# A mode-converted (P-SV) synthetic seismogram

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## ABSTRACT

A method has been developed for calculating a 'zero-offset' converted-wave (P-SV) synthetic seismogram. The algorithm is based on the simplifying assumption of layer thicknesses with constant interval times. These interval times represent the zero-offset travel time of a downgoing compressional wave and an upgoing shear wave. A reflected amplitude is computed for each offset used in the data acquisition, using the exact expression for a P-SV reflection. These reflection amplitudes are stacked to create the zero offset trace. Offsets are mapped to incident angles by raytracing the model. Computation of the synthetic requires either a full waveform sonic log, or a compressional-wave sonic log and a time variant estimate of the $V_p/V_s$ ratio.

The algorithm has been tested on a full waveform sonic log from southern Alberta, and compared to an offset mode-converted VSP. In general the correlation between the synthetic and the real data is good, as the major reflections are successfully predicted in both time and amplitude.

## INTRODUCTION

Imaging the subsurface with seismic data is facilitated by positioning events in their zero-offset equivalent locations which, in regions of flat or gently dipping reflectors, depicts normal incidence. Thus it is reasonable to generate synthetic seismograms by integrating the sonic and density logs into zero-offset two-way, compressional-wave travel time, and by computing reflection coefficients at normal incidence based on the impedance contrast derived from integration of the well logs. On the other hand, it is well-known that at normal incidence, a compressional wave incident at the interface between two elastic media will not undergo mode conversion (Pilant, 1979). Nevertheless a number of authors have described techniques for gathering, applying normal moveout corrections, and stacking mode-converted (P-SV) reflection data (Fromm et al., 1985; Tessmer and Behle, 1988; Slotboom, 1990; and Eaton et al., 1990). Thus, it is possible to depict P-SV data as a conventional zero-offset stacked section, while at the same time generating results that do not have physical analogues. Under these circumstances it is clear that the standard zero-offset approach used for calculating synthetic seismograms for P-P data is inadequate for the P-SV case.

Identifying events, establishing polarity on P-SV data, and correlating these to the P-P sections are tasks that are fundamental to understanding converted-wave data. Presently the only technique available to the interpreter is the composite L-plot (Gaiser et al., 1984). Geis et al. (1990) have produced an L-plot using full waveform sonic logs, VSP extracted traces for P-P and P-SV reflections, and P-P synthetic and surface data. These authors found significant differences in the reflectivity between the P-P and P-SV data in some intervals. Hence it would seem that a P-SV synthetic seismogram could enhance interpretation of these data. As such we feel that it is an essential tool for undertaking an interpretation, both in cases where VSP data are available, as a component of a composite L-plot, and by itself when only surface seismic data are available. The present work is an effort to develop a technique by which such a seismogram can be computed.

## ALGORITHM DESIGN

### Normal incidence P synthetic seismograms

A number of authors have reviewed the various processes by which a synthetic compressional-wave seismic trace can be constructed, provided the geologic layering is known from well logs (Peterson et al., 1955, Durschner, 1958; and Dennison, 1960.). The standard approach is to assume normal incidence, in which case the reflected amplitude at the interface between the ith and (i+1)th layer is given by

$$R_{pp} = \frac{V_{p_{i+1}}\rho_{i+1} - V_{p_i}\rho_i}{V_{p_{i+1}}\rho_{i+1} + V_{p_i}\rho_i} \tag{1}$$

where $V_p$ is the compressional-wave velocity and $\rho$ is the density. Using Equation 1 and sonic and density logs, a reflectivity series in time can be estimated and convolved with a wavelet to give the modelled seismic response of the Earth. Often only sonic logs are available, in which case the density structure is assumed to be either constant, or to be a function of the velocity structure. In the simplest scenario, it is assumed that the amplitude of the seismic wave is influenced only by transmission and reflection, and multiples are ignored. It should be noted that a number of authors have described techniques to model multiply reflected energy (Waters, 1978; and Vetter, 1981).

The approach that we have adopted to extend to the P-SV case is based on the simplifying assumption that the Earth is composed of layers of constant travel time (i.e. $\Delta Z/.\Delta V_p$ is equal to a constant) In this case the well log is integrated into discrete time intervals of two-way travel time, typically 1 or 2 ms. For each interval an average velocity and, if available, density is calculated. These values are then used to construct the reflectivity versus time series from equation 1 (Grant and West, 1965).

## P-SV synthetic seismograms

The focus of the present work is to develop an algorithm by which a zero-offset P-SV synthetic seismogram can be computed. At the present stage of the work several of the aforementioned assumptions have been invoked. Specifically, these are: amplitude is dependent only on reflection; variation of the density of the sedimentary section is insignificant when compared to the variation of the velocity, and multiples are not present.

Two fundamental modifications to the technique used for the computation of P-P synthetic seismograms are required: the sonic log must be integrated in terms of a zero-offset travel time for a downgoing compressional wave and an upgoing shear wave; and offset dependence must be introduced into the computation of the reflection coefficients. The two-fold problem can then be stated as: correctly position events in the reflectivity versus time series, where time represents the simulated zero-offset P-SV travel time; and numerically model the amplitudes of these events. Thus, this method requires shear-wave velocities, and it is desirable to have a full waveform log. If these data are not available, then the shear-wave velocity log must be obtained from the compressional-wave log and an estimate, preferably depth- or time-variant, of the $V_p/V_s$ ratio. There are two methods by which this function can be approximated. The first approach, initially suggested by McCormack et. al. (1984), is to construct a lithologic log, and to assign values from a priori knowledge. A second approach is to derive a $V_p/V_s$ function by correlating events between the P-P and P-SV stacked sections, and using the relationship:

$$V_p/V_s = (2\Delta t_{ps}/\Delta t_{pp}) - 1 \qquad (2)$$

where $\Delta t_{pp}$ and $\Delta t_{ps}$ are the zero-offset two-way P-P and P-SV travel times. This estimation procedure is possible only if correlations can be made between a number of reflectors on the P-P and P-SV data. An inherent weakness in this approach is that discontinuities, based on observed reflected events, exist in the estimated $V_p/V_s$ function. These discontinuities in turn give rise to events on the synthetic. Accordingly, the approach is circular at correlative events, but tenable in the intervals.

## Computational procedure

The fundamental step in constructing a conventional zero-offset synthetic is to transform the series of transit times and densities in depth, to a series of impedance contrasts in time. For the P-SV case, if it is assumed that density variations are negligible, the fundamental step is to transform the compressional- and shear-wave transit time versus depth series to compressional- and shear-wave velocity versus time series. From these series it is possible to estimate the reflectivity, which when convolved with a wavelet gives the modelled response of the Earth in the vicinity of the borehole. In order to convolve the reflectivity series with a wavelet, it is essential that the sampling (i.e. integration) interval be constant. That is, for N layers

$$R_{ps}(t_n) = R_{ps}(n\Delta t) \tag{3}$$

where Δt is the integration interval. This requires a model comprising a number of layers of constant zero-offset travel time thicknesses, each having constant shear- and compressional- wave velocities. This can be accomplished by repeatedly summing the transit times over successive samples until the total is equal to the integration interval. Then, for each layer, knowing the thickness and the transit times makes it is possible to compute the average compressional- and shear-wave velocity.

After defining the model, the amplitude of the reflections from each interface must be computed. Since at zero-offset there is no mode-conversion, reflections that occur as a result of non-vertical raypaths must be computed, and positioned in the reflectivity series at the corresponding zero-offset travel time. To do this, three processes must be decided upon: selecting appropriate offset; mapping offset to incident angle; and calculating the corresponding reflection. Munoz (1989) devised a scheme whereby the amplitudes are computed from an approximation for P-SV reflections, similar to the approximation for P-P reflections derived by Shuey (1985), and a constant incident angle at each interface. Our approach is more exact, and more computationally intensive. We incorporate all the offsets in the data acquisition, and determine incident angles for each by raytracing the model. Then, for each offset the exact reflected amplitudes are calculated from Zoeppritz's equations using the algorithm from Young and Braile (1976). Averaging these amplitudes at each interface gives a series of composite coefficients that is comparable to the reflectivity of the stacked section. The mapping, coefficient calculation, and simulated stacking are shown schematically in Figure 1.

## RESULTS

The full waveform sonic log from the study by Geis et. al. (1990), from the Rolling Hills region of south central Alberta was used for testing the algorithm. P-SV surface seismic has not been acquired in the area, however there is a three component VSP available. These data were acquired using a P-wave vibrator, with an 8-100 Hz sweep, offset 325 m from the well. The full waveform sonic log is from 2700' (823 m) to 6020' (1835 m).

The P-SV VSP has been mapped to two-way P-P time. In order to compare the synthetic with the VSP, it is necessary to generate a P-SV trace in terms of two-way P-P time. This requires modifying the synthetic algorithm so that integration is done in terms of P-P time. Thus a model consisting of layers of constant zero-offset two-way compressional wave travel time thicknesses must be constructed. In addition, because a single offset was used for acquiring the data, it is necessary only to compute reflection coefficients for a single offset; hence no averaging is necessary.

Shown in Figure 2 is the P-P and P-SV extracted traces, and the P-SV synthetic seismogram computed using a 10 ms Ricker wavelet. Shown in Figure 3 is the P-SV synthetic as a component of an L-plot. The polarity convention for these data is a trough for an increase in acoustic impedance, and a peak for a decrease. It can be seen that the

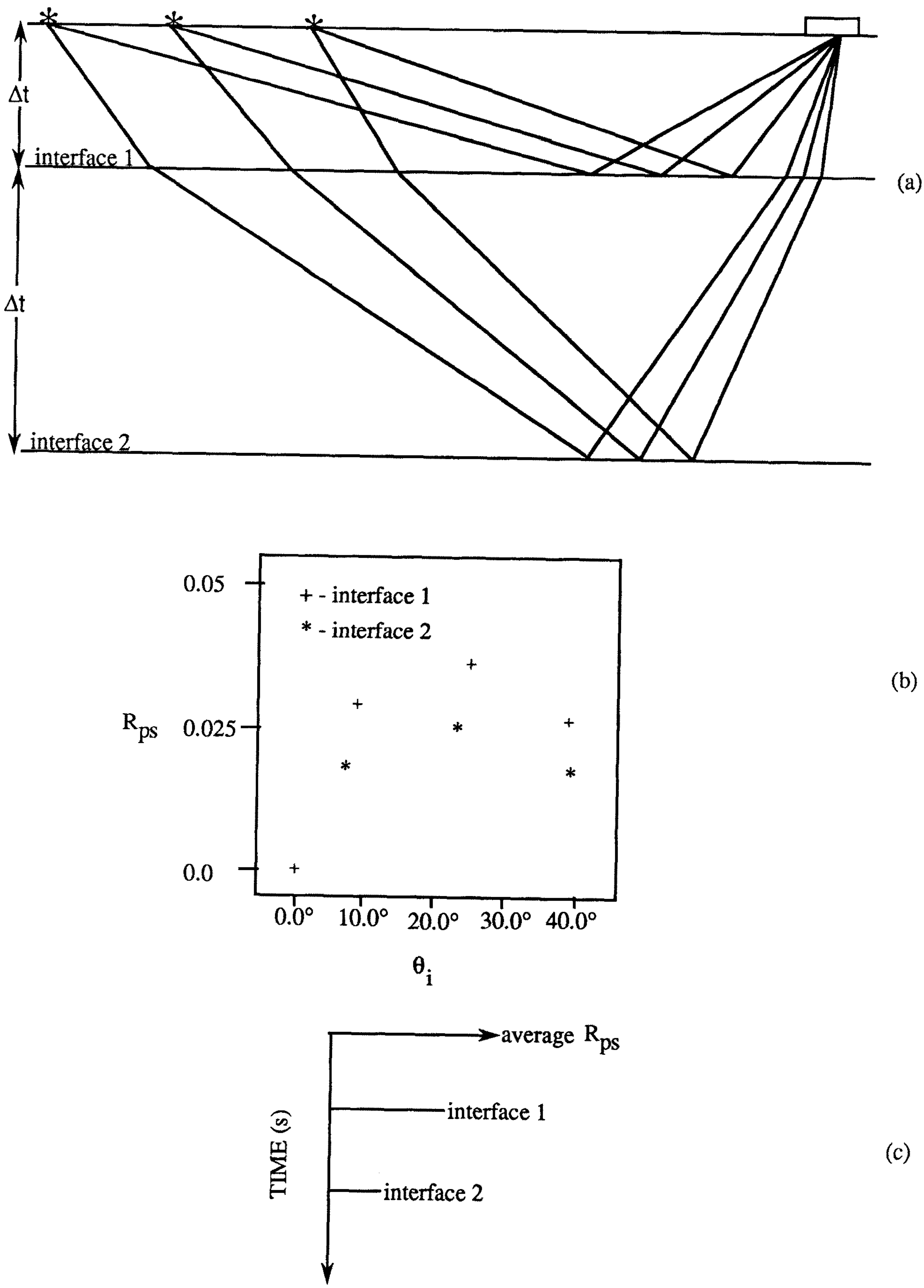


Figure 1. Schematic showing P-SV synthetic caculation for a two layer model. (a) The model of constant travel times is raytraced. (b) At each interface a reflection coefficient is calculated for each incident angle. (c) The average reflection coefficent is used to construct the reflectivity time series.

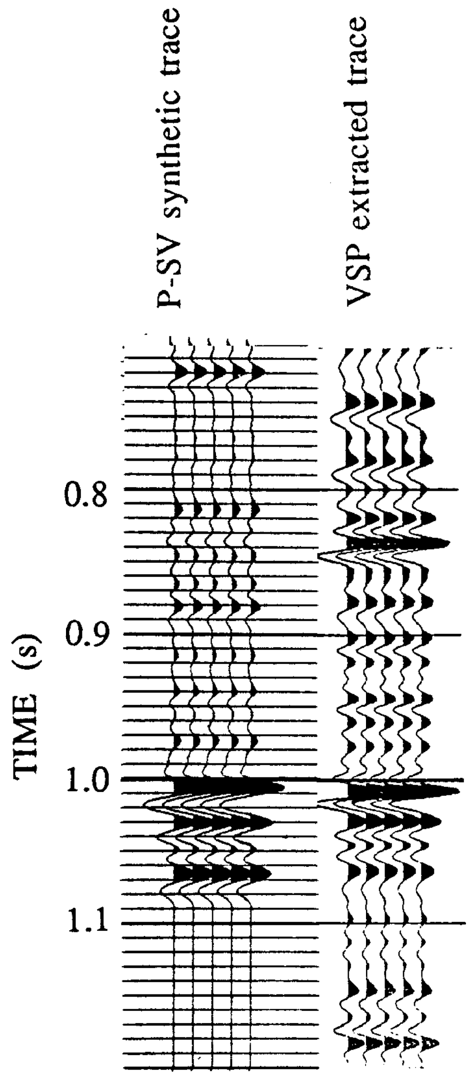


Figure 2. Zero-offset P-SV synthetic seismogram correlated with the P-SV extracted trace. The major horizons are successfully predicted with the synthetic trace.

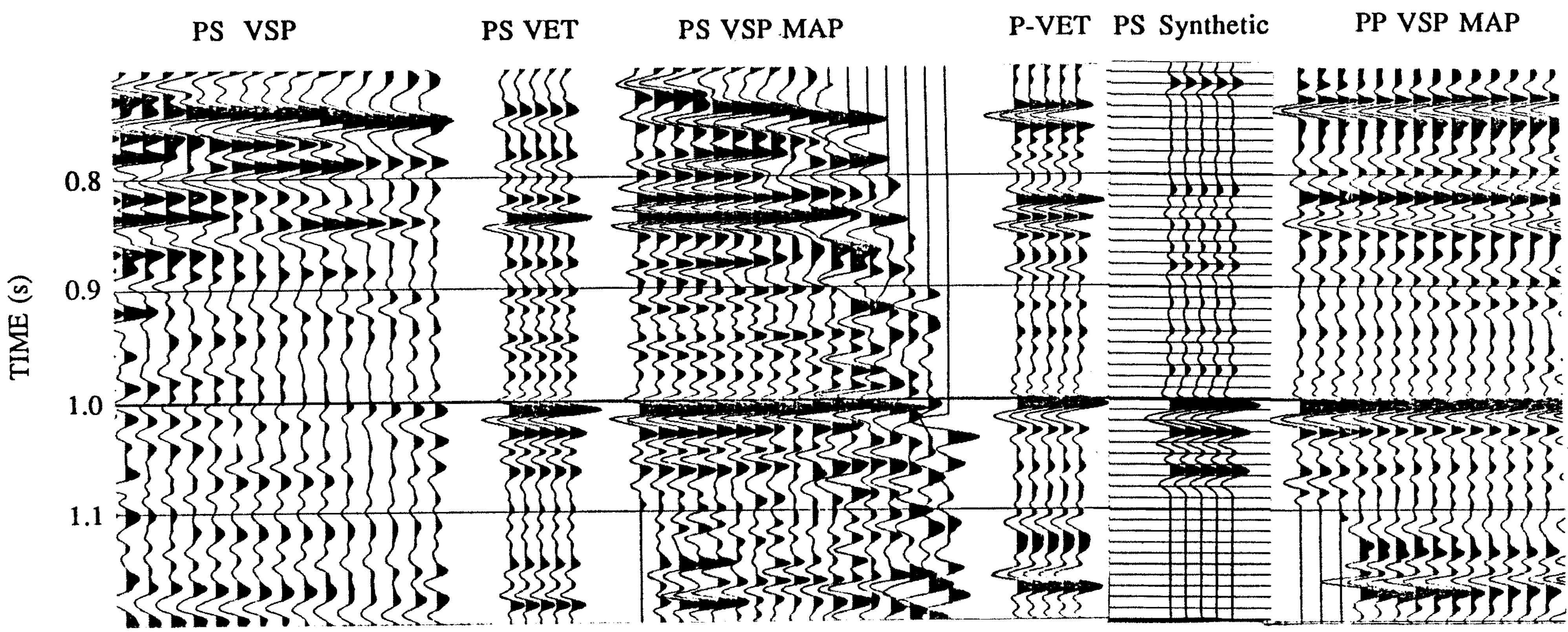


Figure 3. Portion of a composite L-plot, with the P-SV synthetic trace included (VET denotes VSP extracted trace). VSP data are after Geis et al. (1990).

major reflections on the VSP extracted trace are coincident with the major events on the synthetic trace, and in general the amplitude, and character of the synthetic reflections match favourably with the VSP data. It should also be noted that there are some less prominent events on the real data that do not have counterparts on the synthetic trace.

## CONCLUSION

A technique for computing P-SV zero-offset synthetic seismograms has been described and evaluated using a full waveform sonic log from southern Alberta. The technique requires mapping the offset dependent reflection coefficients to their zero-offset equivalent locations in the reflectivity versus time series. The algorithm can construct this series in terms of zero-offset P-SV time (appropriate for surface seismic data) or zero-offset P-P time (appropriate for VSP data). The good correlation between the synthetic data and the P-SV VSP extracted trace indicates that the proposed algorithm for computing mode-converted synthetics is valid.

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