Automatic blind deconvolution with Toeplitz-structured Sparse Total Least Squares Algorithm

Nasser Kazemi*
nasser.kazeminojadeh@ucalgary.ca

Background

Earth can be modeled as an LTI system, and the seismic recordings follow the convolution theorem.

Deconvolution aims at removing the wavelet from the seismic data.

Blind deconvolution aims at estimating the wavelet and reflectivity series simultaneously.

Blind deconvolution is an ill-posed and under-determined problem.

Total Least Squares (TLS) is a type of linear regression that solves for a fully perturbed linear model.

TLS does not provide consistent estimators when the problem at hand is ill-posed and under-determined.

TLS does not consider structured matrices in its formulation.

TLS does not consider sparsity in the coefficients.

More constraints are needed to make the algorithm suitable for real-world applications.

Aims and Motivations

Aims:

Developing a promising single channel blind deconvolution algorithm based on TLS method.

The proposed algorithm should be automatic and preserve the small amplitude reflections in the reflectivity series.

The algorithm should model and handle the noise component properly.

The algorithm should not be confined to minimum phase wavelets.

Motivations:

TLs is a promising algorithm; however, in real-world applications, it usually performs poorly.

Structured TLS assumes that the data matrix has some structures and results in reducing the model domain (number of unknowns).

Structured TLS does not consider sparsity of the reflectivity series and results in poor estimation when the signal of interest is a sparse series.

Assumptions of the proposed algorithm:

No phase assumption about the wavelet.

Toeplitz structure of the convolutional matrices.

Noise is Gaussian.

The desired reflectivity is a sparse series.

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Methodology

Seismic recordings can be written as

$$\mathbf{d} = \mathbf{W}\,\mathbf{r} + \mathbf{n}, \quad \mathbf{W} = \begin{pmatrix} w(0) & & & & \\ w(1) & w(0) & & & \\ w(2) & w(1) & w(0) & & \\ & \vdots & & \ddots & \\ & & & w(L-1) & w(L-2) \\ & & & & w(L-1) \end{pmatrix}$$

Assume that we have an initial estimate about the wavelet

$$\mathbf{w} = \mathbf{w}_0 + \mathbf{w}_{purt} \quad \text{or} \quad \mathbf{W} = \mathbf{W}_0 + \mathbf{E}$$

In other words, the data can be cast as $\mathbf{d} = (\mathbf{W}_0 + \mathbf{E})\,\mathbf{r} + \mathbf{n}$,

Now, TLS solves $\{\hat{\mathbf{r}}, \hat{\mathbf{E}}\} = \underset{\mathbf{r},\mathbf{E}}{\text{argmin}} \quad ||\mathbf{d} - (\mathbf{W}_0 + \mathbf{E})\,\mathbf{r}||_2^2$,

Considering Toeplitz structure for the wavelet, we solve the Structured TLS as

$$\{\hat{\mathbf{r}}, \hat{\mathbf{E}}\} = \underset{\mathbf{r},\mathbf{E}}{\text{argmin}} \quad ||\mathbf{d} - (\mathbf{W}_0 + \mathbf{E})\,\mathbf{r}||_2^2, \quad \text{s.t.} \quad \mathbf{E}\,\mathbf{r} = \mathbf{R}\,\mathbf{w}_{purt}$$

Structured TLS is more efficient than TLS in solving the blind deconvolution problem; however, it does not take advantage of a priori information about the reflectivity series (i.e., sparsity).

We propose to solve

$$\{\hat{\mathbf{r}}, \hat{\mathbf{E}}\} = \underset{\mathbf{r},\mathbf{E}}{\text{argmin}} \quad J(\mathbf{r},\mathbf{E}) = \underset{\mathbf{r},\mathbf{E}}{\text{argmin}} \quad ||\mathbf{d} - (\mathbf{W}_0 + \mathbf{E})\,\mathbf{r}||_2^2 + \lambda||\mathbf{r}||_1, \quad \text{s.t.} \quad \mathbf{E}\,\mathbf{r} = \mathbf{R}\,\mathbf{w}_{purt}$$

To solve the problem efficiently, we expand the proposed cost function around $\mathbf{w}_{purt}$ and $\mathbf{r}$

$$\mathbf{w}_{purt}\{\hat{\Delta\mathbf{r}}, \hat{\Delta\mathbf{w}_{purt}}\} = \underset{\Delta\mathbf{r},\Delta\mathbf{w}_{purt}}{\text{argmin}} \quad J(\mathbf{r}+\Delta\mathbf{r}, \mathbf{w}_{purt}+\Delta\mathbf{w}_{purt}) \quad \text{s.t.} \quad \mathbf{E}\,\mathbf{r} = \mathbf{R}\,\mathbf{w}_{purt}$$

and, by ignoring the small terms, we get

$$\{\hat{\Delta\mathbf{r}}, \hat{\Delta\mathbf{w}_{purt}}\} = \underset{\Delta\mathbf{r},\Delta\mathbf{w}_{purt}}{\text{argmin}} \quad ||\mathbf{res} - \mathbf{R}\,\Delta\mathbf{w}_{purt} - \mathbf{P}\,\Delta\mathbf{r}||_2^2 + \lambda||\mathbf{r}+\Delta\mathbf{r}||_1$$

where $\mathbf{P} = \mathbf{W}_0 + \mathbf{E}$ and $\mathbf{res} = \mathbf{d} - \mathbf{P}\,\mathbf{r}$.

We solve the mentioned above cost function using an alternating minimization algorithm.

Algorithm 1 Alternating algorithm for Toeplitz-structured sparse total least squares

Require: $\mathbf{d}$, $\mathbf{w}_0$, λ, α, N

Initialize: $\mathbf{w}_{purt}^0 = \mathbf{0}$, $\Delta\mathbf{r}^0 = \mathbf{0}$, $k = 1$

1: *Solve* $\mathbf{r}^0 = \underset{\mathbf{r}}{\text{argmin}}\, ||\mathbf{d} - \mathbf{W}_0\,\mathbf{r}||_2^2 + \lambda||\mathbf{r}||_1$

While not converged

For i=1, 2, .., N

2: *Solve* $\Delta\mathbf{w}_{purt}^i = \underset{\Delta\mathbf{w}_{purt}}{\text{argmin}}\, ||\mathbf{res}^{k-1} - \mathbf{P}^{k-1}\Delta\mathbf{r}^{i-1} - \mathbf{R}^{k-1}\Delta\mathbf{w}_{purt}||_2^2$

3: *Solve* $\Delta\mathbf{r}^i = \underset{\Delta\mathbf{r}}{\text{argmin}}\, ||\mathbf{res}^{k-1} - \mathbf{R}^{k-1}\Delta\mathbf{w}_{purt}^i - \mathbf{P}^{k-1}\Delta\mathbf{r}||_2^2 + \lambda||\mathbf{r}^{k-1} + \Delta\mathbf{r}||_1$

endFor

4: *Update* $\mathbf{r}^k \leftarrow \mathbf{r}^{k-1} + \alpha\,\Delta\mathbf{r}^N$

5: *Update* $\mathbf{w}_{purt}^k \leftarrow \mathbf{w}_{purt}^{k-1} + \alpha\,\Delta\mathbf{w}_{purt}^N$

6: *Update* $k \leftarrow k+1$

If converged

Output

$\mathbf{r} \leftarrow \mathbf{r}^k$

$\mathbf{w}_{purt} \leftarrow \mathbf{w}_{purt}^k$

$\mathbf{w} \leftarrow \mathbf{w}_0 + \mathbf{w}_{purt}$

Parameter selection:

The main parameter is λ.

We use Generalized Cross Validation method and pick the optimal value of λ as a minimizer of

$$\text{GCV}(\lambda) = \frac{||\mathbf{res} - \mathbf{R}\,\Delta\mathbf{w}_{purt}(\lambda) - \mathbf{P}\,\Delta\mathbf{r}(\lambda)||_2^2}{(N - C||\mathbf{r}+\Delta\mathbf{r}(\lambda)||_0)^2}$$

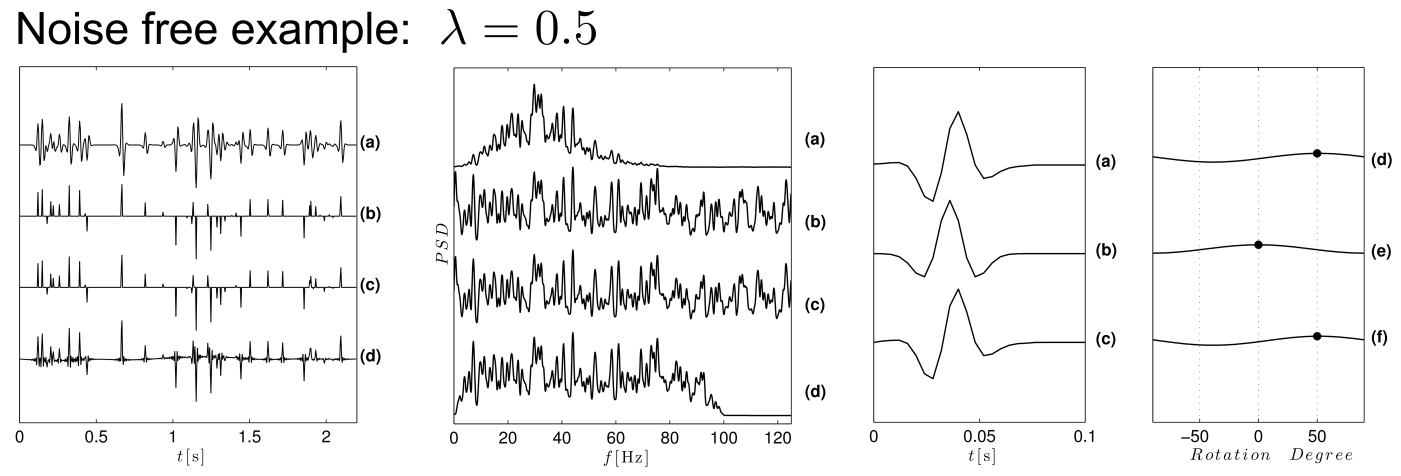
We fix $N = 5$ and $\alpha = 0.1$.

Step 2 is a least squares problem and has a closed form solution.

Steps 1 and 3 are *L2-L1* problems. We use *FISTA* algorithm to solve these steps.

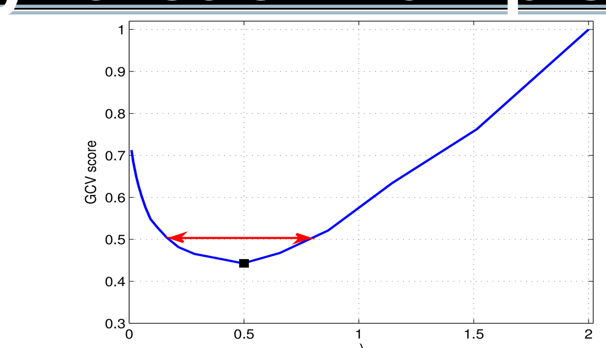
Synthetic Examples

Noise free example: $\lambda = 0.5$



Left) Estimated reflectivity, Middle) Spectrum of the estimated reflectivity, and Right) Estimated wavelet and its corresponding Kurtosis.

Synthetic Examples

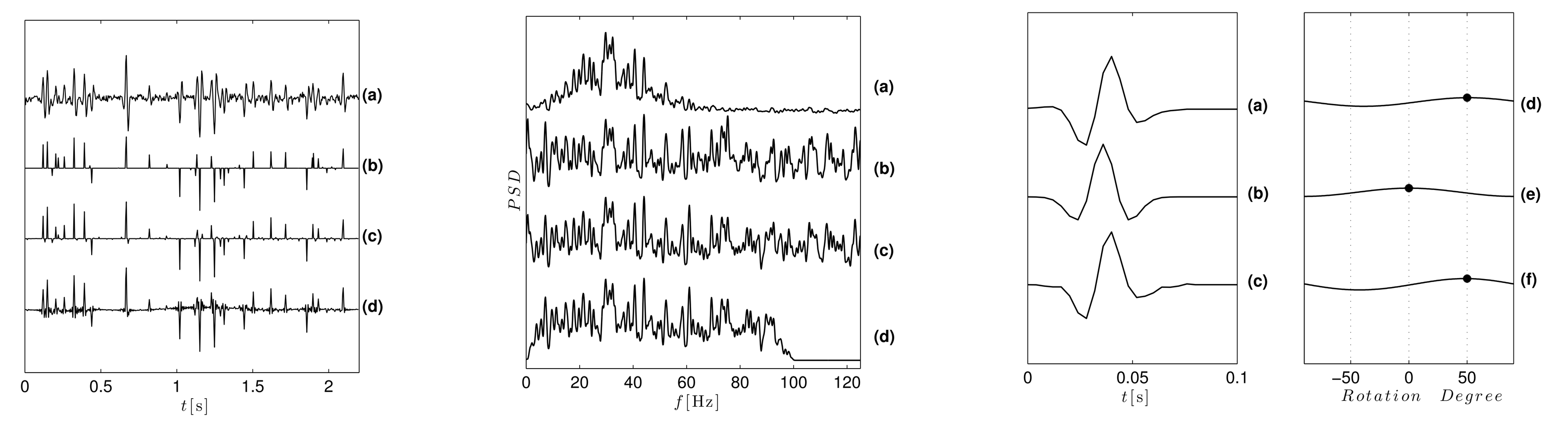


Generalized cross validation score.

Sensitivity of the algorithm to regularization parameter.

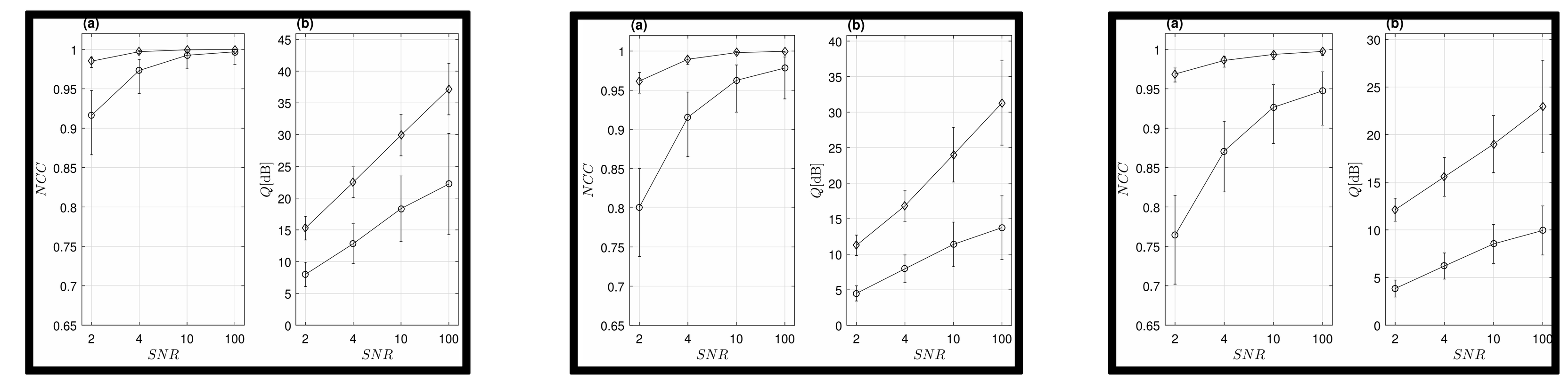
λ	Q_w	Q_r	NCC_w	NCC_r
0.2	39.52	24.20	0.9999	0.9981
0.5	39.76	25.64	0.9999	0.9986
0.8	39.64	24.67	0.9999	0.9983

Noisy data: $SNR = 4$ and $\lambda = 0.65$



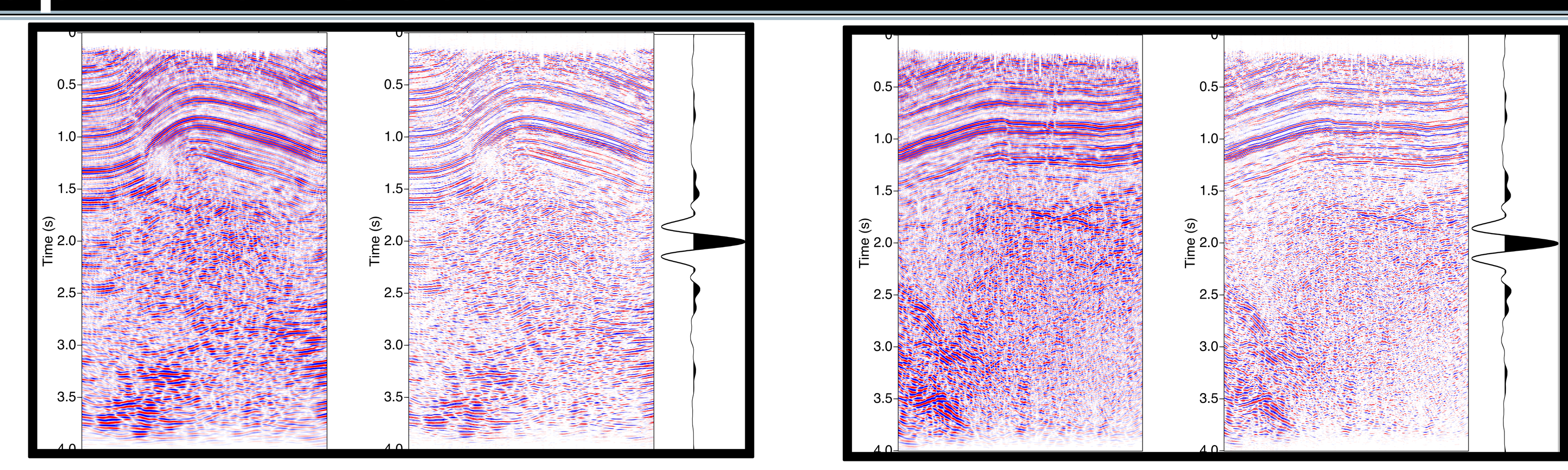
Left) Estimated reflectivity, Middle) Spectrum of the estimated reflectivity, and Right) Estimated wavelet and corresponding Kurtosis.

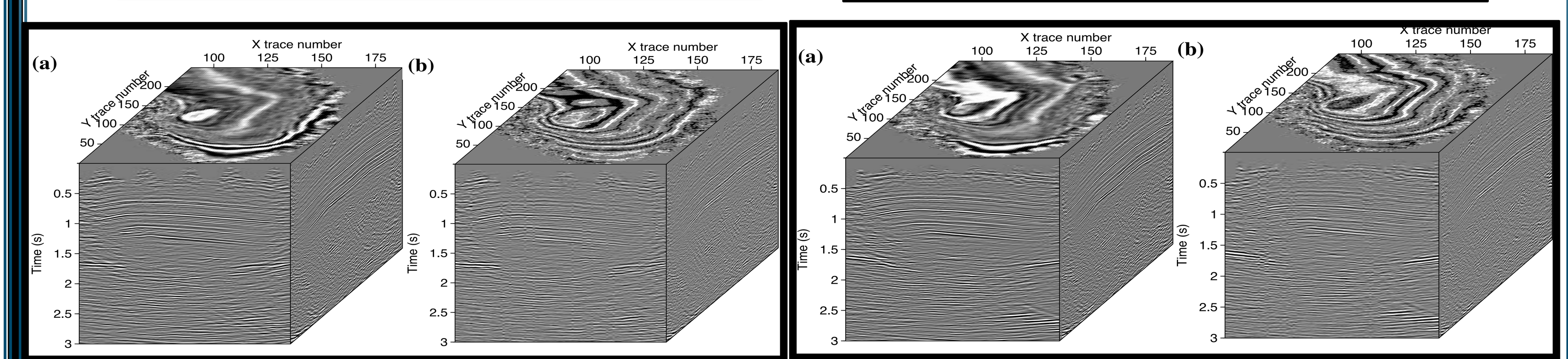
Sensitivity analysis



a) Mean and standard error of normalized correlation coefficients versus SNR. (b) Mean and standard error of the quality of the reconstruction versus SNR. These results were obtained by running a Monte Carlo simulation on 1000 traces with reflectivity series consist of left) 25, middle) 50 and right) 75 coefficients.

Teapot Dome data





3D post-stack data: $Inline = 110$ and $t = 1.03\ s$ and $Xline = 146$.

3D post-stack data: $Inline = 100$ and $t = 1.072\ s$ and $Xline = 143$.

Conclusions

- We developed an efficient and reliable single channel bind deconvolution technique.
- The proposed algorithm is based on Total least squares method.
- There is no assumption about the phase of the wavelet.
- The algorithm is equipped with sparsity constraint on the reflectivity series and preserves the Toeplitz structure of the perturbed data matrix for the wavelet estimation part.
- The proposed method simultaneously recovers the reflectivity series and the wavelet without compromising the small amplitude events in the case of seismic recordings with high SNR.
- The proposed algorithm is successfully applied on real data.