P-SV synthetic seismograms

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ABSTRACT

A method has been developed for calculating a zero-offset converted-wave synthetic seismograms using a sonic log and a time-variant estimate of the V_p/V_s ratio. The algorithm is based on the Goupillaud model, where it is assumed that the layer thicknesses are such that all interval times are constant. It is based on the modification of an existing algorithm, used to calculate zero-offset compressional-wave synthetic seismograms, by integration of the sonic log in terms of a vertical traveltime for a downgoing compressional wave and an upgoing shear wave, and by introducing incident-angle dependence into the reflection coefficient calculation. This requires knowledge of the total depth to the base of each constant time interval (obtained from the velocity structure), and the source-receiver offset (based on the acquisition parameters).

The algorithm has been tested on a sonic log from central Alberta, and compared to a processed converted shear-wave stacked section.

INTRODUCTION

Imaging the sub-surface with seismic data is facilitated by positioning events in their zero-offset equivalent locations which, in regions of flat or gently dipping reflectors, depicts normal incidence. Since conventional compressional-wave reflection data could, in theory, be recorded in such a format, such a model can be represented by a physical phenomenon. On the other hand, it is well known that in the vicinity of normal incidence, a compressional wave incident at the interface between two elastic media will not undergo mode conversion (Pilant, 1979). However, Tessmer and Behle (1988) have developed a technique for gathering, applying normal moveout corrections, and stacking mode-converted reflection data, thus making it possible to map such events to their zero-offset equivalent positions. Hence, applying modified conventional processing algorithms transforms the data to its most conveniently interpreted format, that of zero offset, while at the same time generating results that do not have a physical analogue. This distortion does not give rise to erroneous interpretations, and therefore does not represent a serious obstacle. Nevertheless it is worthwhile to address complications that do arise. This study has addressed the computation of "normal incident" synthetic seismograms for P-SV mode-converted reflection data.

Geis et al. (1988) have proposed that the fundamental, but often difficult, task of correlating P-P and P-SV events can be accomplished using a VSP extracted trace. If VSP data are unavailable this task can be accomplished by generating synthetic zero-offset P-SV seismograms from well log data. The present work is an effort to develop a technique by which such a seismogram can be computed.

THEORETICAL BACKGROUND AND ALGORITHM DESIGN

Normal Incidence P-P Synthetic Seismograms and the Convolutional Model

Given the geologic layering, as determined from well logs, it is possible to construct a synthetic P-P seismic trace (Peterson et al., 1955). Such a trace is established by assuming normal incidence, in which case the reflected amplitude is given by Equation 1,

$$R_{pp} = \frac{V_{p2}\rho_2 - V_{p1}\rho_1}{V_{p2}\rho_2 + V_{p1}\rho_1} \qquad (1)$$

where V_p is the P-wave velocity, and ρ is the density. Based on Equation 1, density and sonic logs can be used to construct a reflectivity sequence in time which is convolved with a wavelet, thus generating the seismic response of the Earth.

Establishing a reflectivity sequence in time is greatly simplified if the so-called Goupillaud model is used. That is, layer thicknesses are assumed to give a constant interval time (i.e. $\delta Z/V_p$ is equal to a constant). In this case the well log is integrated into discrete intervals of two-way traveltime, typically 1 or 2 ms. For each interval an average velocity and density is calculated based on the appropriate well logs, from which a reflection coefficient is computed at the boundary between each layer. In general the velocity function of the sedimentary section varies by several orders of magnitude greater than does the density function, and the computation of synthetic seismograms is often based solely on sonic logs.

The reflectivity sequence, as a function of vertical two-way traveltime, is then convolved with the time domain expression of a source wavelet to obtain the synthetic zero-offset seismic response in the absence of random noise. Multiples can also be incorporated into the seismogram (Waters, 1978).

Modifications Necessary to Compute Zero-Offset Synthetic Seismograms

The present work is an effort to derive a zero-offset P-SV synthetic seismogram from a sonic log and an estimate of the time-dependent (i.e depth-dependent) V_p/V_s function. At the present stage of this work, density variations are assumed to be insignificant.

Two fundamental modifications to the technique used for the computation of a P-P synthetic seismograms are required: first, the sonic log must be integrated in terms of a vertical traveltime for a downgoing compressional wave, and an upgoing shear wave; secondly, offset dependence must be introduced into the computation of the reflection coefficients. The two-fold problem can then be stated as: correctly position events in terms of two-way traveltime; and numerically model the amplitudes of these events.

Obviously, both components of this modelling technique require knowledge of shear wave velocities. In the absence of a full waveform log, these velocities can be estimated in each medium from the P-wave sonic velocities, and the V_p/V_s function. In this study, this function was estimated by correlating events between the compressional and converted-wave stacked sections, and using the relationship:

$$V_p/V_s = \delta t_{pp}/\delta t_{ps}$$

where δt_{pp} and δt_{ps} are the zero-offset interval two-way P-P and P-SV travel times. This estimation procedure is possible only if correlations for a number of reflectors on the compressional and converted-wave sections can be established with confidence prior to the generation of a synthetic seismogram. In general, Poisson's ratio, and therefore the V_p/V_s ratio, varies significantly with depth. Hence a time-variant V_p/V_s function (denoted $V_p/V_s(t)$) is required in the modelling procedure.

Procedure

After specifying an integration interval, the reciprocal of the transit times per depth unit from the sonic log are calculated to give a P-wave velocity in each sampling interval. In addition, the one-way P- wave transit times are converted to two-way P-SV wave traveltimes. This is effected by the following relations. Vertical P-P one-way interval traveltime (δt_{pp1}) for a layer of thickness δh is given by:

$$\delta t_{pp1} = \delta Z/V_p \tag{2a}$$

In addition, the vertical P-SV two-way interval traveltime in the same layer is given by:

$$\delta t_{pv} = \delta Z/V_p + \delta Z/V_s \tag{2b}$$

From equations 2a and 2b it can be seen that:

$$\delta t_{pv} = \delta t_{pp1}[(1+V_p/V_s(t)] \tag{2c}$$

Hence, using the transit times per sampling interval from the sonic log and Equation 2c, the appropriate traveltime can be computed. In addition, the P-wave velocity is computed in each sampling interval by dividing the sampling interval by the P-wave transit time. The P-SV traveltimes are then summed together until a value equal to, or greater than, the integration interval is reached. Then, the average P-wave velocity is computed. This procedure is repeated until the entire sonic log has been divided into discrete intervals of two-way vertical P-SV traveltime.

The next step is to assign an amplitude to the reflections that occur at each travel-time interface; that is, the reflectivity sequence in terms of vertical two-way traveltime. Reflection and transmission of plane waves at the boundary between two elastic media is governed by Zoeppritz's equations for displacement. For a P-SV conversion the exact expression for the reflection amplitude is given by Pilant (1979) and Aki and Richards, (1980). Since at zero-offset there is no mode conversion, reflections that occur as a result of a non-vertical raypath must be mapped to their zero-offset two-way traveltime locations. This is accomplished by transforming the reflection coefficient formula, given by Zoeppritz's equations, to a time-dependent function. This transformation requires an incident angle dependent offset expression, and an offset-dependent time function. The problem is simplified considerably by recognizing the fact that the moveout time is unimportant. For a layered medium in which vertical traveltimes and velocities are known, the incident and reflected angles are determined for a given offset and reflector depth. A reflection coefficient is then calculated, based on these angles and Zoeppritz's equations. Currently, a straight raypath assumption is made for computation of the incident angle.

For a single layer of thickness Z,

$$\theta_i = \arctan(X_s/Z) \tag{3a}$$

and

$$\theta_r = \arctan(X_r/Z) \tag{3b}$$

where X_s and X_r are respectively the distance from the conversion point to the source, and the distance from the conversion point to the receiver. Fromm et al., (1985) have derived a first order approximation relating X_s and X_r to X, the source receiver offset. These are:

$$X_s = X/(1 + V_s/V_p) \quad (4a)$$

and

$$X_r = X/(1 + V_p/V_s) \quad (4b)$$

Substitution of Equations 4a and 4b into 3a and 3b respectively, yields:

$$\theta_i = \arctan\{X/[(1 + V_s/V_p(t))Z]\} \quad (5a)$$

and

$$\theta_r = \arctan\{X/[(1 + V_p/V_s(t))Z]\} \quad (5b)$$

Thus if the depth to a reflector is known, the source-receiver offset can be specified, and using the straight-ray approximation, the incident and reflected angles are computed, from which a reflection coefficient is derived. This reflection coefficient is then incorporated into the reflectivity-time sequence at the time corresponding to the two-way vertical traveltime to the base of the layer. To apply this technique, the total depth to the base of each layer of vertical traveltime is required. Since the thickness of each time interval is obtained from the average velocities in each interval, the depth to the base of each interval can thus be obtained easily.

To obtain a realistic comparison between the P-SV synthetic seismograms and the stacked section, the procedure must take into account the acquisition and processing parameters used to obtain the converted-wave section.

Because the reflectivity is dependent on incident angle, the source-receiver offset range of the data is an important parameter to consider. The non-linear variation in reflection coefficient versus incident angle (and hence offset) requires that accurate synthetic results be generated so as to imitate the acquisition parameters of the data. Hence, in this procedure, reflection coefficients are calculated between the near and far offsets at intervals equal to the group interval. A composite coefficient is then derived from the average of these values, thereby modelling a full-offset range stacked section.

A second important influence on the amplitude of events in the stacked section is the mute pattern. Production processing of mode-converted data is generally conducted using common reflection point gathering algorithms that do not account for the variation of conversion point with depth. This tends to smear far offset reflections (i.e. events with large incident angles). To eliminate this, a harsh mute pattern is applied to the data. Thus the present algorithm allows for a user-defined mute pattern, allowing the synthetic reflection amplitudes to be comprised only of reflection coefficients that correspond to offsets (i.e. incident angles) that are present in the stacked data.

RESULTS

The algorithm has been tested on P-SV data from central Alberta (Figure 1). For this survey the far offset was 2540 m, the near offset was 180 m, and the group interval was 30 m. A digitized sonic log, located at shotpoint 276 on line 2 (6-35-52- 13 w5) was available.

From the P-P and P-SV sections, a V_p/V_s versus time function was determined. This function did not vary significantly from line 1 to line 2, and discrepancies between the two have been attributed to inaccuracies in event picking. Hence the V_p/V_s function, shown in Figure 2, is an average of the function determined from each line. In addition to this, petrophysical measurements conducted at reservoir conditions for a Cardium core sample from well 6-12-53-13 w5, were available. These indicated a V_p/V_s ratio of 1.75. This value has been included in the V_p/V_s

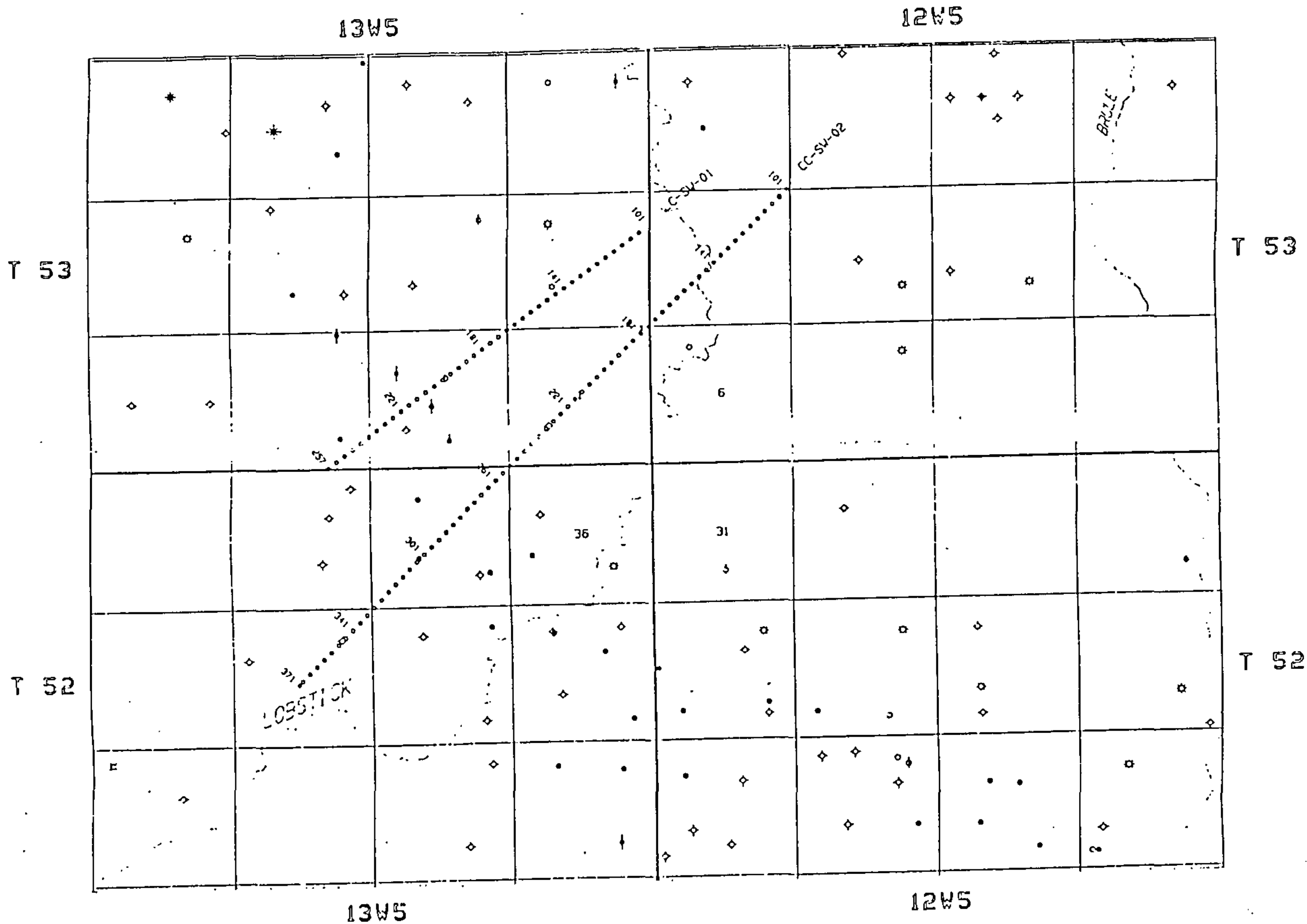

FIG. 1. Map showing location of data used to test the P-SV zero-offset synthetic algorithm.

function in the time window over which the Cardium occurs.

The P-SV synthetic, tied to the P-SV seismic data, is shown in Figure 3. In general, the correlation is good, especially in the first 1.5 s. In the later part of the data, the modelled reflections are generally of lower relative amplitude than their real data counterparts. This may be caused by the lack of time-variant trace scaling of the synthetic trace.

CONCLUSION

A technique has been proposed for computing a P-SV synthetic seismogram based on a sonic log and a time-variant estimate of the V_p/V_s ratio. To model travel times the sonic log is integrated in terms of a downgoing compressional wave, and an upgoing shear wave at normal incidence. Amplitudes are modelled by computing an average of the offset-dependent reflection coefficients, given by Zoeppritz's equations, at each receiver location across the spread.

While these initial results are encouraging, there are two modifications to the algorithm that are planned. First, an exact ray trace could replace the straight-ray approximation used to compute the incident angle at each interface, for each offset. Secondly, estimation of the shear wave velocity from the V_p/V_s function could be replaced by using a full waveform log. Nevertheless, it appears the zero-offset synthetic seismogram, a fundamental interpretation tool, can be computed for mode-converted data.

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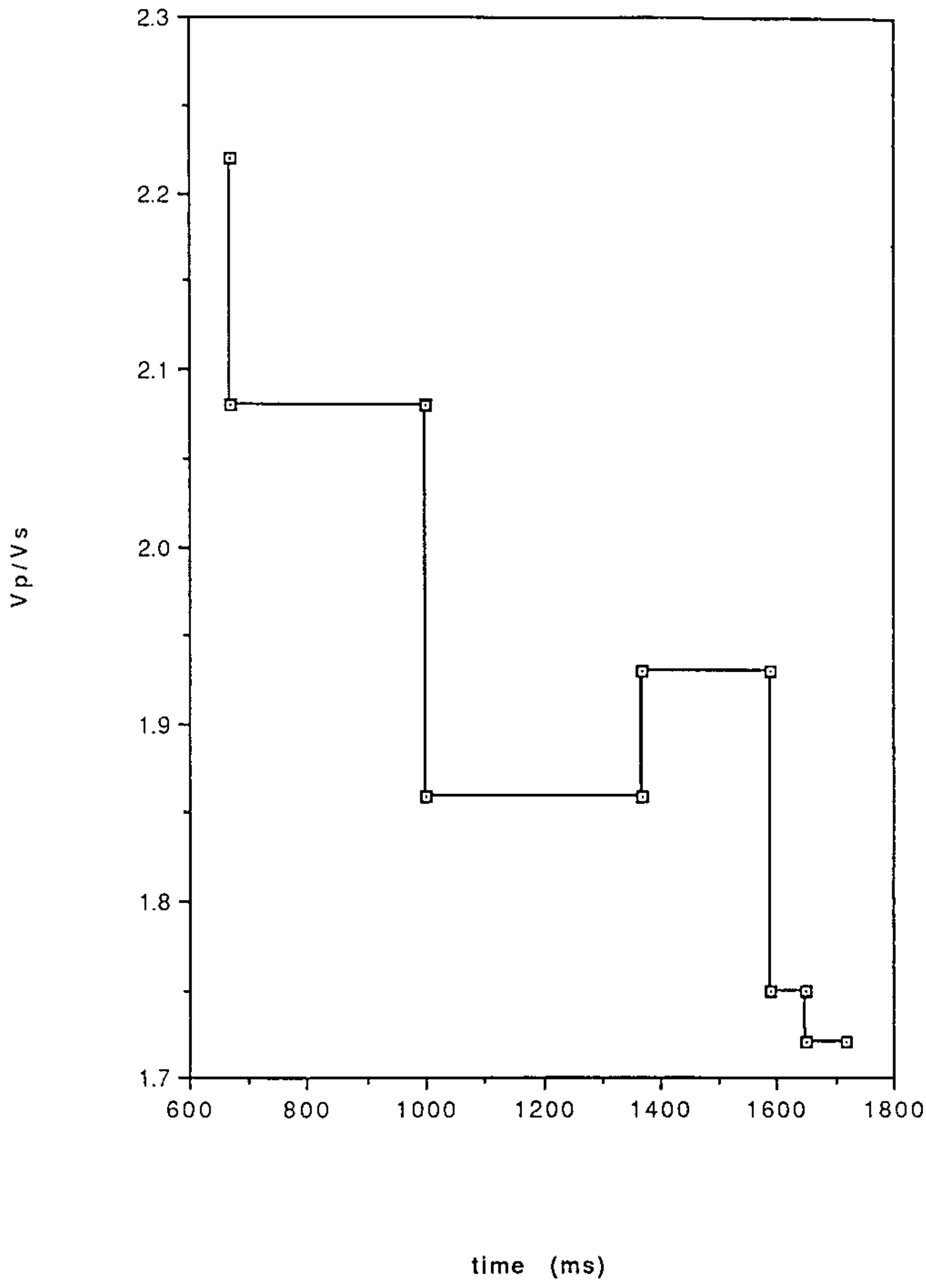

FIG. 2. The time variant Vp/Vs function used in conjunction with the P-wave sonic data to generate the P-Sv synthetic shown in Figure 4. The values are obtained from correlating isochrons on the compres- sional and mode-converted data. In addition, the value between 1590 and 1650 ms was obtained from simulated in-situ petrophysical measurements.

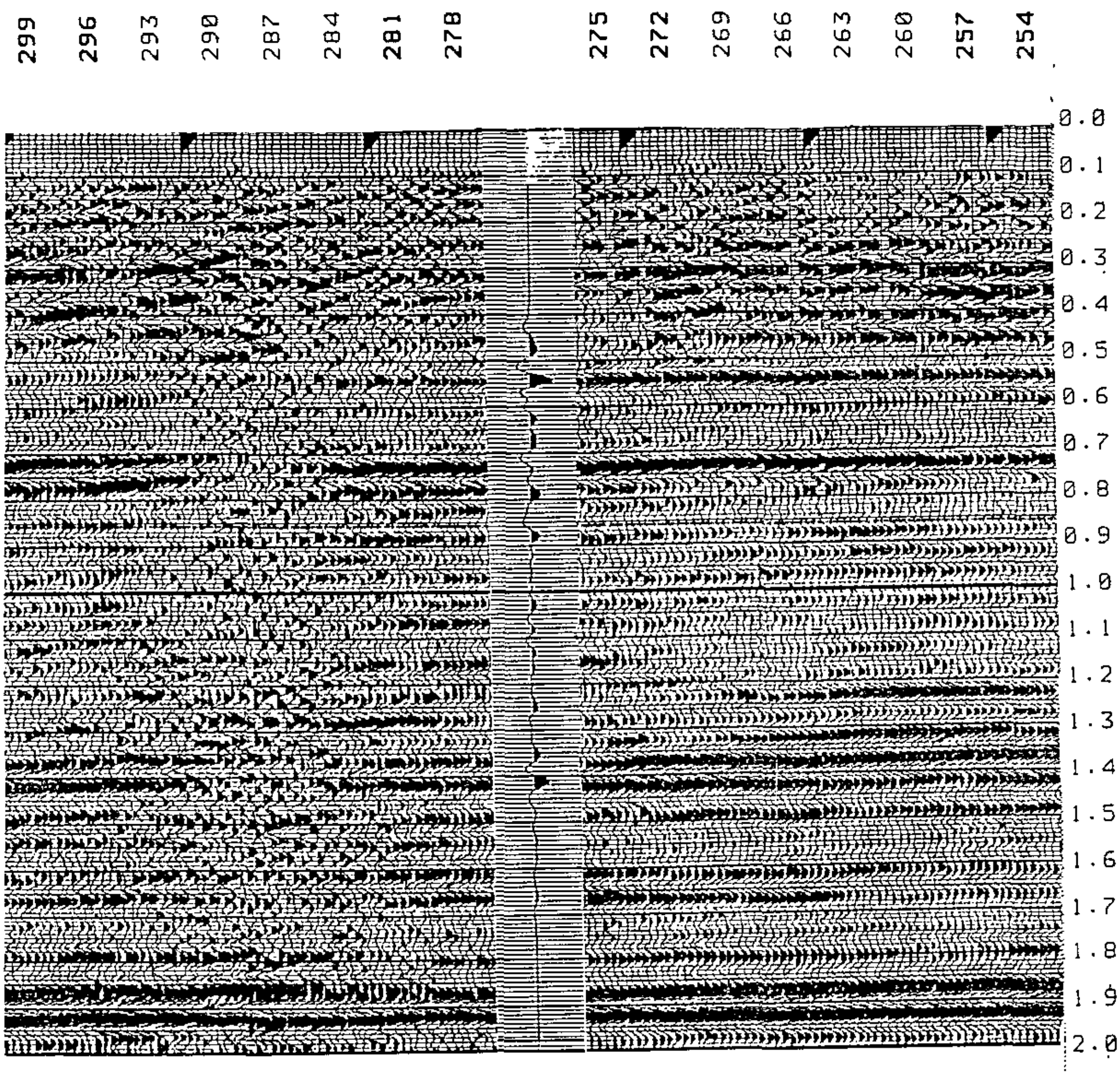

FIG. 3. P-Sv synthetic seismogram calculated for the sonic log from the 6-35-52-13 13 w5 well. The shotpoints are shown along the horizontal axis. The tie location is at shotpoint 276.