Physical parameter estimation for sandstone reservoirs

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ABSTRACT

Through theoretical derivation and numerical modeling, this report shows that Biot-Gassman's theory can be used for predicting some physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus) as functions of porosity or water saturation. Also, it is shown that oil and gas have different Poisson's ratio curve as a function of water saturation. Several plots in this report suggest the importance of obtaining good initial velocity, porosity and dry rock Poisson's ratio estimates when modeling gas-water or oil-water saturated reservoirs.

INTRODUCTION

The Biot-Gassman theory has drawn many authors' attention (Geertsma and Smit, 1961; Domenico, 1974; Gregory, 1977; Hampson and Russell, 1990). Gregory (1977) suggests that the Biot-Gassman theory is a technique through which some physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus) as functions of porosity or water saturation can be obtained. Based on this theory, some modified equations were developed in order to obtain Poisson's ratio for AVO (amplitude versus offset) studies. Comparing the Biot-Gassman's equations (Gregory, 1977) with modified ones, one may feel that the modified equations are a more direct approach to get some parameters such as Poisson's ratio. Several numerical models show the application of this theory.

THEORY

The main issue of the modified Biot-Gassman theory is to obtain Poisson's ratio ($\sigma$) as a function of porosity ($\phi$) or water saturation ($S_w$). Following are the list of symbols:

- $V$ - velocity of the rock;
- $V_p$ - P-wave velocity;
- $V_s$ - S-wave velocity;
- $V_{p0}$ - initial P-wave velocity (at $\phi_0$ and $S_{w0}$);
- $M$ - space modulus (or P-wave modulus);
- $M_d$ - dry rock space modulus;
- $M_0$ - initial space modulus;
- $\mu$ - rigidity modulus (or S-wave modulus);
It is well known that \( \sigma \) can be obtained by:

\[
\sigma = \frac{\left( \frac{V_P}{V_s} \right)^2 - 2}{2 \left( \frac{V_P}{V_s} \right)^2 - 1}
\]

(e.g., Sheriff, 1991), (1)

and \( V_P, V_s \) can be obtained:

\[
V_P = \sqrt{\frac{M}{\rho}} \quad \text{(e.g., Gregory, 1977),} \quad (2)
\]

\[
V_s = \sqrt{\frac{\mu}{\rho}} \quad \text{(e.g., Sheriff, 1991),} \quad (3)
\]

Therefore, \( \sigma \) can be obtained if \( M, \mu \) and \( \rho \) are all known.
Calculation of $\rho$

It is known that:
$$\rho = (1-\phi)p_s + \phi p_f,$$
and
$$p_f = S_w p_w + (1-S_w)p_h \quad \text{(e.g., Gregory, 1977),}$$
then we have:
$$\rho = (1-\phi)p_s + \phi S_w p_w + \phi (1-S_w)p_h, \quad (4)$$
$p_s$, $p_w$ and $p_h$ are known. Equation (4) tells us that bulk density $\rho$ is the function of porosity $\phi$ and water saturation $S_w$.

Calculation of $M$

Geertsma and Smit (1961) gave the equation to obtain space modulus ($M$).

$$M = M_d + \frac{(1 - K_d)^2}{K_s} \cdot \frac{\phi}{K_f} \cdot \frac{1 - \phi}{K_s} \cdot \frac{K_d}{K_s} \quad (5)$$

Gregory (1977) gave the equation to get $K_f$:
$$C_f = S_w C_w + (1-S_w)C_h,$$
and
$$K_f = \frac{1}{S_w + 1-S_w} \cdot \frac{K_w}{K_h} \quad \text{(6)}$$

In equation (5),
$$M_d = \frac{4}{3} \mu_d + K_d \quad \text{(e.g., Sheriff, 1991),}$$
and
$$\mu_d = \frac{3(1-2\sigma_d)}{2(1+\sigma_d)}K_d \quad \text{(e.g., Sheriff, 1991),}$$
then (5) becomes:
$$M = \frac{4}{3} \mu_d + K_d + \frac{(1 - K_d)^2}{K_s} \cdot \frac{\phi}{K_f} \cdot \frac{1 - \phi}{K_s} \cdot \frac{K_d}{K_s} \cdot \frac{K_d}{K_s^2}.$$
\[ M = \frac{4}{3} \times 2 \frac{1-2\sigma_d}{1+\sigma_d} K_d + K_d + \frac{(1 - K_d)^2}{K_s}, \]

\[ \frac{\phi}{K_f} + \frac{1-\phi}{K_s} \frac{K_d}{K_s^2} \]

Here

\[ S = \frac{3(1-\sigma_d)}{1+\sigma_d}. \]

In equation (6), \( K_w, K_h \) are known. For example, \( 2.38 \times 10^{10} \text{dynes/cm}^2 \) and \( 0.0208 \times 10^{10} \text{dynes/cm}^2 \) are suggested values for bulk moduli of water \( K_w \), and gas, respectively (Hilterman; Hampson and Russell, 1990). Hence, \( K_f \) is a function only of \( S_w \). In equation (7), \( K_s, \sigma_d \) are known. Values of \( 40.0 \times 10^{10} \text{dynes/cm}^2 \) and \( 0.12 \) are suggested for sandstone \( K_s \) and \( \sigma_d \), respectively (Hilterman; Hampson and Russell, 1990). From equation (5) and (6), it is clear that space modulus \( M \) is a function of porosity \( \phi \), water saturation \( S_w \) and bulk modulus of dry rock \( K_d \).

However, \( K_d \) can be obtained by knowing \( K_{d0} \). From Appendix A, we can obtain:

\[ C_d = \phi C_p + (1-\phi)C_s, \]

then

\[ \frac{1}{K_d} = \frac{\phi}{K_p} + \frac{1-\phi}{K_s}, \]

\[ K_d = \frac{1}{\phi \frac{1}{K_p} + 1-\phi \frac{1}{K_s}} \]

Similarly,

\[ K_{d0} = \frac{1}{\phi_0 \frac{1}{K_p} + 1-\phi_0 \frac{1}{K_s}} \]

From equation (9), we can obtain \( K_p \):

\[ K_p = \frac{\phi_0}{\frac{1}{K_{d0}} + \frac{1-\phi_0}{K_s}} \]
Substituting (10) into (8):

\[
K_d = \frac{1}{\frac{\phi}{\phi_0} \left( \frac{1}{K_{d0}} - \frac{1-\phi_0}{K_s} \right) + \frac{1-\phi}{K_s}}.
\]  

(11)

Equation (7) can be used to solve for \(K_{d0}\). For the initial porosity \(\phi_0\), initial fluid bulk modulus \(K_f0\), initial bulk modulus of the dry rock \(K_{d0}\) and initial space modulus \(M_0\), equation (7) becomes:

\[
M_0 = S K_{d0} + \frac{(1 - K_{d0})^2}{K_s},
\]

(12)

to separate the factor \(K_{d0}\) in equation (12):

\[
M_0 \frac{\phi_0}{K_f0} + \frac{1-\phi_0}{K_s} \frac{K_{d0}}{K_s^2} = SK_{d0} \left( \frac{\phi_0}{K_{f0}} + \frac{1-\phi_0}{K_s} \frac{K_{d0}}{K_s^2} \right) + (1 - K_{d0})^2,
\]

further written as:

\[
\frac{S-1}{K_s} \left( K_{d0} \right)^2 + 2 \left[ \frac{M_0}{K_s} - \frac{S \phi_0}{K_{f0}} \frac{S (1-\phi_0)}{K_s} \right] K_{d0} + \left[ \frac{M_0 \phi_0}{K_f0} + \frac{M_0 (1-\phi_0)}{K_s} \right] - 1 = 0,
\]

then we have:

\[
A (K_{d0})^2 + BK_{d0} + C = 0,
\]

(13)

where:

\[
A = \frac{S-1}{K_s^2};
\]

\[
B = \frac{2}{K_s} \left( \frac{M_0}{K_s} - \frac{S \phi_0}{K_{f0}} \frac{S (1-\phi_0)}{K_s} \right);
\]

\[
C = \frac{M_0 \phi_0}{K_{f0}} + \frac{M_0 (1-\phi_0)}{K_s};
\]

\[
S = \frac{3 \cdot 1}{1 + \sigma_d};
\]

\[
K_{f0} = \frac{1}{S_w 0 + \frac{1-S_w 0}{K_h}};
\]
\[ M_0 = (V_{p0})^2 \rho_0; \]

and

\[ \rho_0 = (1-\phi_0)\rho_s + \phi_0 S_{w0}\rho_w + \phi_0 (1-S_{w0})\rho_h. \]

By solving the equation (13), we can get \( K_{d0} \):

\[ K_{d0} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \] (14)

Knowing \( K_{d0} \) from equation (14), and by substituting (11) into (7), we can see clearly that \( M \) is only the function of porosity \( \phi \) and water saturation \( S_w \).

**Calculation of \( \mu \)**

Gregory (1977) gave the equation:

\[ \mu = \mu_d = \frac{3(1-2\sigma_d)K_d}{2(1+\sigma_d)}. \] (15)

\( \sigma_d \) is known, and from equation (11), \( K_d \) is the function of porosity \( \phi \). Hence, \( \mu \) is the function of porosity \( \phi \) only.

In equations (2) and (3), \( M, \rho \) and \( \mu \) can be obtained by equations (7), (4) and (15), respectively. From the above, it is clear that P-wave velocity \( (V_p) \) and S-wave velocity \( (V_s) \) are the functions of porosity \( \phi \) and water saturation \( S_w \). By giving one value of \( \phi \) or \( S_w \), we can get the curves of \( V_p \) (\( V_s \)) versus \( S_w \) or \( \phi \). Then from equation (1), Poisson's ratio (\( \sigma \)) versus \( S_w \) or \( \phi \) curves can be obtained.

**DISCUSSION**

Figure 1 and Figure 2 are two examples for the application of the Biot-Gassman theory for water-gas and water-oil saturated sandstone. Parameters (most values were suggested by Hampson and Russell, 1990) are listed in Table 1.

<table>
<thead>
<tr>
<th>( K \times 10^{10} ) dynes/cm²</th>
<th>( \rho ) (g/cm³)</th>
<th>( \sigma_d )</th>
<th>( V_{p0} )</th>
<th>( S_{w0} )</th>
<th>( \phi_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_s )</td>
<td>( K_w )</td>
<td>( K_h )</td>
<td>( \rho_s )</td>
<td>( \rho_w )</td>
<td>( \rho_h )</td>
</tr>
<tr>
<td>Water-gas</td>
<td>40.0</td>
<td>2.38</td>
<td>0.0208</td>
<td>2.65</td>
<td>1.089</td>
</tr>
<tr>
<td>Water-oil</td>
<td>40.0</td>
<td>2.38</td>
<td>1.0</td>
<td>2.65</td>
<td>1.089</td>
</tr>
</tbody>
</table>
Figure 1: Water-gas Saturated Sandstone
Figure 2: Water-oil Saturated Sandstone
Shown in the left side diagrams of Figure 1, it is clear that a small percentage of gas in a sand has a strong effect on the $V_p$ and $\sigma$, which is consistent with Ostrander's (1984) and Hilterman's results. Oil, however, does not have the same effect. In the left side diagrams of Figure 2, $V_p$ and $\sigma$ change smoothly with the change of $S_w$; $\rho$ increases with the increasing of $S_w$, thus causes the $V_s$ to decrease with increasing $S_w$, which can be seen in the plots on the upper left hand side both in Figure 1 and Figure 2. Generally $V_p$ and $V_s$ decrease with the increasing $\phi$, $\sigma$ increases with the increasing of $\phi$. These can be seen in the right-hand side diagrams of both Figure 1 and Figure 2. Also, it can be noticed that $\sigma$ changes nonlinearly with the change of $\phi$ for small values of $\phi$.

**Effect of Initial Estimation of Velocity**

From the theoretical derivation, it is clear that initial velocity $V_{p0}$ is one of the important input parameters for the calculation of Poisson's ratio. Therefore, it is needed to know how would the initial estimation of velocity affect the final results. Figure 3 shows the plots of three models with $V_{p0}$ changed (arbitrary values) and other parameters kept constant. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

- $\rho (g/cm^3)$:
  - $\rho_w = 1.089$, $\rho_h = 0.75$, $\rho_s = 2.65$;

- $K (\times 10^{10} \text{ dynes/cm}^2)$:
  - $K_w = 2.38$, $K_h = 1.0$, $K_s = 40.0$;

- $\sigma_d = 0.12$, $S_{w0} = 0.30$, $\phi_0 = 0.15$;

- $V_{p0} (m/s)$:
  - $V_{p01} = 4000$, $V_{p02} = 3600$, $V_{p03} = 3200$.

The plot on the upper left hand side of Figure 3 represents the $V_p$ change with the change of $S_w$ for different $V_{p0}$, and the plot on the upper right shows the $V_s$ change. From these two, it is clear that for a given $S_w$, decreasing $V_{p0}$ will decrease both $V_p$ and $V_s$. $\sigma$ increases with the decreasing $V_{p0}$ for a given $S_w$, which can be seen on the lower left hand side plot. The lower right hand plot shows that with the decreasing of $V_{p0}$, $K_d$ decreases. This means that the rock is more compressible, in other words, less compatible.

**Effect of Initial Estimation of Porosity**

Initial estimation of porosity ($\phi_0$) is another important factor for the calculation. Figure 4 shows the plots of three models with $\phi_0$ changed (arbitrary values) and other parameters...
Figure 3: Effect of Initial Estimation of Velocity
Figure 4: Effect of Initial Estimation of Porosity
unchanged. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

\[
\rho(\text{g/cm}^3):
\begin{align*}
\rho_w &= 1.089, 
\rho_h &= 0.75, 
\rho_s &= 2.65;
\end{align*}
\]

\[
K(\times 10^{10} \text{ dynes/cm}^2):
\begin{align*}
K_w &= 2.38, 
K_h &= 1.0, 
K_s &= 40.0;
\end{align*}
\]

\[
\sigma_d = 0.12, 
S_{w0} = 0.30, 
V_{p0} = 3600 \text{ m/s};
\]

\[
\phi_0:
\begin{align*}
\phi_{01} &= 0.20, 
\phi_{02} &= 0.15, 
\phi_{03} &= 0.10.
\end{align*}
\]

From the two upper plots, we can see that there is a crossing of the \( V_p \) curve with \( S_w \), and for a given \( S_w \), \( V_s \) decreases with the decreasing value of \( \phi_0 \). The change in \( V_s \) is greater than that of \( V_p \), therefore for a given \( S_w \), \( \sigma \) increases with the decreasing value of \( \phi_0 \).

**Effect of initial estimation of dry rock Poisson's ratio**

Figure 5 are plots of three models with \( \sigma_d \) varying (arbitrary values) while all other parameters remain constant. Parameters (most values were suggested by Hampson and Russell, 1990) are listed below:

\[
\rho(\text{g/cm}^3):
\begin{align*}
\rho_w &= 1.089, 
\rho_h &= 0.75, 
\rho_s &= 2.65;
\end{align*}
\]

\[
K(\times 10^{10} \text{ dynes/cm}^2):
\begin{align*}
K_w &= 2.38, 
K_h &= 1.0, 
K_s &= 40.0;
\end{align*}
\]

\[
S_{w0} = 0.30, 
V_{p0} = 3600 \text{ m/s}, 
\phi_0 = 0.15;
\]

\[
\sigma_d:
\begin{align*}
\sigma_{d1} &= 0.12, 
\sigma_{d2} &= 0.14, 
\sigma_{d3} &= 0.16.
\end{align*}
\]

From the two upper plots in Figure 5, it is obvious that \( V_p \) remains almost constant with the change of \( \sigma_d \), while \( V_s \) decreases with the increasing \( \sigma_d \). Hence, for a given \( S_w \), \( \sigma \) increases with the increasing value of \( \sigma_d \). The lower right hand side plot shows the bulk modulus increasing with the increasing \( \sigma_d \). This infers that the rock is more compacted (or less compressible).

**Assumptions for Biot-Gassman Theory**

The Biot-Gassman theory is only used for water-gas and water-oil saturated reservoirs, but it is not valid for water-oil-gas saturated reservoirs. Also, as Hilterman cited in his notes, the Biot-
Figure 5: Effect of Initial Estimation of Dry Rock Poisson's Ratio
Gassman's theory assumes a homogeneous rock in which the pore fluid is uniformly distributed in the pores, the shear modulus is not affected by the pore fluid, and the pore shapes are spheroidal.

CONCLUSION

Four conclusions can be obtained from this report:
(1): The Biot-Gassman theory can be used for predicting physical parameters (e.g., velocity, density, Poisson's ratio, bulk modulus, etc.) as a function of porosity or water saturation.
(2): Oil and gas have different Poisson's ratio change curve as a function of water saturation for a given porosity.
(3): When modeling gas-water and oil-water saturated reservoirs, it is important to obtain good estimation of initial velocity, initial porosity and dry rock Poisson's ratio.
(4): Generally, decreasing the initial P-wave velocity or initial porosity, or increasing dry rock Poisson's ratio, and keeping the rest of the parameters constant, will cause Poisson's ratio to increase for a given water saturation.

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Gregory, A.R., 1977, Aspects of rock physics from laboratory and log data that are important to seismic interpretations: AAPG Memoir, 26, 15-46.
Hilterman, F., Gassman-Biot-Geerstma Equation: unpublished course notes.
APPENDIX A

According to the concept of the compressibility:

\[ C = \frac{\Delta Z/Z}{\Delta P} = \frac{\Delta Z}{Z \Delta P}, \]  
(A-1)

i.e., the compressibility (C) is the relative change in volume (\( \Delta Z/Z \)) with pressure (\( \Delta P \)). \( \Delta Z \) is the absolute change in volume, and \( Z \) is the total volume of the rock. The total change in volume \( \Delta Z \) is equal to the change in volume of the pore space (\( \Delta Z_p \)) plus the change in volume of the solid matter (\( \Delta Z_s \)), i.e.:

\[ \Delta Z = \Delta Z_p + \Delta Z_s, \]

therefore:

\[ \frac{\Delta Z}{Z} = \frac{\Delta Z_p}{Z} + \frac{\Delta Z_s}{Z}. \]  
(A-2)

According to the concept of the porosity:

\[ Z_p = \phi Z, \quad Z_s = (1-\phi)Z, \]

\[ Z = Z_p, \quad Z = Z_s, \]

\[ \phi \quad 1-\phi \]

where \( Z_p \) is the total volume of pore space and \( Z_s \) is the total volume for solid matter. Then equation (A-2) becomes:

\[ \frac{\Delta Z}{Z} = \frac{\phi \Delta Z_p}{Z_p} + \frac{(1-\phi)\Delta Z_s}{Z_s}, \]

divided by \( \Delta P \):

\[ \frac{\Delta Z}{Z \Delta P} = \frac{\phi \Delta Z_p}{Z_p \Delta P} + \frac{(1-\phi)\Delta Z_s}{Z_s \Delta P}, \]

comparing the above equation with equation (A-1), we can get:

\[ C_d = \phi C_p + (1-\phi)C_s. \]  
(A-3)

Equation (A-3) was used to obtain \( K_d \) in equation (8).

Equation (A-3) can also be written as:

\[ C_d = \phi C_p + C_s - \phi C_s. \]

\( C_s \) is quite small compared to \( C_p \), and \( \phi C_s \) is very small compared with both \( \phi C_p \) and \( C_s \). Therefore the above equation can be simplified as:

\[ C_d = \phi C_p + C_s. \]  
(A-4)

Equation (A-4) is the same equation given by Domenico (1974).