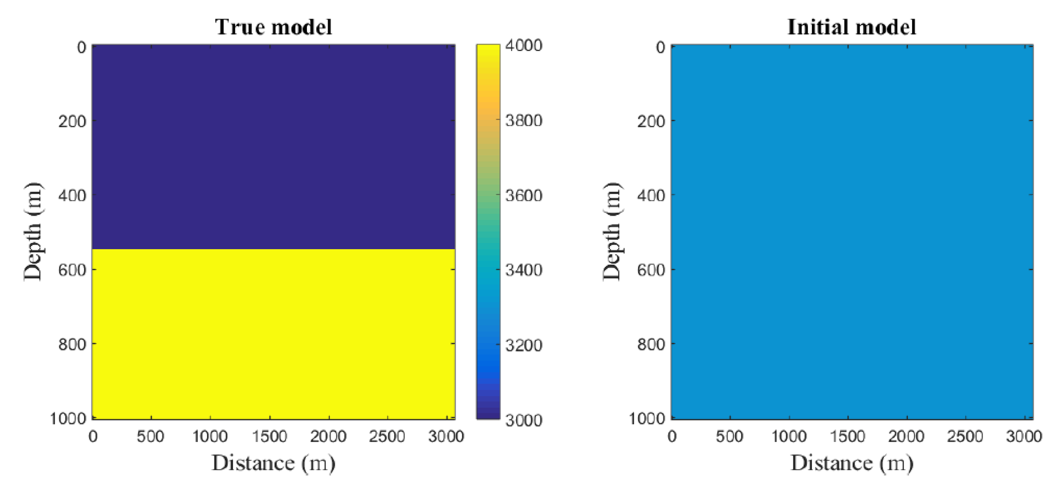
To solve the linear equation $\mathbf{H}\delta\mathbf{m} = -\mathbf{g}$, we use linear conjugate gradient method to get model perturbation iteratively.

Time domain LSRTM

The imaging condition used in time domain LSRTM algorithm is

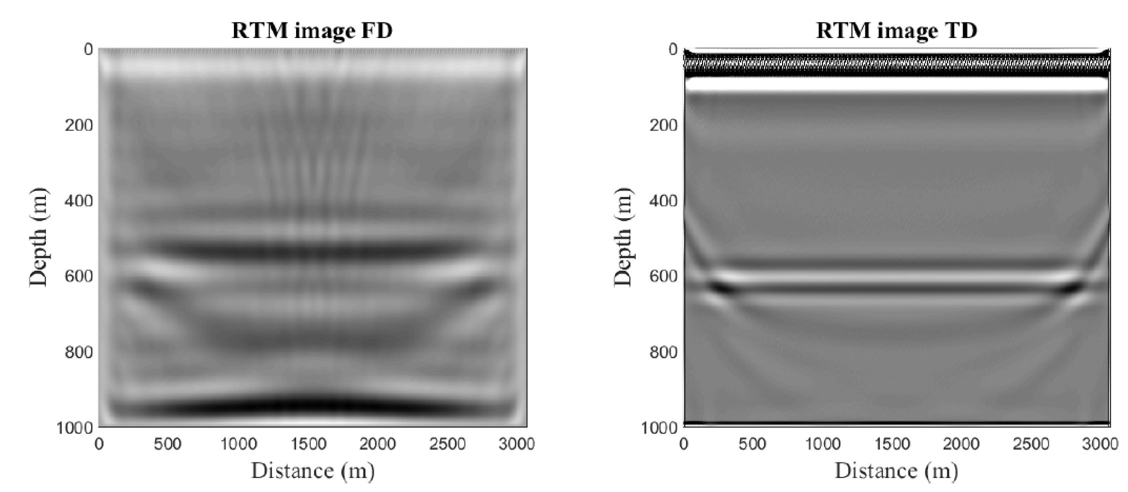
$$\mathbf{m}_{mig}(\mathbf{x}) = \sum_{n_s}\sum_{n_t}\phi_\mathbf{s}(\mathbf{x},\mathbf{t})\phi_\mathbf{r}(\mathbf{x},\mathbf{T}-\mathbf{t}),$$

This imaging condition is the same as the imaging condition in the frequency domain.



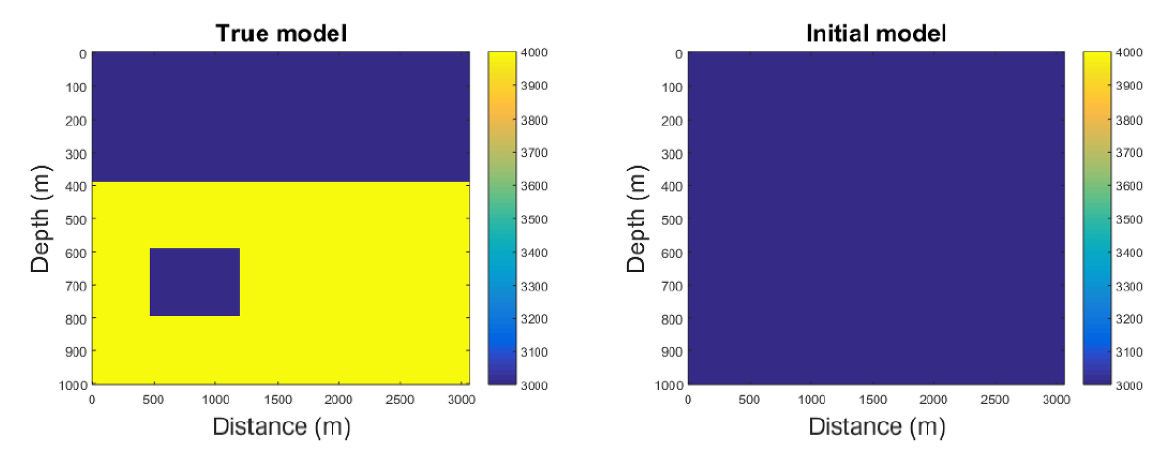
(a) The true model (b) The initial model

Figure 1. The 2-layer model



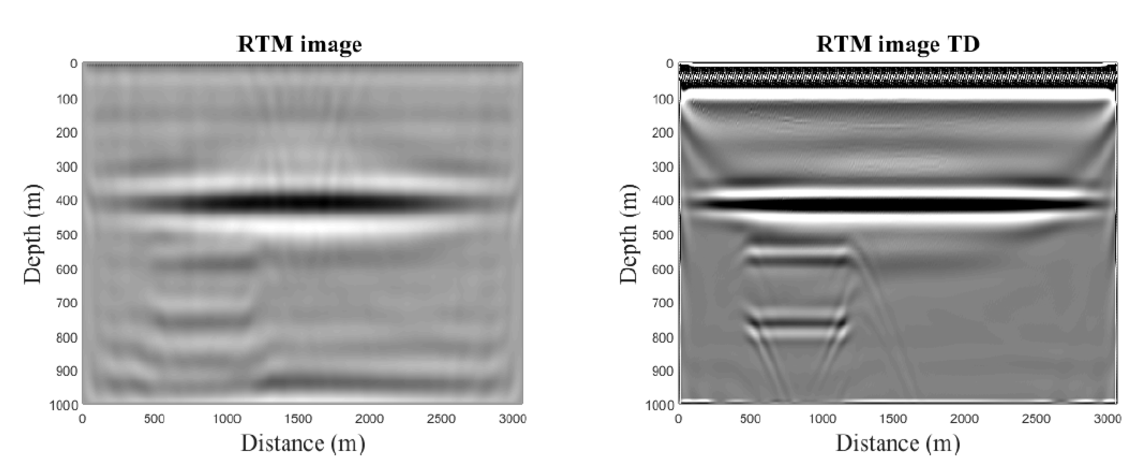
(a) RTM in frequency domain (b) RTM in time domain

Figure 2. The results for time and frequency domain RTM



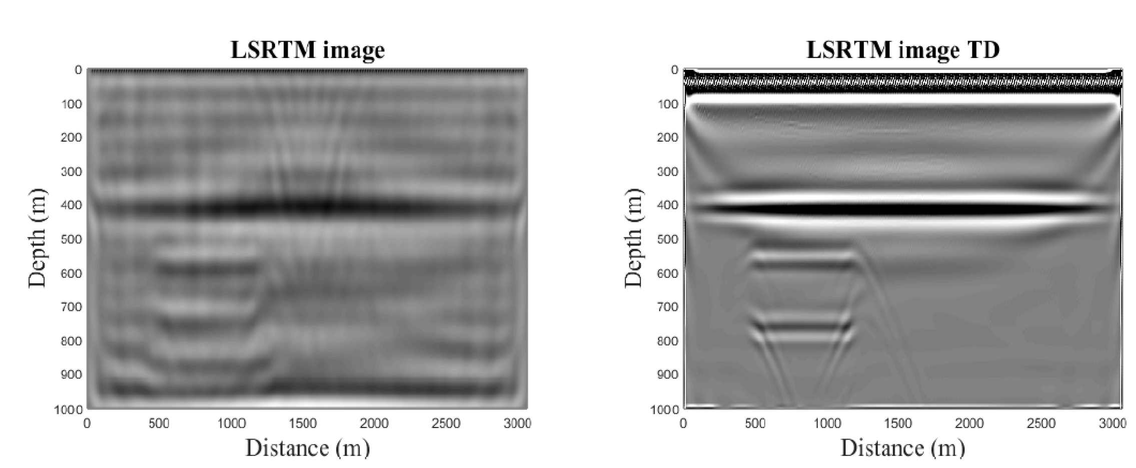
(a) The true model (b) The initial model

Figure 3. The blocky model



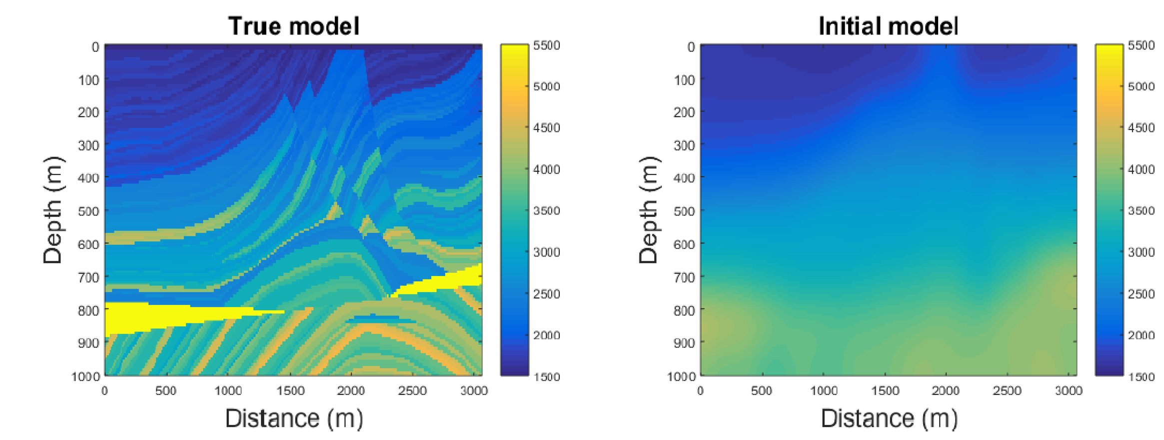
(a) RTM in frequency domain (b) RTM in time domain

Figure 4. The results for time and frequency domain RTM



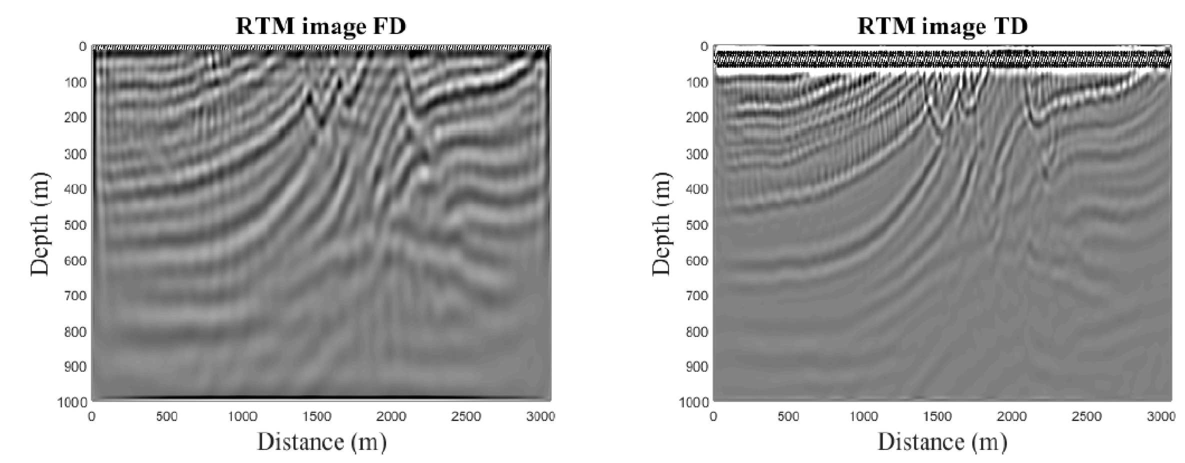
(a) LSRTM in frequency domain (b) LSRTM in time domain

Figure 5. The results for time and frequency domain LSRTM



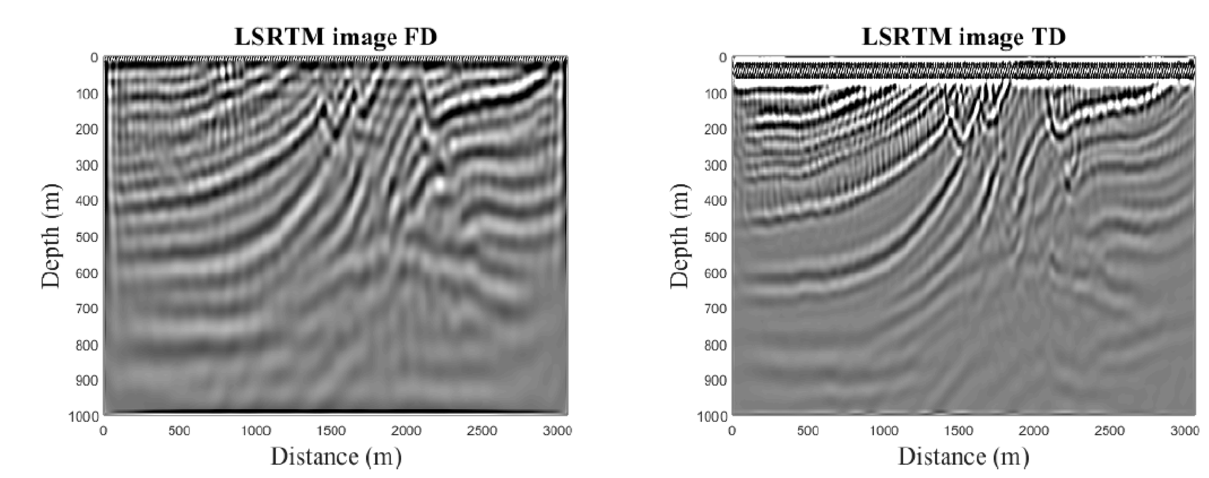
(a) The true model (b) The initial model

Figure 6. The resampled Marmousi model



(a) RTM in frequency domain (b) RTM in time domain

Figure 7. The results for time and frequency domain RTM



(a) LSRTM in frequency domain (b) LSRTM in time domain

Figure 8. The results for time and frequency domain LSRTM

Conclusions

We first compare the LSRTM in frequency and time domains and then propose a FWI-based LSRTM in the frequency domain. This method is based on the objective function of FWI and uses truncated Newton's method to solve for the model perturbation. We used three examples to compare these algorithms and understand the strength and weakness of each method. The conventional time domain LSRTM has a good convergence and sharper image when the velocity model is correct, but when the model is wrong, the reflectors are not correctly located. For the FWI-based LSRTM in the frequency domain with wrong velocities, although it suffers from convergence problems, it seems to locate the reflectors at the correct locations. We speculate that low frequencies seem to be less affected by the wrong velocities. If this is true, we expect to be able to develop a RTM in the frequency domain that uses information from the low frequencies to constrain the high frequencies. Also, we plan to investigate further how the convergence is affected in each case when the velocities are wrong.

Acknowledgements

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