Physical parameter estimation for sandstone reservoirs

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ABSTRACT

Through theoretical derivation and numerical modeling, this report shows that Biot-Gassman's theory can be used for predicting some physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus) as functions of porosity or water saturation. Also, it is shown that oil and gas have different Poisson's ratio curve as a function of water saturation. Several plots in this report suggest the importance of obtaining good initial velocity, porosity and dry rock Poisson's ratio estimates when modeling gas-water or oil-water saturated reservoirs.

INTRODUCTION

The Biot-Gassman theory has drawn many authors' attention (Geertsma and Smit, 1961: Domenico, 1974: Gregory, 1977: Hampson and Russell, 1990). Gregory (1977) suggests that the Biot-Gassman theory is a technique through which some physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus) as functions of porosity or water saturation can be obtained. Based on this theory, some modified equations were developed in order to obtain Poisson's ratio for AVO (amplitude versus offset) studies. Comparing the Biot-Gassman's equations (Gregory, 1977) with modified ones, one may feel that the modified equations are a more direct approach to get some parameters such as Poisson's ratio. Several numerical models show the application of this theory.

THEORY

The main issue of the modified Biot-Gassman theory is to obtain Poisson's ratio (σ) as a function of porosity (ϕ) or water saturation (S_w). Following are the list of symbols:

V - velocity of the rock;
V_p - P-wave velocity;
V_s - S-wave velocity;
V_{p0} - initial P-wave velocity (at ϕ_0 and S_{w0});
M - space modulus (or P-wave modulus);
M_d - dry rock space modulus;
M_0 - initial space modulus;
μ - rigidity modulus (or S-wave modulus);

μ_d - dry rock rigidity modulus;
σ - Poisson's ratio;
σ_d - dry rock Poisson's ratio;
ρ - bulk density;
ρ_0 - initial bulk density (at ϕ_0 and S_{w0});
ρ_s - bulk density of solid matter;
ρ_f - bulk density of fluid;
ρ_{f0} - initial bulk density of fluid (at ϕ_0 and S_{w0});
ρ_w - bulk density of water;
ρ_h - bulk density of hydrocarbon (oil or gas);
ϕ - porosity;
ϕ_0 - initial porosity;
S_w - water saturation;
S_{w0} - initial water saturation;
K - bulk modulus;
K_d - dry rock bulk modulus;
K_{d0} - initial dry rock bulk modulus;
K_s - solid matter bulk modulus;
K_p - pore bulk modulus;
K_f - fluid bulk modulus;
K_w - water bulk modulus;
K_h - hydrocarbon (oil and gas) bulk modulus;
C - compressibility (reciprocal of the bulk modulus);
C_d - dry rock compressibility;
C_s - solid matter compressibility;
C_f - fluid compressibility;
C_w - water compressibility;
C_h - hydrocarbon (oil or gas) compressibility;

It is well known that σ can be obtained by:

$$\sigma = \frac{(\frac{V_p}{V_s})^2 - 2}{2\left[(\frac{V_p}{V_s})^2 - 1\right]} \qquad \text{(e.g., Sheriff, 1991),} \qquad (1)$$

and V_p, V_s can be obtained:

$$V_p = \sqrt{\frac{M}{\rho}} \qquad \text{(e.g., Gregory, 1977),} \qquad (2)$$

$$V_s = \sqrt{\frac{\mu}{\rho}} \qquad \text{(e.g., Sheriff, 1991),} \qquad (3)$$

Therefore, σ can be obtained if M, μ and ρ are all known.

Calculation of ρ

It is known that:

$$\rho = (1-\phi)\rho_s + \phi\rho_f,$$

and

$$\rho_f = S_w\rho_w + (1-S_w)\rho_h \qquad \text{(e.g., Gregory, 1977),}$$

then we have:

$$\rho = (1-\phi)\rho_s + \phi S_w\rho_w + \phi(1-S_w)\rho_h, \tag{4}$$

ρ_s, ρ_w and ρ_h are known. Equation (4) tells us that bulk density ρ is the function of porosity ϕ and water saturation S_w.

Calculation of M

Geertsma and Smit (1961) gave the equation to obtain space modulus (M).

$$M = M_d + \frac{\left(1-\frac{K_d}{K_s}\right)^2}{\frac{\phi}{K_f} + \frac{1-\phi}{K_s} - \frac{K_d}{K_s}}. \tag{5}$$

Gregory (1977) gave the equation to get K_f:

$$C_f = S_wC_w + (1-S_w)C_h,$$

$$\frac{1}{K_f} = \frac{S_w}{K_w} + \frac{1-S_w}{K_h},$$

and

$$K_f = \frac{1}{\frac{S_w}{K_w} + \frac{1-S_w}{K_h}}. \tag{6}$$

In equation (5),

$$M_d = \frac{4}{3}\mu_d + K_d \qquad \text{(e.g., Sheriff, 1991),}$$

and

$$\mu_d = \frac{3(1-2\sigma_d)}{2(1+\sigma_d)}K_d \qquad \text{(e.g., Sheriff, 1991),}$$

then (5) becomes:

$$M = \frac{4}{3}\mu_d + K_d + \frac{\left(1-\frac{K_d}{K_s}\right)^2}{\frac{\phi}{K_f} + \frac{1-\phi}{K_s} - \frac{K_d}{K_s^2}},$$

$$M = \frac{4}{3}\times\frac{3}{2}\,\frac{1-2\sigma_d}{1+\sigma_d}K_d + K_d + \frac{(1-\frac{K_d}{K_s})^2}{\frac{\phi}{K_f}+\frac{1-\phi}{K_s}-\frac{K_d}{K_s^2}},$$

$$M = SK_d + \frac{(1-\frac{K_d}{K_s})^2}{\frac{\phi}{K_f}+\frac{1-\phi}{K_s}-\frac{K_d}{K_s^2}}, \tag{7}$$

here

$$S = \frac{3(1-\sigma_d)}{1+\sigma_d}.$$

In equation (6), K_w, K_h are known. For example, 2.38×10^{10}dynes/cm^2 and 0.0208 x 10^{10} dynes/cm^2 are suggested values for bulk moduli of water(K_w), and gas, respectively (Hilterman; Hampson and Russell, 1990). Hence, K_f is a function only of S_w. In equation (7), K_s, σ_d are known. Values of 40.0 x 10^{10} dynes/cm^2 and 0.12 are suggested for sandstone K_s and σ_d, respectively (Hilterman; Hampson and Russell, 1990). From equation (5) and (6), it is clear that space modulus M is a function of porosity ϕ , water saturation S_w and bulk modulus of dry rock K_d.

However, K_d can be obtained by knowing K_{d0}. From Appendix A, we can obtain:

$$C_d = \phi C_p + (1-\phi)C_s,$$

then

$$\frac{1}{K_d} = \frac{\phi}{K_p} + \frac{1-\phi}{K_s},$$

$$K_d = \frac{1}{\frac{\phi}{K_p}+\frac{1-\phi}{K_s}}, \tag{8}$$

similarly,

$$K_{d0} = \frac{1}{\frac{\phi_0}{K_p}+\frac{1-\phi_0}{K_s}}. \tag{9}$$

From equation (9), we can obtain K_p:

$$K_p = \frac{\phi_0}{\frac{1}{K_{d0}}-\frac{1-\phi_0}{K_s}}. \tag{10}$$

Substituting (10) into (8):

$$K_d = \frac{1}{\frac{\phi}{\phi_0}\left(\frac{1}{K_{d0}} - \frac{1-\phi_0}{K_s}\right) + \frac{1-\phi}{K_s}}. \tag{11}$$

Equation (7) can be used to solve for K_{d0}. For the initial porosity ϕ_0, initial fluid bulk modulus K_{f0}, initial bulk modulus of the dry rock K_{d0} and initial space modulus M_0, equation (7) becomes:

$$M_0 = SK_{d0} + \frac{\left(1 - \frac{K_{d0}}{K_s}\right)^2}{\frac{\phi_0}{K_{f0}} + \frac{1-\phi_0}{K_s} - \frac{K_{d0}}{K_s^2}}, \tag{12}$$

to separate the factor K_{d0} in equation (12):

$$M_0\left(\frac{\phi_0}{K_{f0}} + \frac{1-\phi_0}{K_s} - \frac{K_{d0}}{K_s^2}\right) = SK_{d0}\left(\frac{\phi_0}{K_{f0}} + \frac{1-\phi_0}{K_s} - \frac{K_{d0}}{K_s^2}\right) + \left(1 - \frac{K_{d0}}{K_s}\right)^2,$$

further written as:

$$\frac{S-1}{K_s^2}(K_{d0})^2 + \left[\frac{2}{K_s} - \frac{M_0}{K_s^2} - \frac{S\phi_0}{K_{f0}} - \frac{S(1-\phi_0)}{K_s}\right]K_{d0} + \left[\frac{M_0\phi_0}{K_{f0}} + \frac{M_0(1-\phi_0)}{K_s} - 1\right] = 0,$$

then we have:

$$A(K_{d0})^2 + BK_{d0} + C = 0, \tag{13}$$

where:

$$A = \frac{S-1}{K_s^2};$$

$$B = \frac{2}{K_s} - \frac{M_0}{K_s^2} - \frac{S\phi_0}{K_{f0}} - \frac{S(1-\phi_0)}{K_s};$$

$$C = \frac{M_0\phi_0}{K_{f0}} + \frac{M_0(1-\phi_0)}{K_s} - 1;$$

$$S = 3\frac{1-\sigma_d}{1+\sigma_d};$$

$$K_{f0} = \frac{1}{\frac{S_{w0}}{K_w} + \frac{1-S_{w0}}{K_h}};$$

$$M_0 = (V_{p0})^2\rho_0;$$

and

$$\rho_0 = (1-\phi_0)\rho_s + \phi_0 S_{w0}\rho_w + \phi_0(1-S_{w0})\rho_h.$$

By solving the equation (13), we can get K_{d0}:

$$K_{d0} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \tag{14}$$

Knowing K_{d0} from equation (14), and by substituting (11) into (7), we can see clearly that M is only the function of porosity ϕ and water saturation S_w.

Calculation of μ

Gregory (1977) gave the equation:

$$\mu = \mu_d = \frac{3(1-2\sigma_d)}{2(1+\sigma_d)}K_d. \tag{15}$$

σ_d is known, and from equation (11), K_d is the function of porosity ϕ. Hence, μ is the function of porosity ϕ only.

In equations (2) and (3), M, ρ and μ can be obtained by equations (7), (4)and (15), respectively. From the above, it is clear that P-wave velocity (V_p) and S-wave velocity (V_s) are the functions of porosity ϕ and water saturation S_w. By giving one value of ϕ or S_w, we can get the curves of V_p (V_s) versus S_w or ϕ. Then from equation (1), Poisson's ratio (σ) versus S_w or ϕ curves can be obtained.

DISCUSSION

Figure 1 and Figure 2 are two examples for the application of the Biot-Gassman theory for water-gas and water-oil saturated sandstone. Parameters (most values were suggested by Hampson and Russell, 1990) are listed in Table 1.

Table 1: Physical Parameters used for Sandstone

	K(x 10^{10} dynes/cm^2)			ρ(g/cm^3)			σ_d	V_{p0}	S_{w0}	ϕ_0
	K_s	K_w	K_h	ρ_s	ρ_w	ρ_h		(m/s)		
Water-gas	40.0	2.38	0.0208	2.65	1.089	0.103	0.12	2500	1.0	0.33
Water-oil	40.0	2.38	1.0	2.65	1.089	0.75	0.12	3000	1.0	0.33

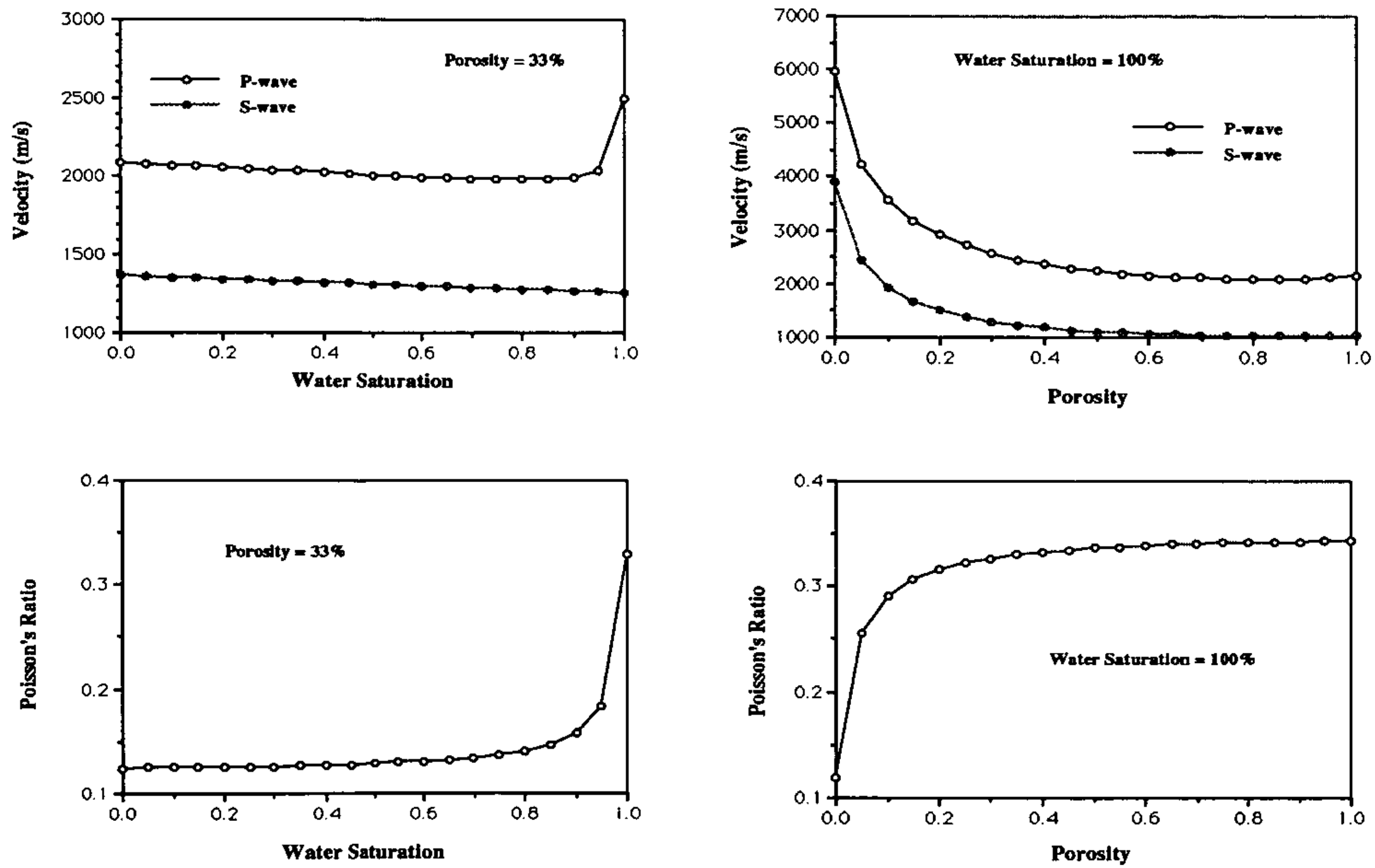

Figure 1: Water-gas Saturated Sandstone

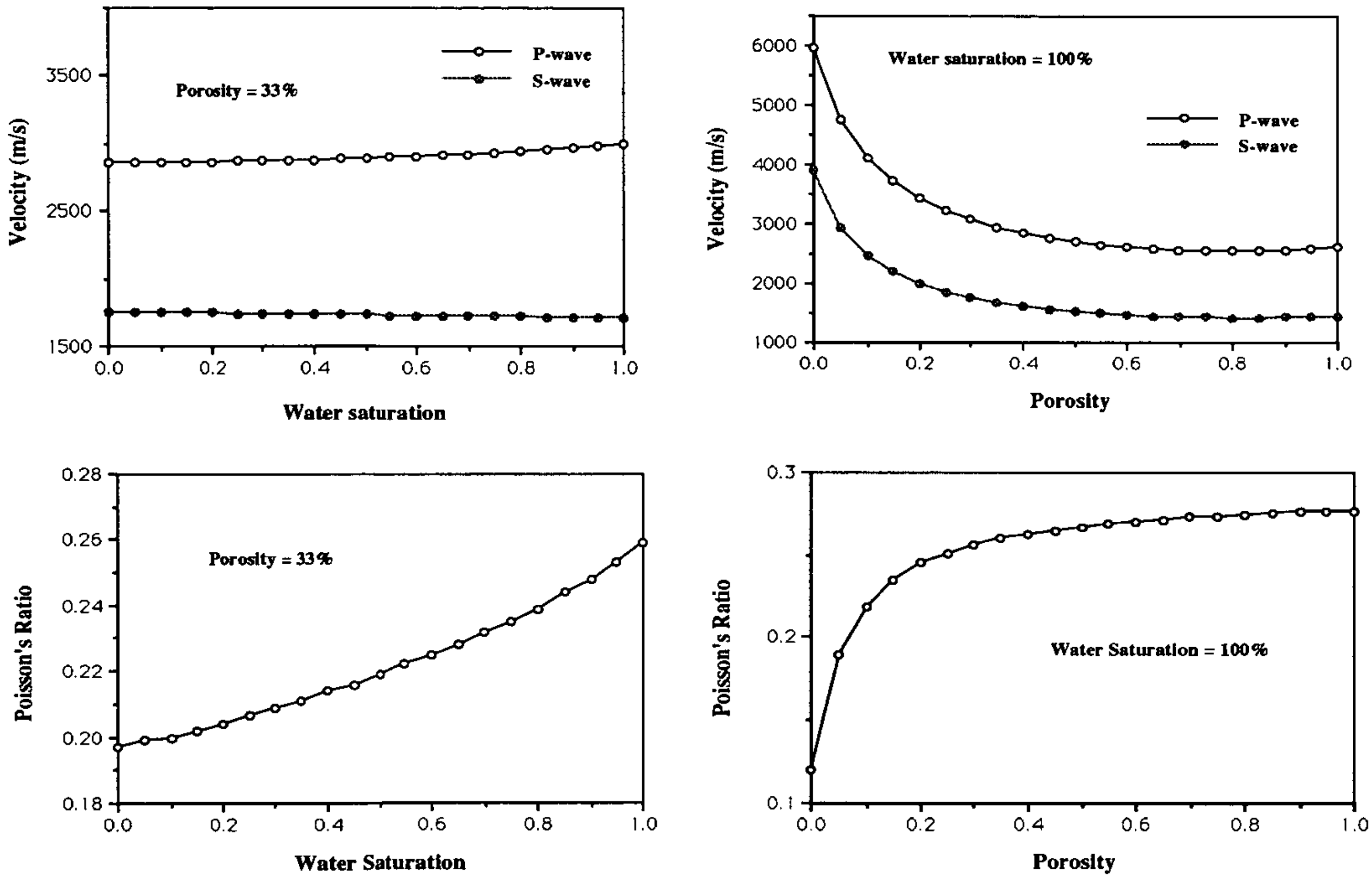

Figure 2: Water-oil Saturated Sandstone

Shown in the left side diagrams of Figure 1, it is clear that a small percentage of gas in a sand has a strong effect on the V_p and σ , which is consistent with Ostrander's (1984) and Hilterman's results. Oil, however, does not have the same effect. In the left side diagrams of Figure 2, V_p and σ change smoothly with the change of S_w; ρ increases with the increasing of S_w, thus causes the V_s to decrease with increasing S_w, which can be seen in the plots on the upper left hand side both in Figure 1 and Figure 2. Generally V_p and V_s decrease with the increasing ϕ, σ increases with the increasing of ϕ. These can be seen in the right-hand side diagrams of both Figure 1 and Figure 2. Also, it can be noticed that σ changes nonlinearly with the change of ϕ for small values of ϕ.

Effect of Initial Estimation of Velocity

From the theoretical derivation, it is clear that initial velocity V_{p0} is one of the important input parameters for the calculation of Poisson's ratio. Therefore, it is needed to know how would the initial estimation of velocity affect the final results. Figure 3 shows the plots of three models with V_{p0} changed (arbitrary values) and other parameters kept constant. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

ρ(g/cm^3):
$\rho_w = 1.089$, $\rho_h = 0.75$, $\rho_s = 2.65$;

K($\times 10^{10}$ dynes/cm^2):
$K_w = 2.38$, $K_h = 1.0$, $K_s = 40.0$;

$\sigma_d = 0.12$, $S_{w0} = 0.30$, $\phi_0 = 0.15$;

V_{p0} (m/s):
$V_{p01} = 4000$, $V_{p02} = 3600$, $V_{p03} = 3200$.

The plot on the upper left hand side of Figure 3 represents the V_p change with the change of S_w for different V_{p0}, and the plot on the upper right shows the V_s change. From these two, it is clear that for a given S_w, decreasing V_{p0} will decrease both V_p and V_s. σ increases with the decreasing V_{p0} for a given S_w, which can be seen on the lower left hand side plot. The lower right hand plot shows that with the decreasing of V_{p0}, K_d decreases. This means that the rock is more compressible, in other words, less compatible.

Effect of Initial Estimation of Porosity

Initial estimation of porosity (ϕ_0) is another important factor for the calculation. Figure 4 shows the plots of three models with ϕ_0 changed (arbitrary values) and other parameters

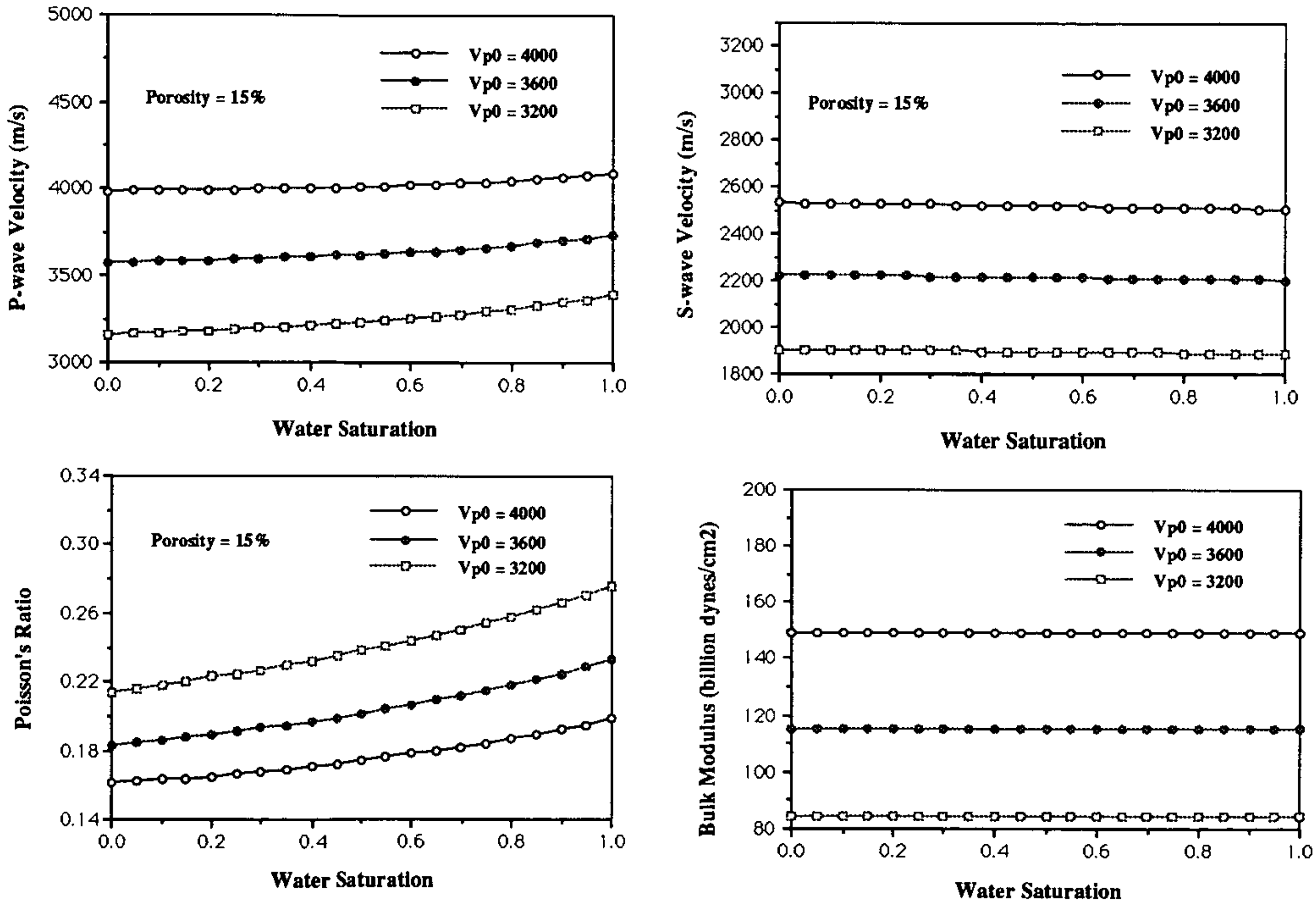

Figure 3: Effect of Initial Estimation of Velocity

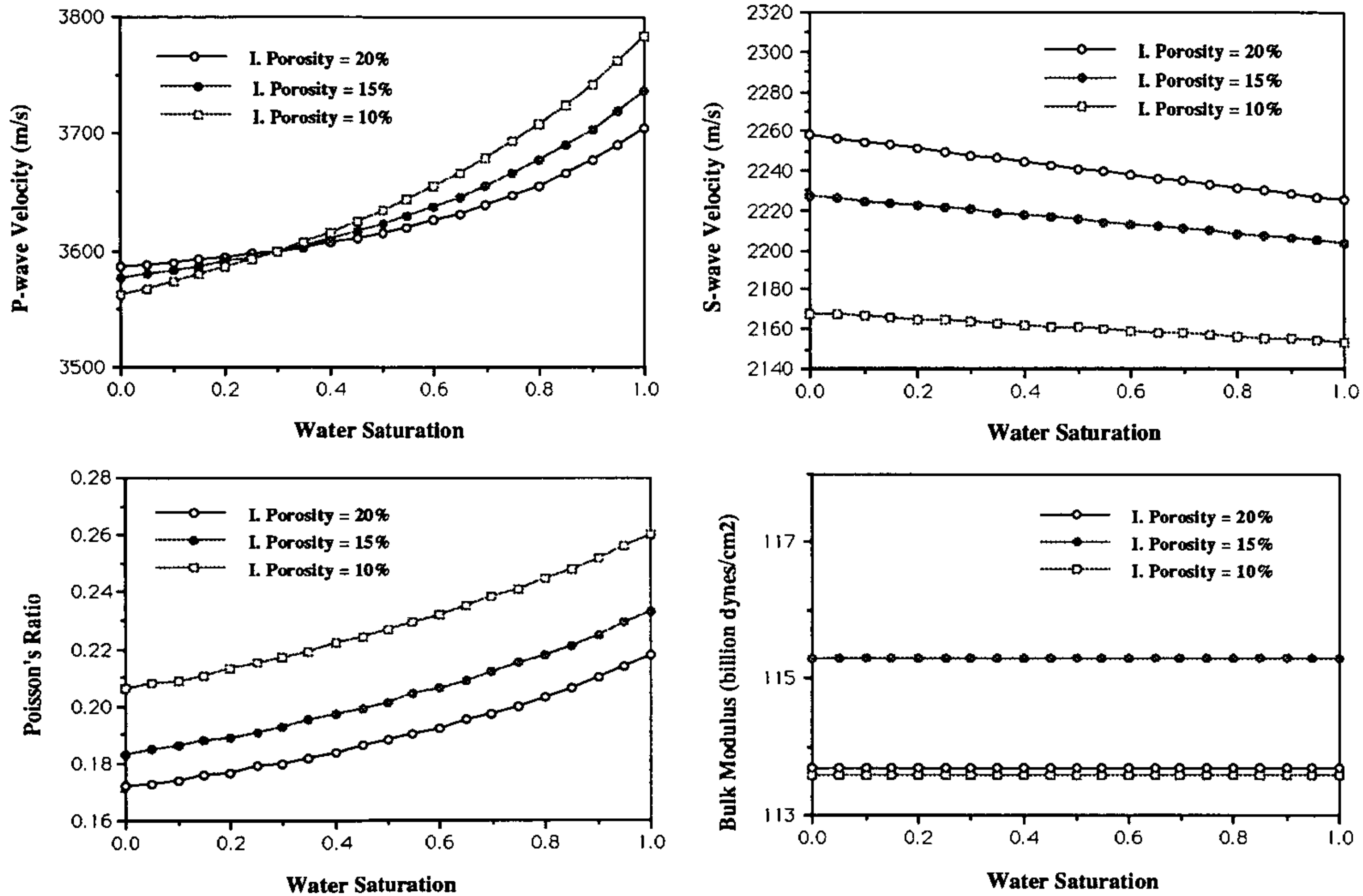

Figure 4: Effect of Initial Estimation of Porosity

unchanged. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

ρ(g/cm^3):
$\rho_w = 1.089$, $\rho_h = 0.75$, $\rho_s = 2.65$;

K($\times 10^{10}$ dynes/cm^2):
$K_w = 2.38$, $K_h = 1.0$, $K_s = 40.0$;

$\sigma_d = 0.12$, $S_{w0} = 0.30$, $V_{p0} = 3600$ m/s;

ϕ_0:
$\phi_{01} = 0.20$, $\phi_{02} = 0.15$, $\phi_{03} = 0.10$.

From the two upper plots, we can see that there is a crossing of the V_p curve with S_w, and for a given S_w, V_s decreases with the decreasing value of ϕ_0. The change in V_s is greater than that of V_p, therefore for a given S_w, σ increases with the decreasing value of ϕ_0.

Effect of initial estimation of dry rock Poisson's ratio

Figure 5 are plots of three models with σ_d varying (arbitrary values) while all other parameters remain constant. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

ρ(g/cm^3):
$\rho_w = 1.089$, $\rho_h = 0.75$, $\rho_s = 2.65$;

K($\times 10^{10}$ dynes/cm^2):
$K_w = 2.38$, $K_h = 1.0$, $K_s = 40.0$;

$S_{w0} = 0.30$, $V_{p0} = 3600$ m/s, $\phi_0 = 0.15$;

σ_d:
$\sigma_{d1} = 0.12$, $\sigma_{d2} = 0.14$, $\sigma_{d3} = 0.16$.

From the two upper plots in Figure 5, it is obvious that V_p remains almost constant with the change of σ_d, while V_s decreases with the increasing σ_d. Hence, for a given S_w, σ increases with the increasing value of σ_d. The lower right hand side plot shows the bulk modulus increasing with the increasing σ_d. This infers that the rock is more compacted (or less compressible).

Assumptions for Biot-Gassman Theory

The Biot-Gassman theory is only used for water-gas and water-oil saturated reservoirs, but it is not valid for water-oil-gas saturated reservoirs. Also, as Hilterman cited in his notes, the Biot-

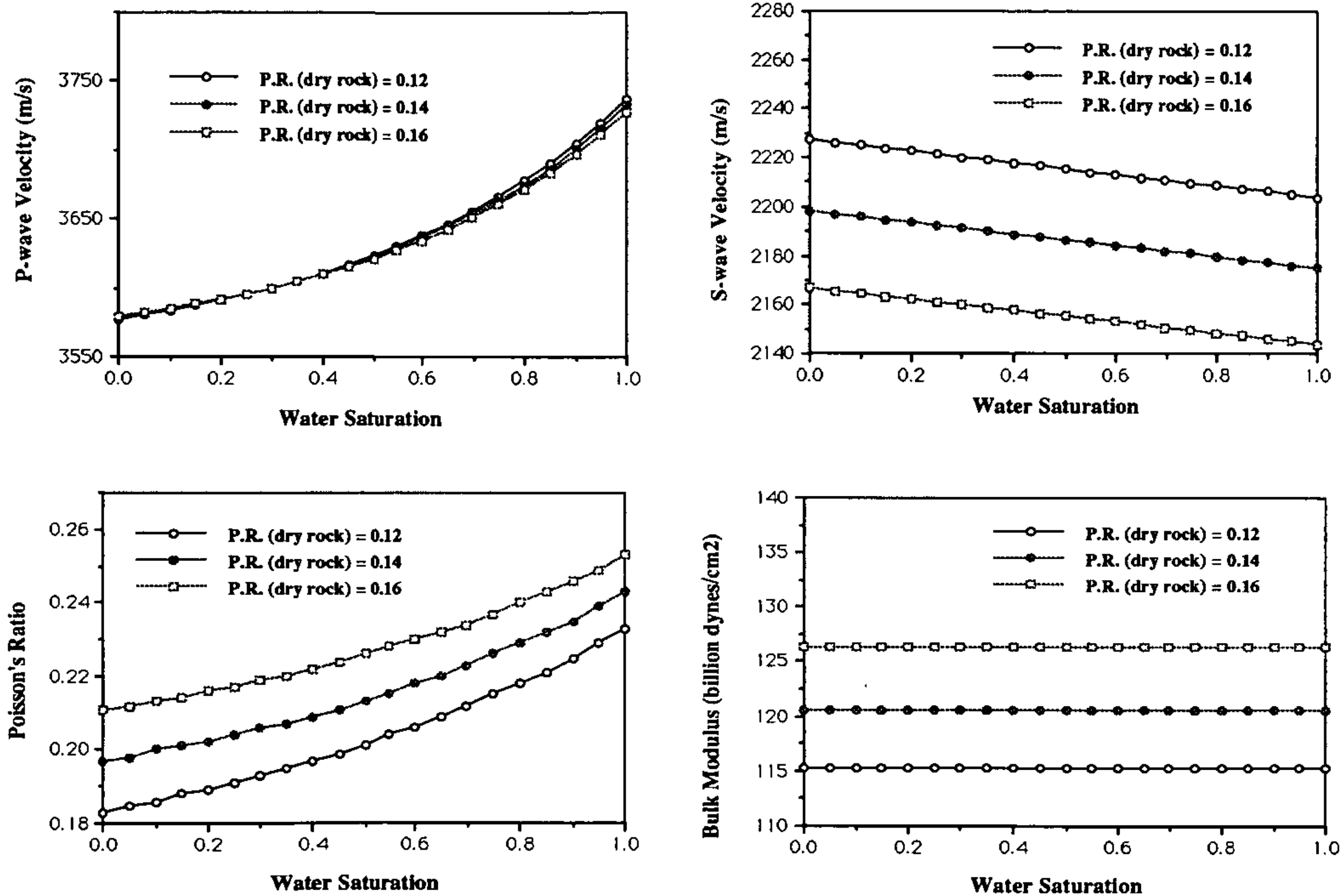

Figure 5: Effect of Initial Estimation of Dry Rock Poisson's Ratio

Gassman's theory assumes a homogeneous rock in which the pore fluid is uniformly distributed in the pores, the shear modulus is not affected by the pore fluid, and the pore shapes are spheroidal.

CONCLUSION

Four conclusions can be obtained from this report:

(1): The Biot-Gassman theory can be used for predicting physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus, etc.) as a function of porosity or water saturation.

(2): Oil and gas have different Poisson's ratio change curve as a function of water saturation for a given porosity.

(3): When modeling gas-water and oil-water saturated reservoirs, it is important to obtain good estimation of initial velocity, initial porosity and dry rock Poisson's ratio.

(4): Generally, decreasing the initial P-wave velocity or initial porosity, or increasing dry rock Poisson's ratio, and keeping the rest of the parameters constant, will cause Poisson's ratio to increase for a given water saturation.

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APPENDIX A

According to the concept of the compressibility:

$$C = \frac{\Delta Z/Z}{\Delta P} = \frac{\Delta Z}{Z\Delta P}, \qquad \text{(A-1)}$$

i.e., the compressibility (C) is the relative change in volume ($\Delta Z/Z$) with pressure (ΔP). ΔZ is the absolute change in volume, and Z is the total volume of the rock. The total change in volume ΔZ is equal to the change in volume of the pore space (ΔZ_p) plus the change in volume of the solid matter (ΔZ_s), i.e.:

$$\Delta Z = \Delta Z_p + \Delta Z_s,$$

therefore:

$$\frac{\Delta Z}{Z} = \frac{\Delta Z_p}{Z} + \frac{\Delta Z_s}{Z}. \qquad \text{(A-2)}$$

According to the concept of the porosity:

$$Z_p = \phi Z, \; Z_s = (1-\phi)Z,$$

$$Z = \frac{Z_p}{\phi}, \; Z = \frac{Z_s}{1-\phi},$$

where Z_p is the total volume of pore space and Z_s is the total volume for solid matter. Then equation (A-2) becomes:

$$\frac{\Delta Z}{Z} = \frac{\phi \Delta Z_p}{Z_p} + \frac{(1-\phi)\Delta Z_s}{Z_s},$$

divided by ΔP:

$$\frac{\Delta Z}{Z\Delta P} = \frac{\phi \Delta Z_p}{Z_p \Delta P} + \frac{(1-\phi)\Delta Z_s}{Z_s \Delta P},$$

comparing the above equation with equation (A-1), we can get:

$$C_d = \phi C_p + (1-\phi)C_s). \qquad \text{(A-3)}$$

Equation (A-3) was used to obtain K_d in equation (8).

Equation (A-3) can also be written as:

$$C_d = \phi C_p + C_s - \phi C_s.$$

C_s is quite small compared to C_p, and ϕC_s is very small compared with both ϕC_p and C_s. Therefore the above equation can be simplified as:

$$C_d \approx \phi C_p + C_s\,. \qquad \text{(A-4)}$$

Equation (A-4) is the same equation given by Domenico (1974).