Traveltime in media with linear velocity-depth functions

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ABSTRACT

An analytic formula for traveltime in media with linear velocity-depth functions is found. This formula can be used in automatic picking of best fitting linear velocity depth functions.

INTRODUCTION

Determining seismic velocity as a function of position is a large part of seismology. Fairly simple functions are often quite good approximations to the real earth. The most commonly used function, because of the simplicity of calculations using it, is one with constant velocity layers, i.e.: no lateral variation, step changes at certain depths. In some regions velocity varying as a linear function of depth with no lateral variations is a better approximation to the real earth (see references in Li & Stewart 1991). We develop below an analytic formula for direct traveltimes between source and receiver at any depths and lateral separations, given a linear velocity depth function. This would be useful in finding velocity depth functions from surface seismic, VSP,or crosswell data.

THEORY

We will now use ray theory to derive properties of seismic wave propagation in a medium in which velocity (v) is a linear function of depth (z) only, i.e.:

$$v = v_0 + kz \tag{1}$$

Ray theory is valid if the dominant frequencies of the seismic energy are high enough that the properties of the medium change little over one wavelength (Grant & West 1965 ch 5).

In any medium in which velocity varies only with depth, for a seismic ray:

$$p = \frac{\sin\theta}{v} = \text{constant} \tag{2}$$

where θ is the angle between the ray and vertical downwards, p is the ray parameter.

From Figure 1 we can see that for a small section of either a down or up going ray ds:

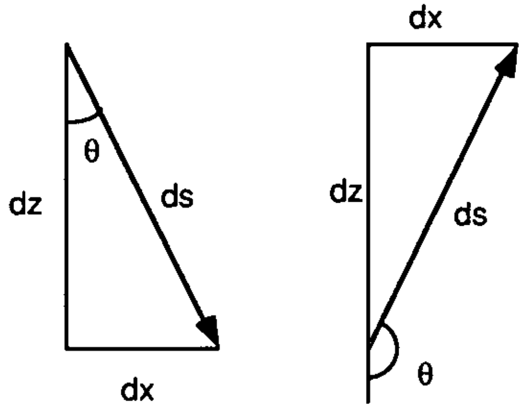

FIG. 1 Small segments of down and up going rays

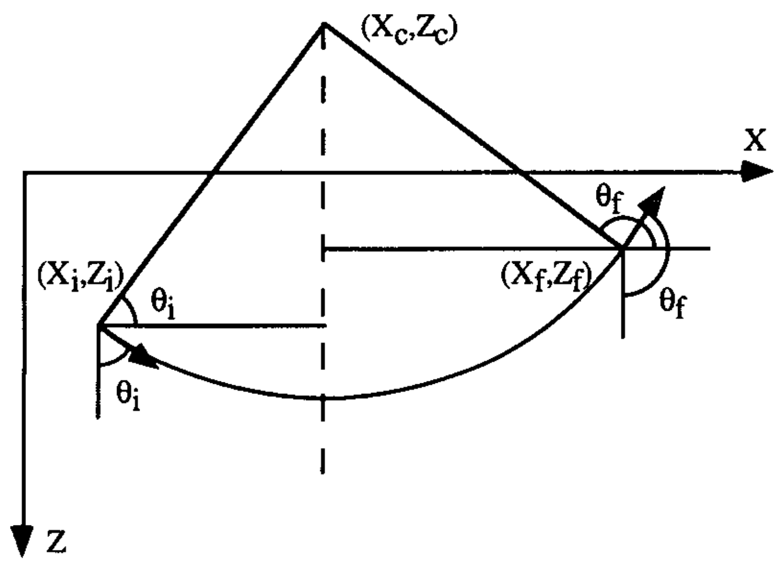

FIG. 2 Full raypath from source to receiver showing relationship between source and receiver coordinates, center and radius of raypath and initial and final angle of incidence

$$dx = \tan\theta \, dz \tag{3}$$

$$dt = \frac{ds}{v} = \frac{dz}{v\cos\theta} \tag{4}$$

from equations (1) and (2)

$$z = \frac{\sin\theta}{pk} - \frac{v_0}{k} \tag{5}$$

$$dz = \frac{\cos\theta}{pk} d\theta \tag{6}$$

we use (2) and (6) to get dx and dt in terms of θ (equations (7) and (10)), and integrate to get equations (9) and (13) relating horizontal position and traveltime to angle of incidence.

$$dx = \tan\theta \frac{\cos\theta}{pk} d\theta = \frac{\sin\theta}{pk} d\theta \tag{7}$$

$$\int_{X_i}^{X_f} dx = \frac{1}{pk} \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta \tag{8}$$

$$X_f - X_i = \frac{\cos\theta_i - \cos\theta_f}{pk} \tag{9}$$

$$dt = \frac{p}{\sin\theta} \frac{1}{\cos\theta} \frac{\cos\theta}{pk} d\theta = \frac{d\theta}{k\sin\theta} \tag{10}$$

$$\int_0^T dt = \frac{1}{k} \int_{\theta_i}^{\theta_f} \frac{d\theta}{\sin\theta} \tag{11}$$

$$T = \frac{1}{k} \left[\ln \left(\frac{\sin\theta}{1 + \cos\theta} \right) \right]_{\theta_i}^{\theta_f} \tag{12}$$

$$T = \frac{1}{k} \ln \left(\frac{\sin\theta_f}{\sin\theta_i} \frac{1+\cos\theta_i}{1+\cos\theta_f} \right) \tag{13}$$

Equations (9) and (13) have been derived differently elsewhere (e.g.: Baerg 1985 p39, Telford et al. p273)

Equations (5) and (9) are parametric equations for a circle of radius $R=1/pk$, and center (X_c, Z_c);

$$X_c = X_i + \frac{\cos\theta_i}{pk} = X_i + R\cos\theta_i \tag{14}$$

$$Z_c = -\frac{v_0}{k} \tag{15}$$

Given the source and receiver positions (X_i,Z_i), (X_f,Z_f) on this circle and the known value of Z_C, X_C and R can be solved for;

$$X_c = \frac{X_f^2 - X_i^2 + (Z_f-Z_c)^2 - (Z_i-Z_c)^2}{2(X_f-X_i)} \tag{16}$$

$$R = \sqrt{(X_i-X_c)^2 + (Z_i-Z_c)^2} \tag{17}$$

From Figure (2) we can see that;

$$\sin\theta_i = \frac{Z_i-Z_c}{R} \tag{18}$$

$$\cos\theta_i = \frac{X_c-X_i}{R} \tag{19}$$

$$\sin\theta_f = \frac{Z_f-Z_c}{R} \tag{20}$$

$$\cos\theta_f = \frac{X_c-X_f}{R} \tag{21}$$

which can be substituted into equation (13) to give;

$$T = \frac{1}{k}\ln\left\{\frac{Z_f-Z_c}{Z_i-Z_c}\;\frac{R+X_c-X_i}{R+X_c-X_f}\right\} \tag{22}$$

Equations (15) (16) (17) and (22) give the traveltime in terms of the source and receiver positions and V_O and k.

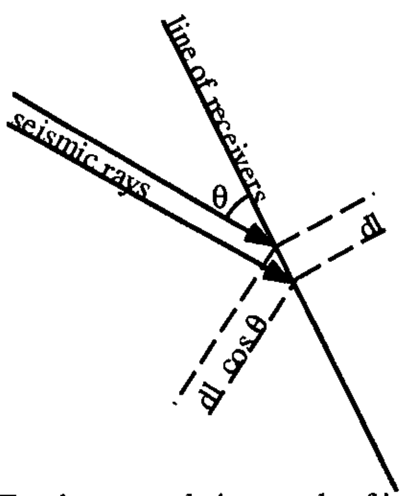

FIG. 3 For theorem relating, angle of incidence of seismic rays to line of receivers, to minimum travel time

A check on the validity of any formula for traveltime is provided by the theorem derived here:

From figure 3: if l = distance along a line of seismic receivers (which may be in a borehole)

θ = angle between, direction of propagation of seismic rays and line of receivers.

$t(l)$ = time for seismic rays to travel to a receiver at l.

v = seismic wave speed at l

then:

$$dt = \frac{dl \cos\theta}{v} \tag{23}$$

$$\frac{dt}{dl} = \frac{\cos\theta}{v} \tag{24}$$

For a local minimum (or maximum) in $t(l)$ $\frac{dt}{dl} = 0$ so

$$\frac{\cos\theta}{v} = 0 \tag{25}$$

$$\theta = 90^o \tag{26}$$

i.e.: seismic rays are perpendicular to a line of receivers at a position along the line at which traveltime is a minimum, regardless of the form of $v(x,y,z)$ or the slant or curvature of the receiver line.

The traveltime given by equations (15), (16), (17) and (22) was checked by finding dT/dZ_f and determining that $dT/dZ_f = 0$ when $X_c = X_f$ i.e.: for a line of receivers in a vertical borehole traveltime is a minimum when the center of curvature of the ray path is directly over the borehole so the rays are perpendicular to the borehole, as would be expected.

CONCLUSION

Equations (15), (16), (17), and (22) can be used in a very simple and fast computer subroutine to allow automatic picking of best fitting linear velocity-depth relationships, from crosswell data as in Li & Stewart 1991 or from VSP or surface refraction data.

FUTURE WORK

Proposed future work is to develop traveltime formulae for several layers each with their own values of V_o and k, and find geometric spreading factors for amplitude calculations.

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